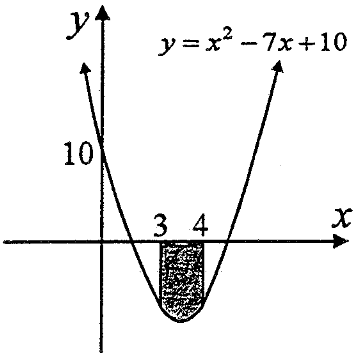
EXERCISE 13A

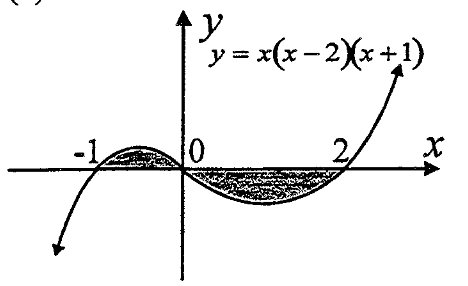
AREA BETWEEN A CURVE AND THE *X*-AXIS

1. Find the area of the shaded regions:

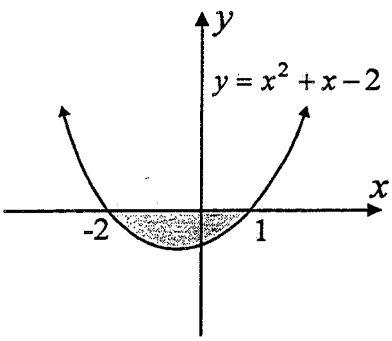
(a)



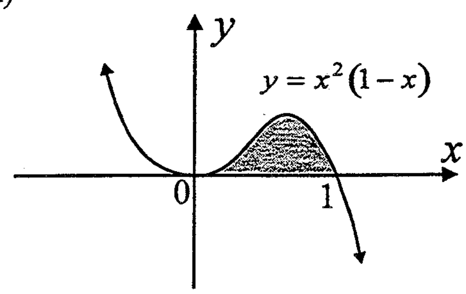
(b)



(c)



(d)



2. Find the area bounded by the graph $y = x^3 + 1$, the x - axis and the ordinates at $x = -2$ and $x = 1$.

3. Find the area bounded by the following graphs and the x - axis.

(a) $y = x^2 - 8x$,

(b) $y = x^3 - x$,

(c) $y = (x+1)^2(x-2)$.

ANSWERS

1. (a) $2\frac{1}{6}$ units^2 (c) $4\frac{1}{2}$ units^2

(b) $3\frac{1}{12}$ units^2 (d) $\frac{1}{12}$ units^2

2. 4.75 units^2

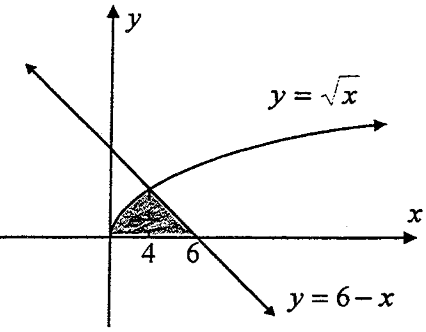
3. (a) $85\frac{1}{3}$ units^2 (b) $\frac{1}{2}$ units^2 (c) 6.75 units^2

EXERCISE 13B

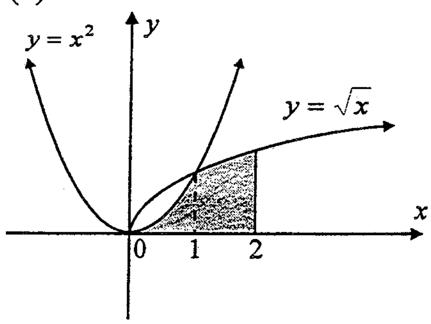
AREA BETWEEN TWO CURVES AND ADDITION OF AREAS

1. Find the area of the shaded region.

(a)



(b)



2. Find the area bounded by the curves $y=x^2$, $y=(x-2)^2$ and the x - axis.

3. A straight line through the origin cuts the parabola $y=4x-x^2$ at the point where $x=3$.

   (a) Find the equation of this line.
   (b) Calculate the area of the region bounded by:
       (i) the parabola and the straight line,
       (ii) the parabola and the x - axis,
       (iii) the parabola, the straight line and the x - axis.

4. Find the equation of the tangent to the parabola $y=x^2+1$ at the point where $x=2$ and find the area enclosed by:

   (a) the parabola, the tangent and the y - axis,
   (b) the parabola, the tangent and the coordinate axes.

5. Find the area of the region bounded by the parabolas $y=x^2$ and $y=4-x^2$ and the x - axis.

ANSWERS

1. (a) $7\frac{1}{3}$ units^2 (b) $\frac{4\sqrt{2}-1}{3}$ units^2
2. $\frac{2}{3}$ units^2
3. (a) $y=x$

   (b) (i) $4\frac{1}{2}$ units^2 (ii) $10\frac{2}{3}$ units^2 (iii) $6\frac{1}{6}$ units^2
4. $y=4x-3$ (a) $2\frac{2}{3}$ units^2 (b) $1\frac{13}{24}$ units^2
5. $\frac{16}{3}\left(2-\sqrt{2}\right)$ units^2

EXERCISE 13A.

1 (a) $A = \left| \int_3^4 x^2 - 7x + 10 \, dx \right|$

$= \left| \left[\frac{1}{3}x^3 - \frac{7}{2}x^2 + 10x \right]_3^4 \right|$ ✓

$= \left| \left(9 - \frac{56}{2} + 40 \right) - \left(\frac{64}{3} - 56 + 30 \right) \right|$ ✓

$= 2\frac{1}{6}$ units.^{2}

(b) $A = \int_{-1}^{0} x(x^2 - x - 2) \, dx - \int_0^2 x^3 - x^2 - 2x \, dx$

$= \left[\frac{1}{4}x^4 - \frac{1}{3}x^3 - x^2 \right]_{-1}^{0} - \left[\frac{1}{4}x^4 - \frac{1}{3}x^3 - x^2 \right]_0^2$ ✓

$= \left(+ \frac{5}{12} \right) - \left(-2\frac{2}{3} \right)$ ✓

$= 3\frac{1}{12}$ units^2

(c) $A = \left| \int_{-2}^{1} x^2 + x - 2 \, dx \right|$

$= \left| \left[\frac{1}{3}x^3 + \frac{1}{2}x^2 - 2x \right]_{-2}^{1} \right|$ ✓

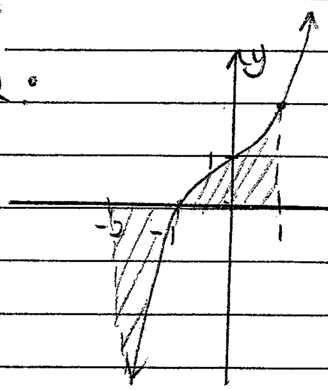
$= \left| \left(\frac{1}{3} + \frac{1}{2} - 2 \right) - \left(\frac{-8}{3} + 2 + 4 \right) \right|$

$= 4\frac{1}{2}$ units^2 ✓

(d) $A = \int_0^1 x^2 - x^3 \, dx = \left[\frac{1}{3}x^3 - \frac{1}{4}x^4 \right]_0^1$

$= \frac{1}{12}$ units^2

Qu2:



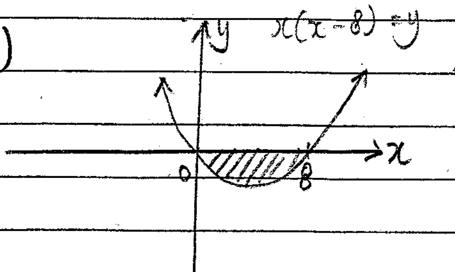
$A = \int_{-1}^{1} x^3 + 1 \, dx + \left| \int_{-2}^{-1} x^3 + 1 \right| dx$

$= \left[\frac{1}{4}x^4 + x \right]_{-1}^{1} + \left| \left[\frac{1}{4}x^4 + x \right]_{-2}^{-1} \right|$ ✓

$= \left(\frac{1}{4} + 1 - \left(\frac{1}{4} - 1 \right) \right) + \left| \left(\frac{1}{4} - 1 \right) - \left(4 - 2 \right) \right|$

$= 2 + 2.75 = 4.75$ units^2 ✓

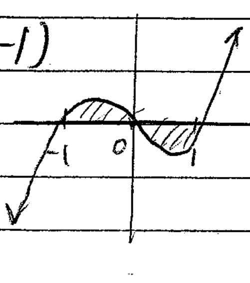
Qu 3 (a)



$A = \int_0^8 x^2 - 8x \, dx = \left| \left[\frac{1}{3}x^3 - 4x^2 \right]_0^8 \right|$ ✓

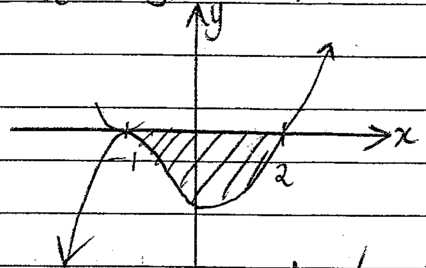
$= 85\frac{1}{3}$ units^2 ✓

(b) $y = x(x+1)(x-1)$



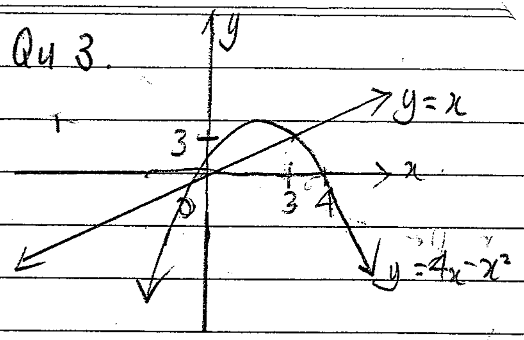
~~A = 0 units~~^{2} This is not symmetrical. Try again.

(c) $y = (x^2+2x+1)(x-2)$
$= x^3+2x^2+x-2x^2-4x-2$
$= x^3-3x-2$ ✓



$A = \left|\int_{-1}^{2} x^3-3x-2\,dx\right|$ ✓
$= \left|\left[\frac{1}{4}x^4-\frac{3}{2}x^2-2x\right]_{-1}^{2}\right|$
$= |(4-6-4)-(\frac{1}{4}-\frac{3}{2}+2)|$
$= 6.75$ units^2

Qu 3.



(a) $x=3$, $y = 12-9 = 3$ ✓
$(3,3)$
$\therefore m = \frac{3-0}{3-0} = 1$ ✓
$\therefore y = x$

(b) (i) $A = \int_0^3 3x-x^2\,dx$ ✓
$= \left[\frac{3}{2}x^2-\frac{1}{3}x^3\right]_0^3$
$= 4\frac{1}{2}$ units^2 ✓

(ii) $A = \int_0^4 4x-x^2\,dx$
$= \left[2x^2-\frac{1}{3}x^3\right]_0^4$ ✓
$= 10\frac{2}{3}$ units^2 ✓

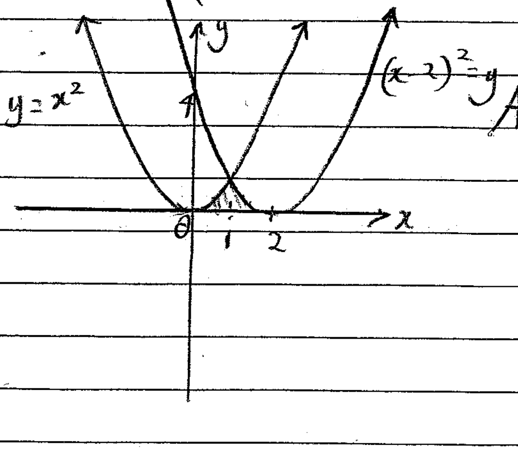
(iii) $A = \int_0^3 x\,dx + \int_3^4 4x-x^2\,dx$
$= \left[\frac{1}{2}x^2\right]_0^3 + \left[2x^2-\frac{1}{3}x^3\right]_3^4$
$= \frac{9}{2} + (10\frac{2}{3}-9)$
$= 6\frac{1}{6}$ units^2 ✓

Exercise 13 B

1. (a) $A = \int_0^4 x^{\frac{1}{2}}\,dx + \int_4^6 6-x\,dx$ ✓
$= \left[\frac{2}{3}x^{\frac{3}{2}}\right]_0^4 + \left[6x-\frac{1}{2}x^2\right]_4^6$
$= 5\frac{1}{3}+(18-16) = 7\frac{1}{3}$ units^2 ✓

(b) $A = \int_1^2 x^{\frac{1}{2}}dx + \int_0^1 x^{\frac{1}{2}}-x^2\,dx$ ✓
$= \left[\frac{2}{3}x^{\frac{3}{2}}\right]_1^2 + \left[\frac{2}{3}x^{\frac{3}{2}}-\frac{1}{3}x^3\right]_0^1$ ✓
$= \left(\frac{2}{3}\cdot 2\sqrt{2}\right)-\left(\frac{2}{3}\right)+\frac{1}{3} = \frac{4\sqrt{2}-1}{3}$ units^2

2.



$A = \int_0^1 x^2\,dx + \int_1^2 (x-2)^2\,dx$
$= \left[\frac{1}{3}x^3\right]_0^1 + \left[\frac{1}{3}x^3-2x^2+4x\right]_1^2$ ✓
$= \frac{1}{3}+\left[(\frac{8}{3}-8+8)-(\frac{1}{3}-2+4)\right]$
$= \frac{2}{3}$ units^2 ✓

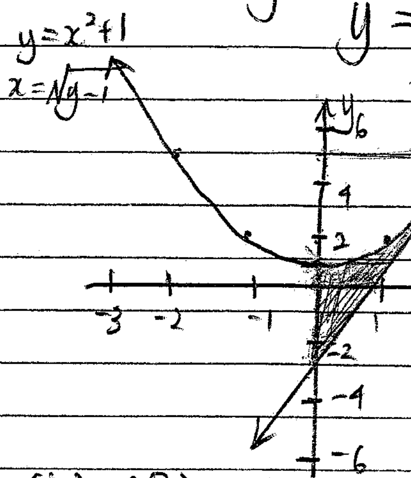
Qu 4. $y = x^2 + 1$

$y' = 2x$

at $x = 2$, $y' = 4$, $y = 5$.

$\therefore y - 5 = 4(x-2)$

$y = 4x - 3$



(b) $\int_0^2 x^2+1\,dx = \left[\frac{1}{3}x^3 + x\right]_0^2$

$= \left(\frac{8}{3}+2\right)$

$= 4\frac{2}{3}$

$A = 4\frac{2}{3} - 2\frac{1}{2} =$

(a) $A = \int_{-2}^{5} \frac{y+3}{4} - (y-1)^{\frac{1}{2}}\,dy$

$= \left[\frac{1}{8}y^2 + \frac{3}{4}y - \frac{2}{3}(y-1)^{\frac{3}{2}}\right]$

$= \left(-\frac{23}{24}\right) - ($

(b) $\int_1^5 \frac{1}{4}(y+3) - (y-1)^{\frac{1}{2}}\,dxy$

$= \left[\frac{1}{8}y^2 + \frac{3}{4}y - \frac{2}{3}(y-1)^{\frac{3}{2}}\right]_1^5$

$= \left(1\frac{13}{24}\right) - \frac{7}{8}$

$= \frac{2}{3}$ units^2

(b) $A = \int_0^2 x^2+1 - 4x+3\,dx$

$= \int_0^2 x^2 - 4x + 4\,dx$

$= \left[\frac{1}{3}x^3 - 2x^2 + 4x\right]_0^2$

$= 2\frac{2}{3}$ units^2.

$A = \int_1^5 (y-1)^{\frac{1}{2}}\,dy$

$= \left[\frac{2}{3}(y-1)^{\frac{3}{2}}\right]_1^5$

$= 5\frac{1}{3} - 0 = 5\frac{1}{3}$ units^2.

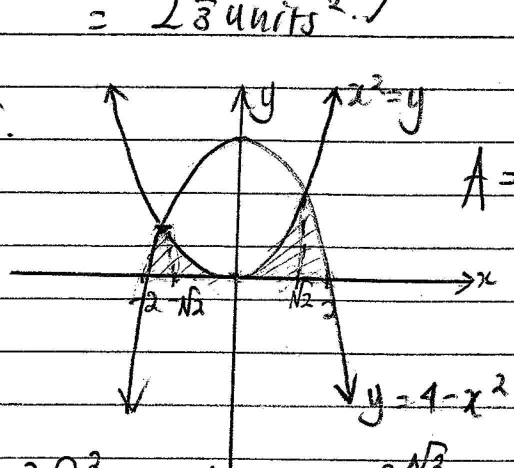
$\therefore$ Total $A = 8 - 5\frac{1}{3}$

$= 2\frac{2}{3}$ units^2.

(b) $A = 2\frac{2}{3} - \left(\frac{1}{2} \times 3 \times \frac{3}{4}\right)$

$= 1\frac{13}{24}$ units^2.

Qu 5.



$A = \int_{-2}^{2} 4 - x^2 - x^2\,dx$

$= \left[4x - \frac{2}{3}x^3\right]_{-2}^{2}$

$=$

$A = 2\int_{\sqrt{2}}^{2} 4 - x^2\,dx + 2\int_0^{\sqrt{2}} x^2 = 2\left[4x - \frac{1}{3}x^3\right]_{\sqrt{2}}^{2} + 2\left[\frac{1}{3}x^3\right]_0^{\sqrt{2}}$

$= 2\left(5\frac{1}{3} - \left(4\sqrt{2} - \frac{2\sqrt{2}}{3}\right)\right) + 2\left(\frac{2\sqrt{2}}{3}\right)$

$= 2\left(\frac{16}{3} - \left(\frac{12\sqrt{2} - 2\sqrt{2}}{3}\right)\right) + \frac{4\sqrt{2}}{3}$

$= 2\left(\frac{16 - 10\sqrt{2}}{3}\right) + \frac{4\sqrt{2}}{3}$

$= \left(10\frac{2}{3} - \frac{16\sqrt{2}}{3}\right)$ units^2.