Differentiation

1   Differentiate with respect to x.

   a x^2 **b** x^7 **c** x^4 **d** x^{-2} **e** x^{-5} **f** x

   g 7 **h** $5x$ **i** $4x^2$ **j** $2x^3$ **k** $6x^{-4}$ **l** $8x^{-3}$

2   Differentiate with respect to t.

   a t^3 **b** $10t$ **c** $2t^{-1}$ **d** 8 **e** $t^{\frac{1}{2}}$ **f** $4t^{\frac{5}{2}}$

   g $6t^{-\frac{1}{2}}$ **h** $3t^{\frac{3}{2}}$ **i** $9t^{\frac{2}{3}}$ **j** $5t^{-\frac{3}{4}}$ **k** $\frac{1}{2}t^{\frac{4}{3}}$ **l** $\frac{5}{3}t^{\frac{6}{5}}$

3   Find $f'(x)$

   a $f(x) \equiv x^3 + 5x^2$ **b** $f(x) \equiv 2x^4 - 3x$ **c** $f(x) \equiv 8x + 4x^{-3}$

   d $f(x) \equiv x^7 + 6x^5 - x^3$ **e** $f(x) \equiv x(x^2 - 3)$ **f** $f(x) \equiv (x+1)(x-5)$

   g $f(x) \equiv (x+7)(3x-1)$ **h** $f(x) \equiv (2x+1)(x-9)$ **i** $f(x) \equiv 9x + 4x^{\frac{1}{2}}$

   j $f(x) \equiv 6x^{\frac{5}{2}} - x^{-1}$ **k** $f(x) \equiv 6x^{\frac{4}{3}} - 2x^{\frac{1}{3}}$ **l** $f(x) \equiv \frac{1}{2}x^{-2} + \frac{2}{3}x^{-\frac{7}{2}}$

4   Differentiate with respect to x.

   a $\frac{3}{x}$ **b** $\frac{1}{3x}$ **c** $\sqrt{x}$ **d** $\frac{6}{\sqrt{x}}$ **e** $\sqrt[3]{x}$ **f** $\sqrt{x^3}$

5   Find $\frac{dy}{dx}$ in each case

   a $y = 18x^2 - x + 2$ **b** $y = x^3 + 4x + 3x^{-2}$ **c** $y = (x+4)^2$

   d $y = x^4 - 2x^2 + 5x^{-3}$ **e** $y = 2x^2(x^3 - 3x^{-1})$ **f** $y = x(x-2)(x-3)$

   g $y = (x^2 - 3)(x-2)$ **h** $y = 3 + 4\sqrt{x}$ **i** $y = \frac{3x^4 - x}{x^2}$

   j $y = \frac{3}{x^2} - \frac{1}{2\sqrt{x}}$ **k** $y = \frac{2x^2 - 6x}{2\sqrt{x}}$ **l** $y = \frac{9x^3 - 2x^2 + 6}{3x}$

6   Find $\frac{d^2y}{dx^2}$ in each case

   a $y = x^4$ **b** $y = x^3 - 7x^2 + 2$ **c** $4x - x^{-1} + 3x^{-2}$

   d $y = (x^2 - 3)^2$ **e** $y = 4x^{\frac{3}{2}} + 2x^{-\frac{1}{2}}$ **f** $y = \frac{4x^2 - 5x + \sqrt{x}}{2x}$

**7**   Find the gradient of each curve at the point where $x = 2$.

   a $y = 3x^2$ **b** $y = x^3 - 9x$ **c** $y = x^2 + 6x^{-1}$

8   Find the gradient of each curve at the point where $x = {}^-1$.

   a $y = 2x^3$ **b** $y = x^2 + x^{-2}$ **c** $y = (x+3)(x+4)$

9   Find an equation of the tangent to each curve at the point with the given x-coordinate.

   a $y = x^2$; $x = 1$ **b** $y = x^3 - 5x$; $x = 2$

   c $y = x^2 + 6x + 3$; $x = {}^-3$ **d** $y = 4 - x^2$; $x = 5$

   e $y = 2\sqrt{x}$; $x = 16$ **f** $y = x - \frac{2}{x}$; $x = 2$

10 Find an equation of the normal to each curve at the point with the given x-coordinate.

  a $y = x^2$; $x = 2$ **b** $y = 3 - 5x - x^2$; $x = {}^-3$

  c $y = (x - 2)^2$; $x = 3$ **d** $y = 5x^2 - x^3$; $x = 4$

  e $y = 3x - \frac{1}{2}x^3$; $x = {}^-1$ **f** $y = 6x^{\frac{1}{2}} - 18x^{-\frac{1}{2}}$; $x = 9$

11 Find in each case any values of x for which $\frac{dy}{dx} = 0$.

  a $y = x^2 + 3x$ **b** $y = x^3 - 27x$ **c** $y = 2x^3 + 3x^2 - 12x$

  d $y = x + 4x^{-1}$ **e** $y = x^3 - 2x^2 + 5$ **f** $y = 7 + 4x + 2x^2 - x^3$

12 Find the set of values for which f(x) is increasing.

  a $f(x) \equiv x^2 - 3x - 7$ **b** $f(x) \equiv x^3 - 3x^2$ **c** $f(x) \equiv x^3 + 9x^2 + 15x$

13 Find the set of values for which f(t) is decreasing.

  a $f(t) \equiv 3t^3 - t + 4$ **b** $f(t) \equiv t(t^2 - 9t - 21)$ **c** $f(x) \equiv 5 + t^2 - t^3$

14 Find the coordinates of the stationary point on the curve $y = x^2 + 4x - 1$

  a   by completing the square,   **b**   by using differentiation.

15 Find the coordinates of any stationary points on each curve. Find whether each stationary point is a maximum, minimum or point of inflexion.

  a $y = 4x^2 + 8x - 5$ **b** $y = x^2 - 5x + 7$ **c** $y = x^3 - 12x$

  d $y = 6 + 3x^2 - x^3$ **e** $y = 4 - x^3$ **f** $y = x^4 - 4x^3 + 4x^2$

  g $y = 4x + \frac{1}{x}$ **h** $y = 6x^{\frac{1}{2}} - x$ **i** $y = 16x - \frac{1}{x^2}$

16 Sketch each curve labelling the coordinates of any stationary points.

  a $y = 3x^3 - 9x - 2$ **b** $y = x^3 + 7x^2 - 5x$ **c** $y = 1 + 24x + 3x^2 - x^3$

17 $f(x) \equiv x^2 - 6x - 1.$

  a   Find $f'(x)$.

  b   Find the equation of the normal to the curve $y = f(x)$ at the point $(2, {}^-9)$

18 The height above ground, h metres, of a ball t seconds after being thrown upwards is given by $h = 8t - 5t^2$.

  a   Find the rate at which the ball is gaining height when $t = 0.5$

  b   Find the maximum height reached by the ball.

19 $f(x) \equiv x^3 - 2x^2 + 3x + 5.$

  a   Find $f'(x)$.

  b   Find the exact coordinates of the points on the curve $y = f(x)$ at which the gradient is 2.

20

x cm [rectangle]

y cm
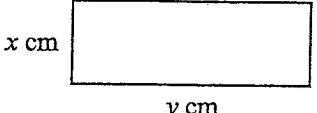
The diagram shows a thin rectangular picture frame measuring x cm by y cm.

Given that the perimeter of the frame is to be 72 cm,

  a   show that $y = 36 - x$,

  b   find an expression in terms of x for the area, A cm^2, enclosed by the frame,

  c   show that the largest value of A occurs when the frame is a square.

21   a Find the coordinates of the points where the curve $y = 4x - x^2$ crosses the x-axis.

     b Find the equation of the tangent to the curve $y = 4x - x^2$ at each of the points found in part **a**.

22               $y = x + \dfrac{9}{x}$

     a State any values of x for which y is undefined.

     b Find the coordinates of any stationary points on the curve $y = x + \dfrac{9}{x}$ and determine their nature.

     c Sketch the curve $y = x + \dfrac{9}{x}$.

23   The size of a spinning cube on a computer screen saver varies so that the length of one side, l cm, t seconds after the start, is given by $l = \frac{1}{3}t^2 - 3t + 10$ for $0 \le t < 9$.

     a Find the rate at which l is changing when $t = 2$.

     b Find the smallest value of l.

Exercise 16S   Skills Practice

1   a $2x$ b $7x^6$ c $4x^3$ d $-2x^{-3}$
    e $-5x^{-6}$ f 1 g 0 h 5
    i $8x$ j $6x^2$ k $-24x^{-5}$ l $-24x^{-4}$

2   a $3t^2$ b 10 c $-2t^{-2}$ d 0
    e $\frac{1}{2}t^{-1/2}$ f $10t^{3/2}$ g $-3t^{-3/2}$ h $\frac{9}{2}t^{1/2}$
    i $6t^{-1/3}$ j $-\frac{15}{4}t^{-7/4}$ k $\frac{2}{3}t^{1/3}$ l $2t^{1/5}$

3   a $3x^2+10x$ b $8x^3-3$
    c $8-12x^{-4}$ d $7x^6+30x^4-3x^2$
    e $3x^2-3$ f $2x-4$
    g $6x+20$ h $4x-17$
    i $9+2x^{-1/2}$ j $15x^{3/2}+x^{-2}$
    k $8x^{1/3}-\frac{2}{3}x^{-2/3}$ l $-x^{-3}-\frac{7}{3}x^{-9/2}$

4   a $-3x^{-2}$ b $-\frac{1}{3}x^{-2}$ c $\frac{1}{2}x^{-1/2}$
    d $-3x^{-3/2}$ e $\frac{1}{3}x^{-2/3}$ f $\frac{3}{2}x^{1/2}$

5   a $36x-1$ b $3x^2+4-6x^{-3}$
    c $2x+8$ d $4x^3-4x-15x^{-4}$
    e $10x^4-6$ f $3x^2-10x+6$
    g $3x^2-4x-3$ h $2x^{-1/2}$
    i $6x+x^{-2}$ j $-6x^{-3}+\frac{1}{4}x^{-3/2}$
    k $\frac{3}{2}x^{1/2}-\frac{3}{2}x^{-1/2}$ l $6x-\frac{2}{3}-2x^{-2}$

6   a $12x^2$ b $6x-14$ c $-2x^{-3}+18x^{-4}$
    d $12x^2-12$ e $3x^{-1/2}+\frac{3}{2}x^{-5/2}$ f $\frac{3}{8}x^{-5/2}$

7   a 12 b 3 c $\frac{5}{2}$

8   a 6 b 0 c 5

9   a $y=2x-1$ b $y=7x-16$ c $y=-6$
    d $y=-10x+29$ e $x-4y+16=0$ f $3x-2y-4=0$

10   a $x+4y-18=0$ b $x+y-6=0$
    c $x+2y-5=0$ d $x-8y+124=0$
    e $4x+6y+19=0$ f $3x+4y-75=0$

11   a $-\frac{3}{2}$ b ±3 c $-2, 1$
    d ±2 e $0, \frac{4}{3}$ f $-\frac{2}{3}, 2$

12   a $x > \frac{3}{2}$ b $x < 0, x > 2$ c $x < -5, x > -1$

13   a $-\frac{1}{3} < t < \frac{1}{3}$ b $-1 < t < 7$ c $t < 0, t > \frac{2}{3}$

14   a $(-2, -5)$ b $(-2, -5)$

15   a $(-1, -9)$, min   b $(\frac{5}{2}, \frac{3}{4})$, min
    c $(-2, 16)$, max; $(2, -16)$, min
    d $(0, 6)$, min; $(2, 10)$, max
    e $(0, 4)$, pt. infl.
    f $(0, 0)$, min; $(1, 1)$, max; $(2, 0)$, min
    g $(-\frac{1}{2}, -4)$, max; $(\frac{1}{2}, 4)$, min
    h $(9, 9)$, max   i $(-\frac{1}{2}, -12)$, max

16   a $(-1, 4), (1, -8)$ b $(-5, 75), (\frac{1}{3}, -\frac{23}{27})$
    c $(-2, -27), (4, 81)$

17   a $2x-6$ b $x-2y-20=0$

18   a 3 m/s   b $\frac{16}{5}$ m

19   a $3x^2-4x+3$ b $(\frac{1}{3}, 5\frac{22}{27}), (1, 7)$

20   b $36x-x^2$

21   a $(0, 0), (4, 0)$ b $y=4x$ c $y=-4x+16$

22   a 0 b $(-3, -6)$, max; $(3, 6)$, min

23   a $-\frac{5}{3}$ cm/s   b $\frac{13}{4}$