Exam Practice - DIFFERENTIATION Exercise 16E

a Find an equation of the tangent to the curve $y = 2 + 3x - 2x^2$ at the point A(2,0).

(4 marks)

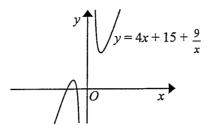
b Find an equation of the tangent to the curve $y = 2 + 3x - 2x^2$ at the point B, where it meets the y-axis.

(3 marks)

c Find the coordinates of the point C, where the tangents you have found in parts a and b intersect.

(3 marks)

2



The diagram shows part of the curve $y = 4x + 15 + \frac{9}{x}$

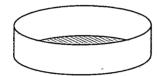
a Find the coordinates of the points where the curve intersects the x-axis.

(4 marks)

b Find the coordinates of the turning points of the curve.

(5 marks)

3



The diagram shows an open-topped cylindrical cake tin.

Given that the volume of the tin is to be 1000π cm³,

a find an expression for the height, h cm, of the tin in terms of the radius of the base of the tin, r cm.

(2 marks)

b show that the area of metal, A cm², used in making the tin is given by $A = \pi r^2 + \frac{2000\pi}{2}$.

(3 marks)

Given that r can vary,

c find the minimum value of A in terms of π ,

(6 marks)

d justify that your value is a minimum.

(3 marks)

$$f(x) \equiv x^3 - x^2 + kx + 2.$$

Given that (x-2) is a factor of f(x),

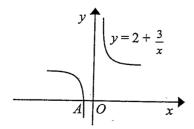
a find the value of the constant k,

(2 marks)

find exactly the set of values of x for which f(x) is decreasing.

(6 marks)

5



The diagram shows part of the curve $y = 2 + \frac{3}{2}$.

The curve intersects the x-axis at the point A.

a Find the coordinates of the point A.

(2 marks)

b Find an equation of the normal to the curve at the point A.

(6 marks)

c Show that the normal intersects the curve again at the point

with coordinates $(\frac{8}{3}, \frac{25}{8})$.

(5 marks)

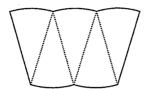
6 The number of people in thousands, P, at an outdoor festival h hours after the gates open is modelled with the formula

$$P = 15h + 6h^2 - h^3$$
, for $h \ge 0$.

Using this model,

- a find an expression in terms of h for the rate at which P increases, (2 marks)
- **b** show that the population is increasing most rapidly when h = 2, (3 marks)
- c find the number of people entering the festival per minute when h = 2, assuming that no one is leaving. (3 marks)

7



The diagram shows the shape of a new biscuit, consisting of five circular sectors, each of radius r cm and angle θ radians.

a Find expressions in terms of r and θ for the perimeter and area of the shape.

(4 marks)

Given that the area of the shape is to be 40 cm²,

b show that the perimeter of the shape, P cm, can be expressed as

$$P = 20 \theta^{\frac{1}{2}} + 8 \theta^{-\frac{1}{2}}$$

(4 marks)

Given that θ can vary,

c find the value of θ for which P is a minimum and find the corresponding value of P in the form $k\sqrt{10}$.

(7 marks)

Exercise 16E **Exam Practice**

- 1 **a** y = -5x + 10 **b** y = 3x + 2 **c** (1, 5)
- 2 **a** (-3, 0), (-3/4, 0) **b** (-3/2, 3), (3/2, 27)
- 3 **a** $h = \frac{1000}{r^2}$ **c** 300π
- 4 **a** -3 **b** $\frac{1}{3}(1-\sqrt{10}) < x < \frac{1}{3}(1+\sqrt{10})$
- 5 **a** (-3/2, 0) **b** 6x 8y + 9 = 0
- 6 **a** $(15+12h-3h^2)$ /hr **c** 450/min
- $a^{5}/_{2}r^{2}\theta \text{ cm}^{2}$ $c^{2}/_{5}$, $8\sqrt{10}$