Exercise 13E Exam Practice

1 The second and fourth terms of a geometric series are 108 and 3 respectively.

a Find the two possible values of the common ratio of the series. **(3 marks)**

b Find the sum to infinity corresponding to each of your values for the common ratio. Give your answers correct to 1 decimal place. **(5 marks)**

2 $2000 is paid into an account at the start of each year.

The account pays 5% per annum compound interest.

a Find the amount in the account at the end of the second year. **(2 marks)**

b Show that the amount in the account at the end of n years is given by $k[(1.05)^n - 1]$ and find the value of the constant k. **(5 marks)**

3 The first term and common ratio of a geometric series are $\frac{1}{2-\sqrt{3}}$ and $(\sqrt{3} - 1)$ respectively.

Find in the form $a + b\sqrt{3}$

a the second term of the series, **(4 marks)**

b the sum to infinity of the series. **(4 marks)**

4 The first three terms of a geometric series are $(3p + 1)$, $(2p - 1)$ and $(p + 1)$ respectively, where p is a non-zero constant.

a Find the value of p. **(5 marks)**

b Find the sum to infinity of the series. **(4 marks)**

5 A ball is projected vertically upwards from ground level reaching a greatest height of 12 m above the ground. It falls to the ground and bounces up and down vertically.

Given that after each bounce the greatest height then reached by the ball is reduced by 25%,

a show that after the second bounce it reaches a height of 6.75 m, **(2 marks)**

b find, correct to 3 significant figures, the total distance that the ball has travelled when it hits the ground for the 10th time. **(5 marks)**

6 The third and sixth terms of a geometric series are 8 and k^3 respectively, where k is a constant.

a Find the common ratio of the series in terms of k. **(3 marks)**

b Find the first term of the series in terms of k. **(2 marks)**

c Given that $k = -4$, find the sum of the first nine terms of the series. **(4 marks)**

Exercise 13E Exam Practice

1 **a** $\pm \frac{1}{6}$ **b** 777.6, -555.4

2 ~~**a** £4305 **b** 42000~~ **a.** $2205 **b.** 40000

3 **a** $1 + \sqrt{3}$ **b** $7 + 4\sqrt{3}$

4 **a** 8 **b** $62\frac{1}{2}$

5 **b** 90.6 m

6 **a** $\frac{k}{2}$ **b** $\frac{32}{k^2}$ **c** 342

Exercise 13E Exam practice

1) $ar = 108$ — ①

(a) $ar^3 = 3$ — ②

② ÷ ① $r^2 = \frac{3}{108}$

$r^2 = \frac{1}{36} = \pm \frac{1}{6}$ ✓

(b) $S_\infty = \frac{a}{1-r}$

$a = 648$ when $r = \frac{1}{6}$

$a = -648$ when $r = -\frac{1}{6}$

$\therefore S_\infty = \frac{648}{5/6} = 777\frac{3}{5}$ ✓

$S_\infty = -555\frac{3}{7}$ ✓

2) 1st Yr: 1.05×2000

2nd Yr: $(1.05)^2 2000$

$T_n = 2000 \times (1.05)^{n-1}$

(a) $T_2 = 2000 \times 1.05^{-2}$

$= \$2205$ ✓

(b) $S_n = \frac{2000(1.05^n - 1)}{0.05}$

$k = 40000$ ✓

3. $a = \frac{1}{2-\sqrt{3}}$

$r = \sqrt{3} - 1$

$\therefore T_n = \frac{1}{2-\sqrt{3}} \times (\sqrt{3}-1)^{n-1}$

(a) $T_2 = \frac{1}{2-\sqrt{3}} \times \sqrt{3} - 1$

$= \frac{\sqrt{3}-1}{2-\sqrt{3}} \times \frac{2+\sqrt{3}}{2+\sqrt{3}}$ ✓

$= 2\sqrt{3} + 3 - 2 - \sqrt{3}$

$= 1 + \sqrt{3}$ ✓

$a = 1 \quad b = 1$ ✓

(b) $S_\infty = \frac{a}{1-r}$

$= \frac{1}{2-\sqrt{3}} \div (1 - \sqrt{3} + 1)$ ✓

$\frac{2+\sqrt{3}}{2-\sqrt{3}}$ $= \frac{1}{2-\sqrt{3}} \times \frac{1}{2-\sqrt{3}}$ ✓

$4 + 4\sqrt{3} + 3 = \frac{1}{4m}$

$= 2 + \sqrt{3} \times \frac{1}{2-\sqrt{3}}$

$= 4 + 4\sqrt{3} + 3 = 7 + 4\sqrt{3}$ ✓

4) $\frac{2p-1}{3p+1} = \frac{p+1}{2p-1}$

$4p^2 - 4p + 1 = 3p^2 + 4p + 1$

$(p^2 - 8p = 0$ ✓

$(p-8)p = 0$

Since p is a non-zero const

p must be 8 ✓

(b) $T_1 = 25$

$T_2 = 15$

$T_3 = 9$

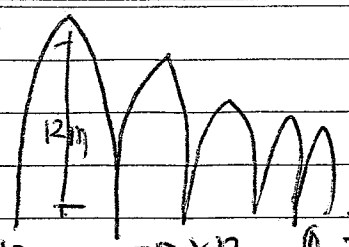
$a = 25$ ✓

$r = 3/5$

$S_\infty = \frac{25}{1 - \frac{3}{5}}$ ✓

$= 62\frac{1}{2}$

5



$12, 0.75 \times 12, (0.75)^2 \times 12, \ldots$

(a) $T_n = 12 \times (0.75)^{n-1}$

$T_2 = 12 \times (0.75)^2 = 6.75\text{m}$ ✓

(b) $S_n = n\left(\frac{12(1-0.75^n)}{0.25}\right)$

$S_{10} = 1002(45.29)$ ✓

$\approx$ ~~952m~~

$\approx 90.58\text{m}$ ✓

$S_\infty = \frac{12}{0.25} = 48$

$5 \quad T_n = 48$

$4 = r^{n-1}$

$2^2 = n-1$

$n = 3$

a u b $ar^2 = 8$

$ar^5 = k^3$

(a) $r^3 = \frac{k^3}{8}$

$r = \frac{k}{2}$ ✓

(b) $a \times \frac{k^2}{4} = 8$

$ak^2 = 32$

$a = \frac{32}{k^2}$ ✓

(c) $r = -2$

$a = 2$

$S_9 = \frac{2((-2)^9 - 1)}{-2-1}$ ✓

$= 342$ ✓