## EXERCISE 13C

## VOLUMES OF SOLIDS OF REVOLUTION

1. What is the volume of the solid formed when the area between $y = x^2 + 1$, the $x$-axis and the ordinates $x = -1$ and $x = 1$ is rotated about the $x$-axis?

2. A shell is formed by rotating the curve $y = \sqrt{x}$ between $x = 0$ and $x = 20$ about the $x$-axis. Find the volume of the shell.

3. Sketch the curve $y = \dfrac{1}{x+1}$. The area under this curve between $x = 0$ and $x = 3$ is rotated about the $x$-axis. Find the volume formed.

4. Find the area bounded by the curve $y = \sqrt{9 - x}$ and the $x$ and $y$ axes. Find the volume when this area is rotated about the $x$-axis.

5. The area of the region bounded by the curves $y = x^2$ and $y = x + 2$ between $x = 0$ and $x = 1$ is rotated about the $x$-axis. Find the volume of the solid generated.

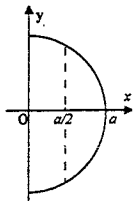
6. Sketch the curves $y = \sqrt{1 - x^2}$ and $y = 2\sqrt{1 - x^2}$, and find their points of intersection. Find the volume when the region enclosed by these curves is rotated about the $x$-axis.

7. By first finding the $x$ and $y$ intercepts, sketch the ellipse $4x^2 + y^2 = 16$. Find the volume when the area enclosed by this ellipse is rotated about the $x$-axis.

8. The area bounded by the parabola $y = 2x - x^2$, the $y$-axis and the line $y = 1$ is rotated about the $x$-axis. Find the volume generated.

9. Sketch the curves $y = x^2$ and $x = y^2$. Determine the volume of the solid formed when the area enclosed by these curves is rotated about the $x$-axis.

*10. A hemispherical bowl of radius $a$ units is filled with water to depth of $\dfrac{a}{2}$ units. Find the volume of water. (Refer diagram below).



## ANSWERS

1. $\frac{56\pi}{15}$ units$^3$

2. $200\pi$ units$^3$

3. $\frac{3\pi}{4}$ units$^3$

4. 18 units$^2$, $\frac{81\pi}{2}$ units$^3$

5. 19.268 units$^3$

6. (1,0) and (-1,0), $4\pi$ units$^3$

7. $\frac{128\pi}{3}$ units$^3$

8. $\frac{7\pi}{15}$ units$^3$

9. $\frac{3\pi}{10}$ units$^3$

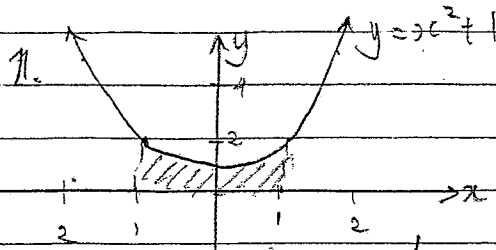
10. $\frac{5\pi a^3}{24}$ units$^3$

TABLE
Area Unde

| z |
|---|
| 0.0 |
| 0.1 |
| 0.2 |
| 0.3 |
| 0.4 |
| 0.5 |
| 0.6 |
| 0.7 |
| 0.8 |
| 0.9 |
| 1.0 |
| 1.1 |
| 1.2 |
| 1.3 |
| 1.4 |
| 1.5 |
| 1.6 |
| 1.7 |
| 1.8 |
| 1.9 |
| 2.0 |
| 2.1 |
| 2.2 |
| 2.3 |
| 2.4 |
| 2.5 |
| 2.6 |
| 2.7 |
| 2.8 |
| 2.9 |
| 3.0 |
| 3.1 |
| 3.2 |
| 3.3 |
| 3.4 |
| 3.5 |
| 3.6 |
| 3.7 |
| 3.8 |
| 3.9 |

# EXERCISE 13C

Angelina

1. 

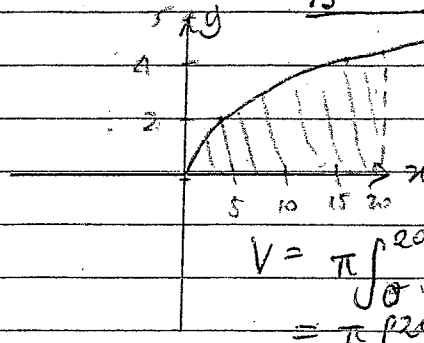

$y = x^2 + 1 \quad y^2 = (x^2+1)^2 = x^4 + 2x^2 + 1$

$V = \pi \int_{-1}^{1} x^4 + 2x^2 + 1 \, dx$

$= \pi \left[ \frac{1}{5}x^5 + \frac{2}{3}x^3 + x \right]_{-1}^{1}$ ✓

$= \pi \left( 1\frac{13}{15} + 1\frac{13}{15} \right)$

$= \frac{56\pi}{15}$ units$^3$ ✗

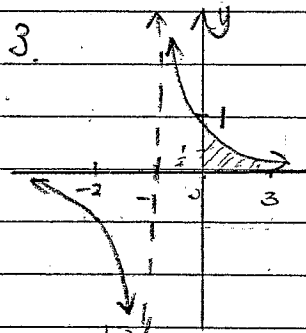
2 


$V = \pi \int_0^{20} y^2 \, dx.$

$= \pi \int_5^{20} x \, dx$ ✓

$= \pi \left[ \frac{1}{2}x^2 \right]_5^{20}$ ✓

$= 200\pi$ units$^3$

3. 


$y = \frac{1}{x+1}$

$y^2 = \frac{1}{x^2+2x+1}$

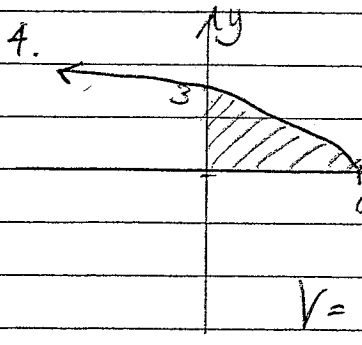
$V = \pi \int_0^3 y^2 \, dx.$

$= \pi \int_0^3 (x+1)^{-2} \, dx$ ✓

$= \pi \left[ -(x+1)^{-1} \right]_0^3$ ✓

$= \pi \left( -\frac{1}{4} + 1 \right)$

$= \frac{3\pi}{4}$ units$^3$ ✗

4. 


$A = \int_0^9 (9-x)^{\frac{1}{2}} \, dx$

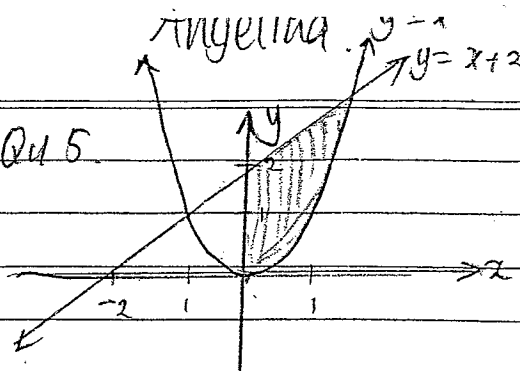
$= \left[ -\frac{2}{3}(9-x)^{\frac{3}{2}} \right]_0^9$ ✓

$= -(-18)$ ✓

$= 18$ units$^2$.

$V = \pi \int_0^9 9 - x \, dx = \pi \left[ 9x - \frac{1}{2}x^2 \right]_0^9$

$= 40\frac{1}{2}\pi$ units$^3$ ✓

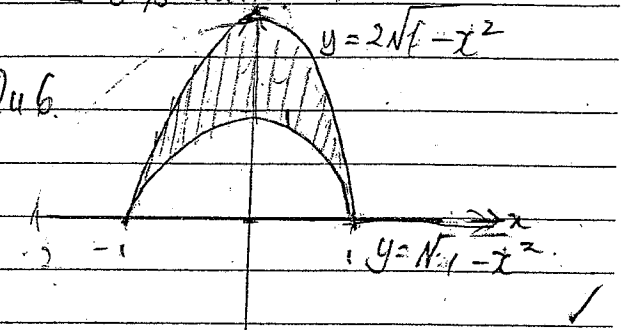
Qu 5 


$V = \pi \int_0^1 (x+2)^2 - x^4 \, dx$

$= \pi \int_0^1 x^2 + 4x + 4 - x^4 \, dx$

$= \pi \left[ \frac{1}{3}x^3 + 2x^2 + 4x - \frac{1}{5}x^5 \right]_0^1$

$= 6\frac{7}{15}\pi$ units$^3$ $= 19.268$ u$^3$

Qu 6. 


points of intersection at $(-1, 0), (1, 0)$ ✓

$V = \pi \int_{-1}^{1} 4x^2 - 4x^2 + x^2 + 1 \, dx$

$= \pi \int_{-1}^{1} -3x^2 + 3 \, dx$ ✓

$= \pi \left[ -x^3 + 3x \right]_{-1}^{1}$

$= \pi \left( 2 - (-1+3) \right)$

$= 4\pi$ units$^3$ ✓

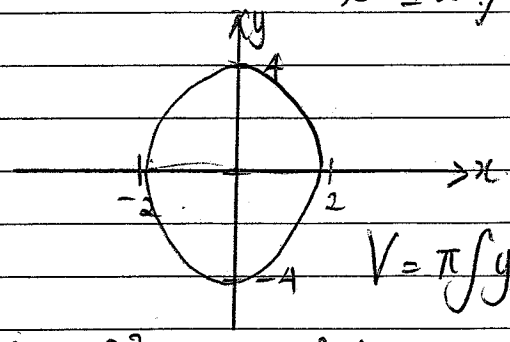
Qu 7. $4x^2+y^2=16$

when $x=0$, $y^2=16$

$\therefore y=\pm 4$

$y=0$, $4x^2=16$

$x=\pm 2$ ✓




$V=\pi\int y^2\,dx$.

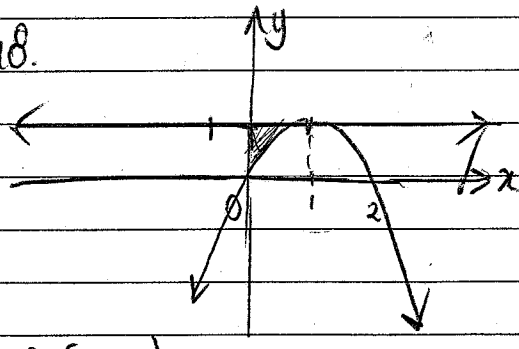
$\therefore V=\pi\int_{-2}^{2} 16-4x^2\,dx$ ✓

$=\pi\left[16x-\frac{4}{3}x^3\right]_{-2}^{2}$

$=\pi\left(21\frac{1}{3}+21\frac{1}{3}\right)$

$=42\frac{2}{3}\pi$ units$^3$ ✓

Qu 8.



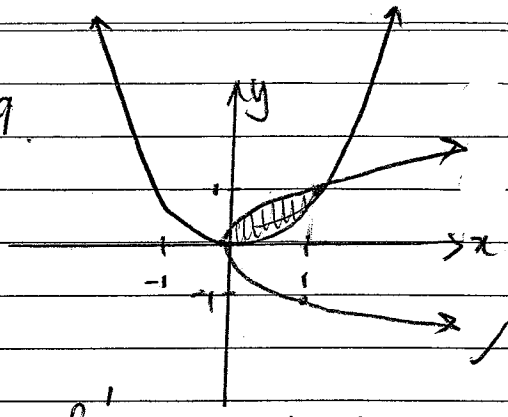

$y=x(2-x)$

$V=\pi\int_0^1 1-(4x^2-4x^3+x^4)\,dx$ ✓

$=\pi\left[x-\frac{4}{3}x^3+x^4-\frac{1}{5}x^5\right]_0^1$

$=\frac{8}{15}$ units$^3$ ✓

Qu 9.




$V=\pi\int_0^1 x-x^4\,dx$ ✓

$=\pi\left[\frac{1}{2}x-\frac{1}{5}x^5\right]_0^1$

$=\frac{3}{10}\pi$ units$^3$ ✓

Qu 10 $x=\sqrt{a^2-y^2}$

$V=\pi\int_0^a a^2-y^2\,dy=$

$\pi\int_0^{\frac{a}{2}} a^2-y^2\,dy$ ✓

$=\pi\left[a^2y-\frac{1}{3}y^3\right]_0^a$

$-\pi\left[a^2y-\frac{1}{3}y^3\right]_0^{\frac{a}{2}}$ ✓

$=\pi\left[\left(a^3-\frac{1}{3}a^3\right)-\left(\frac{a^3}{2}-\left(\frac{a^3}{8}\right)\left(\frac{1}{3}\right)\right)\right]$

$=\pi\left(\frac{2}{3}a^3-\frac{11}{24}a^3\right)$

$=\frac{5\pi a^3}{24}$ units$^3$ ✓