Arithmetic Series

Exercise 12S Skills Practice

1 Write down the first 5 terms of sequences whose nth term, u_n, is given by

a $u_n = 3n + 1$ **b** $u_n = 6n - 5$ **c** $u_n = 11n - 28$

d $u_n = 25 - 4n$ **e** $u_n = 1.5n + 3.5$ **f** $u_n = 5 - 15n$

2 Find an expression for the nth term of sequences beginning

a 7, 9, 11, 13, ... **b** 2, 6, 10, 14, ... **c** 35, 28, 21, 14, ...

d $^-12$, $^-7$, $^-2$, 3, ... **e** 17, 4, $^-9$, $^-22$, ... **f** 0.4, 1.2, 2.0, 2.8, ...

3 Write down the 10th and 25th terms of each of the sequences in question **2**.

4 The sum of the first n terms of a series, S_n, is given by $S_n = 3n^2$.

a Evaluate S_2 and S_3.

b Hence, find the 3rd term of the series

c Find an expression for S_{n-1}, the sum of the first $(n-1)$ terms of the sequence.

d Hence, find an expression for the nth term of the sequence.

5 The sum of the first n terms of a series, S_n, is given by $S_n = 25n - 6n^2$.
Find an expression for the nth term of the series.

6 Use a suitable formula to find the sum of each series.

a $5 + 11 + 17 + 23 + 29 + 35 + 41 + 47 + 53 + 59$

b $22 + 19 + 16 + 13 + 10 + 7 + 4 + 1 + (^-2) + (^-5) + (^-8)$

7 The first term, a, the common difference, d, and the number of terms, n, is given for each of three series. Find the sum of each series.

a $a = 5$; $d = 2$; $n = 10$.

b $a = 17$; $d = 6$; $n = 45$.

c $a = 50$; $d = ^-4$; $n = 14$.

8 The first term, a, the common difference, d, and the last term, l, is given for each of three series. Find the number of terms in each series.

a $a = 3$; $d = 7$; $l = 241$.

b $a = ^-8$; $d = 9$; $l = 46$.

c $a = 387$; $d = ^-11$; $l = 35$.

9 Find the sum of each series in question **8**.

10 Evaluate

a $\sum_{r=1}^{6}(2r+1)$ **b** $\sum_{r=1}^{15}(8r-3)$ **c** $\sum_{r=1}^{34}(3r+9)$

d $\sum_{r=1}^{11}(35-r)$ **e** $\sum_{r=3}^{18}(7r+20)$ **f** $\sum_{r=8}^{40}(14-2r)$

11 An arithmetic series begins $2\frac{1}{2} + 4\frac{1}{4} + 6 + 7\frac{3}{4} + 9\frac{1}{2} + \ldots$.

a Find an expression for the nth term of the series.

b Find the sum of the first 20 terms of the series.

12 **a** Find the sum of the series $1 + 2 + 3 + 4 + \ldots + 100$.

b Find the sum of the series $5 + 10 + 15 + 20 + \ldots + 100$.

c Find the sum of the integers, between 1 and 100 inclusive, not divisible by 5.

13 Find the sum of the integers, between 1 and 300 inclusive, not divisible by 3.

14 The first and 5th terms of an arithmetic series are 4 and 16 respectively.

a Find the common difference of the series.

b Find the sum of the first 10 terms of the series.

15 The second and third terms of an arithmetic series are 26.5 and 31 respectively.

a Find the first term and common difference of the series.

b Given that the last term is 445, find the number of terms in the series.

16 The 4th and 7th terms of an arithmetic series are 7 and $^{-}5$ respectively.

a Find the first term and common difference of the series.

b Find an expression in terms of n for the sum of the first n terms of the series.

17 The third term of an arithmetic series is 18. The sum of the first six terms of the series is 132.

a Find the first term and common difference of the series.

b Find the sum of the first 26 terms of the series.

18 The 4th term of an arithmetic series is 23. The sum of the first eight terms of the series is 214.

a Find the first term and common difference of the series.

b Find the least value of n for which the sum of the first n terms of the series is greater than 1000.

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1 a 4, 7, 10, 13, 16 b 1, 7, 13, 19, 25
c -17, -6, 5, 16, 27 d 21, 17, 13, 9, 5
e 5, 6.5, 8, 9.5, 11
f -10, -25, -40, -55, -70

2 a $2n+5$ b $4n-2$ c $42-7n$
d $5n-17$ e $30-13n$ f $0.8n-0.4$

3 a 25, 55 b 38, 98 c -28, -133
d 33, 108 e -100, -295 f 7.6, 19.6

4 a 12, 27 b 15 c $3(n-1)^2$ d $6n-3$

5 $31-12n$

6 a 320 b 77

7 a 140 b 6705 c 336

8 a 35 b 7 c 33

9 a 4270 b 133 c 6963

10 a 48 b 915 c 2091
d 319 e 1496 f -1122

11 a $^7/_4n+^3/_4$ b 382.5

12 a 5050 b 1050 c 4000

13 30000

14 a 3 b 175

15 a 22, 4.5 b 95

16 a 19, -4 b $21n-2n^2$

17 a 2, 8 b 2652

18 a $^1/_2$, $^{15}/_2$ b 17