Sequences and Series Review

Exercise 14E Exam Practice

1 The fourth term of an arithmetic series is 3.

The sum of the first six terms of the series is 6.

a Find the first term and the common difference of the series. **(5 marks)**

b Find the sum of the first 60 terms of the series. **(3 marks)**

2 The third and fourth terms of a geometric series are 24 and 16 respectively.

a Find the first term and the common ratio of the series. **(4 marks)**

b Find the sum to infinity of the series. **(2 marks)**

3 In its business plan, a company predicts that it will make £26000 profit in the year 2001. The plan predicts that in subsequent years the company's profits will increase by £3500 each year.

a Show that the plan predicts a profit of £33 000 in the year 2003. **(2 marks)**

b By summing an appropriate series, find the total profit that the company will make, according to the business plan, in the years 2001 to 2012 inclusive. **(4 marks)**

4 The first term and common difference of an arithmetic series are 79 and $^{-}4$ respectively.

a Find and simplify an expression for the nth term of the series. **(2 marks)**

b By forming a suitable inequality and solving it, find out how many terms of the series are positive. **(3 marks)**

c Hence, or otherwise, find the greatest value of S_n, the sum of the first n terms of the series. **(4 marks)**

5 The first and third terms of a geometric series are 12 and 4 respectively.

Given that the common ratio is positive,

a find the common ratio of the series in the form $k\sqrt{3}$, where k is an exact fraction, **(4 marks)**

b find the ninth term of the series as an exact fraction, **(3 marks)**

c show that the sum to infinity of the series is $6(3 + \sqrt{3})$. **(4 marks)**

6 The sum to infinity of a geometric series is equal to three times the first term.

a Find the common ratio of the series. **(3 marks)**

Given that the sum of the first k terms of the series is equal to twice the first term,

b show that $\left(\frac{2}{3}\right)^k = \frac{1}{3}$. **(4 marks)**

7 The first and third terms of an arithmetic series are 6 and $4x$ respectively.

Find and simplify expressions in terms of x for

a the common difference of the series, **(3 marks)**

b the fifth term of the series, **(3 marks)**

c the sum of the first eight terms of the series. **(3 marks)**

8 **a** Evaluate correct to 4 significant figures

$$\sum_{r=1}^{10} \left(\frac{4}{3}\right)^r .$$ **(4 marks)**

b Find the sum of the even numbers between 101 and 199. **(5 marks)**

9 The nth term of a sequence, u_n, is given by

$$u_n = k^n + n.$$

Given that $u_4 = 2u_2$ and that $k > 0$,

a show that $k = \sqrt{2}$, **(4 marks)**

b find $\frac{u_6}{u_5}$ in the form $a + b\sqrt{2}$. **(5 marks)**

10 The first term of an arithmetic series is 75.

The sum of the first ten terms of the series is 480.

a Find the common difference of the series. **(4 marks)**

b Find the other value of n for which the sum of the first n terms of the series is 480. **(5 marks)**

11 A savings account pays 0.8% interest on the amount in the account at the end of each month.

a Brian invests £1000 in this account.

Show that after the payment of interest at the end of the first year there is £1100.34 in his account, to the nearest penny. **(3 marks)**

b Tahira pays £200 into this account at the start of each month.

Find to the nearest penny the amount in her account after the payment of interest at the end of a two year period. **(6 marks)**

Exercise 14E Exam Practice

1 **a** -9, 4 **b** 6540

2 **a** 54, $^2/_3$ **b** 162

3 **b** £543000

4 **a** $83 - 4n$ **b** 20 **c** 820

5 **a** $^1/_3\sqrt{3}$ **b** $^4/_{27}$

6 **a** $^2/_3$

7 **a** $2x - 3$ **b** $8x - 6$ **c** $56x - 36$

8 **a** 67.03 **b** 7350

9 **b** $8\sqrt{2} - 10$

10 **a** -6 **b** 16

11 **b** £5310.78

Sequences & Series Review.

1. $a + 3d = 3$ — ①

$3(2a + 5d) = 6$ — ②

(a) ②: $2a + 5d = 2$

① × 2: $2a + 6d = 6$ ✓ — ③

③ − ②: $d = 4$. Sub in ①.

$a + 12 = 3$

$a = -9$ ✓

(b) $T_{60} = (-18 + 59(4))30$

$= 6540$ ✓

2. $ar^2 = 24$ — ①

$ar^3 = 16$. — ② ✓

(a) $r = \frac{2}{3}$

$a = 54$ ✓

(b) $S_\infty = \frac{54}{\frac{1}{3}}$

$= 162$ ✓

Qu3 (a) $T_n = 26000 + (n-1)3500$

$= 3500n + 22500$

$T_3 = 3500 \times 3 + 22500$ ✓

$= £33000$

(b) $S_{12} = 6(52000 + (12-1)3500)$

$= £543000$ ✓

Qu4 $a = 79$

$d = -4$

(a) $T_n = 79 + (n-1) \cdot -4$

$= -4n + 83$ ✓

(b) $-4n + 83 > 0$

$n < 20.75$

$\therefore n = 20$ ✓

$\therefore$ 20 terms.

(c) $S_{20} = \frac{20}{2}(79 + 3)$

$= 820$ ✓

Qu5 $a = 12$ — ①

$ar^2 = 4$. — ② ✓

(a) ② ÷ ①: $r^2 = \frac{1}{3}$ ✓

$r = \frac{\pm 1}{\sqrt{3}}$

$= \frac{\pm\sqrt{3}}{3}$ ✓

$k = \frac{1}{3}$ ✓

$r = \frac{1}{3} \pm \sqrt{3}$

$= \frac{1}{3}\sqrt{3}$ ✓ (since +ve)

(b) $T_n = 12 \times \left(\frac{1}{3}\sqrt{3}\right)^{n-1}$

$T_9 = 12 \times \left(\frac{1}{3}\sqrt{3}\right)^8$

$= 12 \times \left(\frac{\sqrt{3}}{3}\right)^8$ ✓

$= 12 \times \frac{81}{6561}$

$= \frac{4}{27}$ ✓

(c) $S_\infty = \frac{12}{1 - \frac{1}{3}\sqrt{3}}$

$= \frac{12}{1 - \frac{\sqrt{3}}{3}} = \frac{12}{\frac{3-\sqrt{3}}{3}}$ ✓

$= \frac{36}{3-\sqrt{3}} \times \frac{3+\sqrt{3}}{3+\sqrt{3}}$

$= \frac{108 + 36\sqrt{3}}{6}$ ✓

$= 18 + 6\sqrt{3} = 6(3 + \sqrt{3})$

Qu6 $S_\infty = 3a$.

$\frac{a}{1-r} = 3a$.

(a) $a = 3a - 3ar$ ✓

$3ar = 3a - a$.

$r = \frac{3a - a}{3a}$

$= \frac{2a}{3a}$

$= \frac{2}{3}$ ✓

(b) $S_k = 2a$.

$r = \frac{2}{3}$

$\frac{a(1-(\frac{2}{3})^k)}{\frac{1}{3}} = 2a$.

$a(1-(\frac{2}{3})^k) = \frac{1}{3}a$ ✓

$1-(\frac{2}{3})^k = \frac{1}{3}$

$(\frac{2}{3})^k = \frac{2}{3}$

$\therefore (\frac{2}{3})^k = \frac{1}{3}$ ✓

Qu7 $a = 6$ —①

$a+2d = 4x$ —②

(a) ② − ①: $2d = 4x-6$

$d = 2x-3$ ✓

(b) $T_n = 6+(n-1)(2x-3)$

$T_5 = 6+4(2x-3)$ ✓

$= 6+8x-12$

$= 8x-6$ ✓

(c) $S_5 = \frac{5}{2}(6+8x-6)$

$= \frac{5}{2}(8x)$

$= 20x$ ✓

Qu8 (a) $\sum_{r=1}^{10} (\frac{4}{3})^r$ ✓

$n = 10$

$T_n = (\frac{4}{3})^n$

$= \frac{4}{3} \times (\frac{4}{3})^{n-1}$ ✓

$a = \frac{4}{3}$

$r = \frac{4}{3}$

$S_{10} = \frac{\frac{4}{3}((\frac{4}{3})^{10}-1)}{\frac{4}{3}-1}$ ✓

$= 67.03$ ✓

(b) 102, 104, 106, ... 196, 198.

$a = 102$

$d = 2$ $T_n = 102+(n-1)2$

$= 2n+100$

$198 = 2n+100$ ✓

$n = 49$

$T_{49} = \frac{49}{2}(102+198)$

$= 7350$ ✓

Qu9 $T_n = n+k^n$.

$T_4 = 2T_2$ $k>0$

(a) $T_1 = 1+k$

$T_2 = 2+k^2$

$T_4 = 4+k^4$ ✓

$4+k^4 = 2(2+k^2)$

$4+k^4 = 4+2k^2$.

$k^4-2k^2 = 0$

$k^2(k^2-2) = 0$ ✓

$k^2 = 2$

$k = \sqrt{2}$ since $k>0$.

(b) $T_6 = 6+k^6$

$T_5 = 5+k^5$

$\frac{6+k^6}{5+k^5} = \frac{6+(\sqrt{2})^6}{5+(\sqrt{2})^5}$ ✓

$= \frac{14}{5+4\sqrt{2}} \times \frac{5-4\sqrt{2}}{5-4\sqrt{2}}$

$= \frac{70-56\sqrt{2}}{25-32}$

$= \frac{7(10-8\sqrt{2})}{-7}$

$= -10+8\sqrt{2}$ ✓

Qu10 $a = 75$

(a) $S_{10} = 480$.

$S_{10} = 5(150+9d)$.

$480 = 5(150+9d)$

$d = -6$ ✓

(b) $S_n = \frac{n}{2}(150+(n-1)\cdot-6)$

$960 = n(156-6n)$

$= -6n^2+156n$ ✓

$\therefore 6n^2-156n+960 = 0$

$n^2-26n+160 = 0$

$(n-10)(n-16) = 0$

$\therefore n = 16$ ✓

Q11. (a) ~~0.8xP, (0.8)²P, (0.8)³P...~~

~~a = 0.8P~~

~~r = 0.8~~

~~$T_n = 0.8P \times (0.8)^{n-1}$~~

~~$= P \times 0.8^n$~~

~~$T_{12} =$~~

$T_1 = 1.008 \times P$

$T_2 = (1.008)^2 \times P$

$T_n = (1.008)^n \times P$

$\therefore T_{12} = (1.008)^{12} \times 1000$ ✓

$= £1100.34$

(b) $T_{24} = (1.008)^{24} \times 200$

~~$= £242.15$~~

Total amount $= T_1 + T_2 + \ldots + T_{24}$

$= \underbrace{1.008P + (1.008)^2 P + \ldots + (1.008)^{24} P}$

$S_n = \frac{a(r^n - 1)}{r - 1}$

$= \frac{1.008P\left[(1.008)^{24} - 1\right]}{1.008 - 1}$ where $P = £200$

$= £5310.78$