Geometric Series

Exercise 13S Skills Practice

1 Write down the first 4 terms of sequences whose nth term, u_n, is given by

a $u_n = 3^n$ **b** $u_n = 7^n$ **c** $u_n = (^-4)^n$

d $u_n = 3 \times 2^n$ **e** $u_n = 6^{n-1}$ **f** $u_n = (\frac{2}{3})^n$

2 Find an expression for the nth term of sequences beginning

a 5, 25, 125, 625, ... **b** $\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{8}$, $\frac{1}{16}$, ... **c** 12, 36, 108, 324, ...

d $^-2$, 4, $^-8$, 16, ... **e** 12, 4, $\frac{4}{3}$, $\frac{4}{9}$, ... **f** $\frac{1}{2}$, 1, 2, 4, ...

3 Use the formula to find the sum of each series.

a $5 + 10 + 20 + 40 + 80 + 160 + 320 + 640$

b $(^-3) + 9 + (^-27) + 81 + (^-243) + 729 + (^-2187)$

c $96 + 48 + 24 + 12 + 6 + 3 + \frac{3}{2} + \frac{3}{4} + \frac{3}{8} + \frac{3}{16}$

4 The sum of the first n terms of a geometric series, S_n, is given by $S_n = (2^{n+1} - 2)$.

a Evaluate S_1, S_2 and S_3.

b Find the first three terms of the series.

c Find the 6th term of the series.

5 The sum of the first n terms of a geometric series, S_n, is given by $S_n = 4(3^n - 1)$.

Find the 5th term of the series.

6 The first term, a, the common ratio, r, and the number of terms, n, are given for each of three series.

Find the sum of each series giving non-exact answers correct to 2 dp.

a $a = 5$; $r = 2$; $n = 10$.

b $a = 180$; $r = \frac{1}{3}$; $n = 8$.

c $a = 0.16$; $r = ^-2.5$; $n = 12$.

7 The first term, a, and the common ratio, r, are given for each of three series.
Find the sum to infinity of each series giving non-exact answers correct to 2 dp.

a $a = 10$; $r = \frac{1}{2}$.

b $a = 360$; $r = 0.85$.

c $a = 92$; $r = ^-\frac{3}{4}$.

8 Evaluate correct to 4 sf

a $\sum_{r=1}^{16} 2^r$ **b** $\sum_{r=1}^{10} (4 \times 3^r)$ **c** $\sum_{r=1}^{8} (\frac{1}{2})^r$

d $\sum_{r=1}^{8} 5^{n-2}$ **e** $\sum_{r=3}^{10} 3^r$ **f** $\sum_{r=6}^{12} \left[80 \times (\frac{4}{5})^r\right]$

9 A geometric series begins $2 + 3 + 4.5 + 6.75 + 10.125 + \ldots$.

a Find an expression for the nth term of the series.

b Explain why you cannot calculate the sum to infinity of this series.

10 The first term of a geometric series is 15 and its sum to infinity is 40.

a Find the common ratio of the series.

b Find the sum of the first six terms of the series correct to 2 dp.

11 The first term of a geometric series is 252 and its sum to infinity is 216.

a Find the common ratio of the series.

b Find as an exact fraction the 5th term of the series.

12 The first and 4th terms of a geometric series are 80 and 10 respectively.

a Find the common ratio of the series.

b Find the sum to infinity of the series.

13 The second and 5th terms of a geometric series are $^{-}15$ and 405 respectively.

a Find the first term and common ratio of the series.

b Find the sum of the first nine terms of the series.

14 There are estimated to be 20 000 fish in a lake.
It is assumed that this number will increase by 10% each year.

a How many fish will there be in the lake after one year?

b How many fish will there be in the lake after three years?

c By what percentage will the number of fish in the lake increase in four years?

15 $5000 is invested in an account giving 6% per annum compound interest.

a How much will be in the account after three years?

b How much interest will have been paid into the account after eight years?

16 The second term of a geometric series is 36.
The sum of the first two terms of the series is $^{-}18$.

a Find the first term and common ratio of the series.

b Find the sum to infinity of the series.

Exercise 13S Skills Practice

1 **a** 3, 9, 27, 81 **b** 7, 49, 343, 2401
c -4, 16, -64, 256 **d** 6, 12, 24, 48
e 1, 6, 36, 216 **f** $^{2}/_{3}, {}^{4}/_{9}, {}^{8}/_{27}, {}^{16}/_{81}$

2 **a** 5^n **b** $(^1/_2)^n$ **c** 4×3^n
d $(-2)^n$ **e** $36\times(^1/_3)^n$ **f** 2^{n-2}

3 **a** 1275 **b** -1641 **c** 191.8125

4 **a** 2, 6, 14 **b** 2, 4, 8 **c** 64

5 648

6 **a** 5115 **b** 269.96 **c** -2724.74

7 **a** 20 **b** 2400 **c** 52.57

8 **a** 131100 **b** 354300 **c** 0.9961
d 19530 **e** 88560 **f** 82.87

9 **a** $2\times(^3/_2)^{n-1}$ **b** $r > 1$

10 **a** $^5/_8$ **b** 37.62

11 **a** $-^1/_6$ **b** $^7/_{36}$

12 **a** $^1/_2$ **b** 160

13 **a** 5, -3 **b** 24605

14 **a** 22000 **b** 26620 **c** 46.41%

15 **a** £5955.08 **b** £2969.24

16 **a** $-54, -^2/_3$ **b** -32.4

Geometric Series

Qu 1. (a) $u_1 = 3$, $u_2 = 9$, $u_3 = 27$, $u_4 = 81$ ✓
(b) $u_1 = 7$, $u_2 = 49$, $u_3 = 343$ ✓, $u_4 = 2401$
(c) $u_1 = -4$, $u_2 = 16$, $u_3 = -64$ ✓, $u_4 = 256$ ✓
(d) $u_1 = 6$, $u_2 = 12$, $u_3 = 24$ ✓, $u_4 = 48$ ✓
(e) $u_1 = 1$, $u_2 = 6$, $u_3 = 36$ ✓, $u_4 = 216$ ✓
(f) $u_1 = \frac{2}{3}$, $u_2 = \frac{4}{9}$, $u_3 = \frac{8}{27}$ ✓, $u_4 = \frac{16}{81}$ $\quad 12 \times \frac{3^n}{8}$

Qu 2 (a) $T_n = 5^n$ ✓ (b) $T_n = \left(\frac{1}{2}\right)^n$ ✓ (c) $T_n = 12 \times 3^{n-1} = 4 \times 3^n$
(d) $T_n = (-2)^n$ ✓ (e) $T_n = 12 \times 3^{-n+1}$
$= 12 \times \frac{3}{3^n}$
$= \frac{36}{3^n} = 36\left(\frac{1}{3}\right)^n$
(f) $T_n = \frac{1}{2} \times 2^{n-1}$ ✓
$= 2^{-1} \times 2^{n-1}$
$= 2^{n-2}$ ✓

Qu 3 (a) $T_n = 5 \times 2^{n-1}$ $\quad a = 5$, $r = 2$
$640 = 5 \times 2^{n-1}$ $\quad n = 8$
$\therefore S_8 = \frac{5(2^8 - 1)}{2 - 1}$
$= 1275$ ✓
(b) $a = -3$, $r = -3$
$\therefore S_7 = \frac{-3(-3^7 - 1)}{-3 - 1}$ ✓
$= -1641$ ✓
$T_n = -3 \times (-3)^{n-1}$
$-2187 = -3 \times -3^{n-1}$
$n = 7$ ✓
(c) $a = 96$, $r = \frac{1}{2}$ $\quad \therefore T_n = 96 \times \left(\frac{1}{2}\right)^{n-1}$ ✓
$\frac{3}{16} = 96 \times \left(\frac{1}{2}\right)^n$
$\frac{1}{512} = \left(\frac{1}{2}\right)^n$ $\quad n = 10$
$\therefore S_{10} = \frac{96\left(1 - \left(\frac{1}{2}\right)^{10}\right)}{1 - \frac{1}{2}}$ ✓
$= 191\frac{13}{16}$

Qu 4 (a) $S_1 = 2^2 - 2 = 2$ ✓ $\quad S_2 = 2^3 - 2 = 6$ ✓ $\quad S_3 = 2^4 - 2 = 14$ ✓
(b) $T_1 = 2$ ✓ $\quad T_2 = 4$ ✓ $\quad T_3 = 8$ ✓
(c) $T_n = 2 \times (2)^{n-1}$
$T_6 = 2 \times 2^5 = 64$ ✓

Qu 5 $S_1 = 4 \times (3 - 1) = 8$ ✓ $\quad S_2 = 4(3^2 - 1) = 32$ $\quad \therefore T_1 = 8$, $T_2 = 24$ $\quad a = 8$, $r = 3$
$\therefore T_n = 8 \times 3^{n-1}$
$T_5 = 8 \times 3^4 = 648$ ✓

Qu6 (a) $S_n = \frac{5(2^{10}-1)}{2-1}$

$= 5115.00$ ✓

(b) $S_8 = \frac{180(1-\frac{1}{3}^8)}{2-\frac{1}{3}}$

$= 269.96$ ✓

(c) $S_{12} = \frac{0.16(1-2.5^{12})}{1+2.5}$

$= -2724.74$ ✓

(b) $\sum_{r=1}^{10} (4 \times 3^r)$

$n = 10$.

$T_n = 4 \times 3^n$ ✓

$= 4 \times 3 \times 3^{n-1}$

$\therefore a = 12$ ✓

$r = 3$

$\therefore S_{10} = \frac{12(3^{10}-1)}{3-1}$

$= 354288$

$= 354300$ ✓

(c) $\sum_{r=1}^{8} (\frac{1}{2})^r$

$n = 8$

$T_n = (\frac{1}{2})^n$

$= \frac{1}{2} \times (\frac{1}{2})^{n-1}$ ✓

$\therefore a = \frac{1}{2}$ ✓

$r = \frac{1}{2}$

$\therefore S_8 = \frac{\frac{1}{2}(1-(\frac{1}{2})^8)}{1-\frac{1}{2}}$

$= 0.9961$ ✓

Qu7 (a) $S_\infty = \frac{10}{1-\frac{1}{2}}$

$= 20$ ✓

(b) $S_\infty = \frac{360}{1-0.85}$

$= 2400$ ✓

(c) $S_\infty = \frac{92}{1+\frac{3}{4}}$

$= 52.57$ ✓

(d) $\sum_{r=1}^{8} 5^{n-2}$.

$n = 8$ ✓

$T_n = 5^{n-2}$

$= 5^{-1} \times 5^{n-1}$

$a = \frac{1}{5}$ $r = 5$ ✓

$\therefore S_8 = \frac{\frac{1}{5}(1-(5)^8)}{1-5}$

$= \frac{0.2500}{}$

19531 ✓

(e) $\sum_{r=3}^{10} 3^r$.

$n = 8$.

$T_n = 3^n$

$= 3^{n+2} \times 3^{n-1}$

$a = 3$

$r = 3$

$\therefore S_8 = \frac{3(3^8-1)}{3-1}$

$= 9840$ ✗

Note: $\sum_{r=3}^{10} 3^r = 3^3 + 3^4 + \ldots + 3^{10}$

$a = 27$, $r = 3$

$\therefore S_8 = \frac{27(3^8-1)}{3-1} = 88560$.

Qu8 (a) $\sum_{r=1}^{16} 2^r$ ✓

$n = 16$.

$T_n = 2^n$.

$= 2 \times 2^{n-1}$.

$\therefore a = 2$ ✓

$r = 2$

$\therefore S_{16} = \frac{2(2^{16}-1)}{2-1}$

$= 131070$

$= 131100$ ✓

(f) $\sum_{r=6}^{12} \left[80 \times \left(\frac{4}{5}\right)^r\right]$

$n = 7$.

$T_n = 80 \times \left(\frac{4}{5}\right)^n$

$= 80 \times \frac{4}{5} \times \left(\frac{4}{5}\right)^{n-1}$

$a = 64$

$r = \frac{4}{5}$

Similarly $a = 80 \times \left(\frac{4}{5}\right)^6$

$= \frac{64}{3125}$ ... try again.

$\therefore S_7 = \frac{64\left(1 - \left(\frac{4}{5}\right)^7\right)}{1 - \frac{4}{5}}$

$= 252.9$

Qu 9. (a) $a = 2$

$r = \frac{3}{2}$

$T_n = 2 \times \left(\frac{3}{2}\right)^{n-1}$ ✓

(b) $r > 1$ ✓

and for a limiting sum, ✓

the ratio must be, $-1 < r < 1$.

Qu 10 (a) $a = 15$.

$S_\infty = 40$

$40 = \frac{15}{1-r}$ ✓

$40 - 40r = 15$

$25 = 40r$ ✓

$r = 5/8$

(b) $S_6 = \frac{15(1-(5/8)^6)}{3/8}$ ✓

$= 37.62$ ✓

Qu 11 (a) $a = 252$

$S_\infty = 216$

$216 = \frac{252}{1-r}$ ✓

$216 - 216r = 252$.

$\frac{-36}{216} = r$

$r = -\frac{1}{6}$ ✓

(b) $T_n = 252 \times (-\frac{1}{6})^{n-1}$ ✓

$T_5 = 252 \times (-\frac{1}{6})^4$.

$= \frac{7}{36}$ ✓

Qu 12. $a = 80$

$ar^3 = 10$.

(a) $r^3 = \frac{1}{8}$.

$r = \frac{1}{2}$ ✓

(b) $S_\infty = \frac{80}{\frac{1}{2}}$

$= 160$ ✓

Qu 13 (a) $ar = -15$

$ar^4 = 405$

$r^3 = -27$

$r = -3$ ✓

$a = 5$ ✓

(b) $S_9 = \frac{5(1-(-3)^9)}{4}$.

$= 2460$ ✓

Qu 14. (a) $1.1 \times 20000 = 22000$ ✓

(b) $T_3 = 20000 \times (1.1)^3$

$= 26620$ ✓

(c) $T_4 = 29282$

$\therefore \frac{9282}{20000} \times \frac{100}{1}\%$.

$= 46.41\%$ ✓

Qu 15 5000×1.06, $5000 \times (1.06)^2$...

(a) $(1.06)^3 \times 5000 = T_3$

$T_3 = £5955.08$ ✓

(b) $T_8 = 7969.24$

$\therefore £7969.24$ ✓

Qu 16 $ar = 36$

$a \times ar = -18$.

(a) $a = -54$ ✓

$r = -2/3$ ✓

(b) $S_\infty = \frac{-54}{5/3}$.

$= -32\frac{2}{5}$ ✓