EXERCISE 23.1

1. For the following parabolas, written in the standard form $x^2 = 4ay$, find the value of 'a' and hence the parametric equations for each parabola:
   (a) $x^2 = 4y$
   (b) $x^2 = 8y$
   (c) $x^2 = y$
   (d) $x^2 = 2y$
   (e) $x^2 = -4y$
   (f) $x^2 = -16y$

2. Find the Cartesian equations for the following parabolas:
   (a) $\left.\begin{aligned} x &= 4t \\ y &= 2t^2 \end{aligned}\right\}$
   (b) $\left.\begin{aligned} x &= 6t \\ y &= 3t^2 \end{aligned}\right\}$
   (c) $\left.\begin{aligned} x &= 8t \\ y &= 4t^2 \end{aligned}\right\}$
   (d) $\left.\begin{aligned} x &= \tfrac{1}{2}t \\ y &= \tfrac{1}{4}t^2 \end{aligned}\right\}$
   (e) $\left.\begin{aligned} x &= 8t \\ y &= -4t^2 \end{aligned}\right\}$
   (f) $\left.\begin{aligned} x &= t \\ y &= -\tfrac{1}{2}t^2 \end{aligned}\right\}$
   (g) $\left.\begin{aligned} x &= -\tfrac{1}{4}t \\ y &= -\tfrac{1}{8}t^2 \end{aligned}\right\}$
   (h) $\left.\begin{aligned} x &= 2ap \\ y &= -ap^2 \end{aligned}\right\}$

3. For the parabolas (a) $x = t,\ y = \frac{1}{2}t^2$
   (b) $x = 6t,\ y = -3t^2$
   (i) tabulate the values of x and y for values of t in the domain $-3 \leqslant t \leqslant 3$
   (ii) sketch the curve
   (iii) find the Cartesian equation
   (iv) state the coordinates of the vertex and of the focus
   (v) state the equation of the directrix.

Eliminate 't' from each of the following pairs of parametric equations and so form the corresponding Cartesian equation. Make a rough sketch of each curve.
   (a) $\left.\begin{aligned} x &= 2t \\ y &= 2 - t \end{aligned}\right\}$
   (b) $\left.\begin{aligned} x &= 3\cos t \\ y &= 3\sin t \end{aligned}\right\}$
   (c) $\left.\begin{aligned} x &= 1 + 2t \\ y &= 4t^2 + 4t \end{aligned}\right\}$

$\cos t = \frac{x}{3}$ $\cos^2 t = \frac{x^2}{9}$

$\sin t = \frac{y}{3}$ $\sin^2 t = \frac{y^2}{9}$

EQUATION OF CHORD

EXERCISE 23.2

$\cos^2 t + \sin^2 t = 1$

1. Find, from first principles, the equation of the chord joining:
   (a) $t_1 = 2,\ t_2 = -1$ on $x = 2t,\ y = t^2$
   (b) $t_1 = 2,\ t_2 = -\frac{1}{2}$ on $x = 6t,\ y = 3t^2$.
   Show that this chord passes through the focus.

2. (a) (i) Find the equation of the chord joining the points $t_1 = 2$ and $t_2 = -\frac{1}{2}$ on the parabola $x^2 = y$.
   (ii) Show that the coordinates of the focus satisfy the equation.
   (b) One extremity of a focal chord in the parabola $x^2 = 8y$ is $t = 3$.
   Find: (i) the gradient of the chord
   (ii) the equation of the chord
   (iii) the Cartesian coordinates of the other extremity of the chord
   (iv) the parameter t at this point.

3. (a) Show that the chord joining $t_1 = 1$ and $t_2 = -1$ on the parabola $x = 2at,\ y = at^2$ is:
   (i) a focal chord
   (ii) is parallel to the x axis.
   (b) Find the length of this chord.
   (c) Show that the gradient of a focal chord joining the points $(2ap, ap^2)$ and $(2aq, aq^2)$ on the parabola $x^2 = 4ay$, is $\dfrac{p^2 - 1}{2p}$.

4. For the parabola $x^2 = 4ay$.
   (a) Show that the gradient of the chord joining the vertex to a point $P(2ap, ap^2)$ is $\frac{p}{2}$.
   (b) If Q is the point $(2aq, aq^2)$ state the gradient of OQ and hence show that the chords OP, OQ are perpendicular if $pq = -4$.
   (c) What is the condition for PQ to be perpendicular to the axis of the parabola?

5. (a) Find the equation of the chord joining the points $t = 3$ and $t = -\frac{1}{3}$ on the parabola $x^2 = 4ay$.
Show: (i) that it is a focal chord
(ii) that it cuts the directrix at the point $(-\frac{3a}{2}, -a)$.
(b) Prove that the chord joining the points p, q on the parabola $x^2 = 4ay$ cuts the directrix at the point $\left(\dfrac{2a(pq-1)}{p+q}, -a\right)$.
Use this result to check your answer to part (a).

6. A secant which passes through the point $(0, -a)$, cuts the parabola $x^2 = 4ay$ at P, Q with parameters p, q.
(a) Prove that $pq = 1$.
(b) If S is the focus show that $\dfrac{1}{PS} + \dfrac{1}{QS} = \dfrac{1}{a}$.

TANGENTS & NORMALS TO A PARABOLA

EXERCISE 23.3

1. Find the equations of the tangents and normals to the following parabolas:
(a) $x^2 = 4y$ at $(-2, 1)$
(b) $x^2 = 8y$ at $(2, \frac{1}{2})$
(c) $x^2 = 2y$ at $(2, 2)$
(d) $x = 2t, y = t^2$ at $t = 2$
(e) $x = 4t, y = 2t^2$ at $t = -1$
(f) $x = 6t, y = 3t^2$ at $t = 1$
(g) $y = x^2 + 1$ at $(1, 2)$
(h) $y = x^2 - x - 2$ at $(-2, 4)$
(i) $x = 2at, y = at^2$ at $t = 2$
(j) $x = 2at, y = at^2$ at $t = \frac{p}{4}$.

2. Find: (i) the equation of the tangents
(ii) the equations of the normals
(iii) the point of intersection of the two tangents
(iv) the point of intersection of the two normals
for the following parabolas at the points indicated:
(a) $x^2 = 2y$ at $(-2, 2)$ and $(2, 2)$
(b) $x^2 = 16y$ at $(4, 1)$ and $(-8, 4)$
(c) $x = 2t, y = t^2$ at $t = 1$ and $t = -1$
(d) $x = 6t, y = 3t^2$ at $t = 2$ and $t = 0$.

3. (a) Prove that the line $x - 2y - 2 = 0$ is a tangent to the parabola $x^2 = 16y$ and state the coordinates of the point of contact.
(b) Prove that the line $x + y + 3 = 0$ is a tangent to the parabola $x^2 = 12y$. What is the point of contact?
(c) Prove that the line $x - y - 5 = 0$ is tangential to the parabola $x = 10t, y = 5t^2$.
Find the coordinates and the value of the parameter 't' at the point of contact.

4. Given the parabola $x^2 = \frac{1}{2}y$, for what values of m does the line $y = m(x + 1)$:
(a) have two points of intersection?
(b) one point of intersection?
(c) no points of intersection?

5. For the parabola $x = 6t, y = 3t^2$ find:
(a) the equation of the tangent at $t = 3$
(b) the equation of the normal at $t = -\frac{1}{3}$.
What can be said about the two lines?

6. The line $x - 2y + 8 = 0$ cuts the parabola $x^2 = 8y$ at P and Q. Find:
(a) the coordinates of P and Q
(b) the equations of the tangents at P and Q
(c) their point of intersection.

7. (a) Prove that the tangents at $t = p$ and $t = q$ on $x = 4t, y = 2t^2$ meet at $(2(p + q), 2pq)$.
(b) Prove that the normals at these points meet at $(-2pq(p + q), 4 + 2(p^2 + pq + q^2))$.

8. Show that for the points 'p' and 'q' on the parabola $x = 2at, y = at^2$:
(a) the tangents meet at $(a(p + q), apq)$
(b) the normals meet at $(-apq(p + q), a(2 + p^2 + pq + q^2))$.
(c) If $pq = 2$, show by substitution into $x^2 = 4ay$ that the point of intersection of the normals lies on the parabola.

(d) Write down the points of intersection of:
(i) the tangents
(ii) the normals
to the parabola $x = t, y = \frac{1}{2}t^2$ at the points $t = 2$ and $t = -1$.

9. Find the equations of the tangents and normals to the parabola $x = 2at$, $y = at^2$ at the points $t = 1$ and $t = -1$.
   Show that these four lines form a square with diagonals intersecting at the focus.

10. The parabola $x^2 = 4y$ is cut by the straight line $3x - 2y - 4 = 0$ at P and Q.
    Find: (a) the coordinates of P and Q
    (b) the equations of the normals at P and Q
    (c) show that the normals intersect on the parabola.

11. The points A, B, C and D are the points $t = -2$, $t = -1$, $t = 2$ and $t = -\frac{1}{2}$ respectively on the parabola $x = 2t$, $y = t^2$.
    (a) (i) Find the equations of the tangents at C and D.
    (ii) Show that their point of intersection lies on the directrix.
    (b) (i) Find the equations of the normals at A and B.
    (ii) Show that their point of intersection lies on the parabola.

THE CHORD OF CONTACT OF TANGENTS DRAWN TO A PARABOLA FROM AN EXTERNAL POINT

EXERCISE 23.4

1. In each of the following:
   (a) check that the given point is external to the parabola
   (b) find the equation of the chord of contact of the tangents drawn to the parabola from the given point.
   (i) $x^2 = 4y$ from (3, 1)
   (ii) $x^2 = 12y$ from (6, 0)
   (iii) $x^2 = y$ from (3, −2)
   (iv) $x = 4t$, $y = 2t^2$ from (2, −1)
   (v) $x = 6t$, $y = 3t^2$ from (4, 1)
   (vi) $x = 2t$, $y = t^2$ from (0, −1)

2. Tangents are drawn to the parabola $x^2 = 8y$ from an external point $P(4, 0)$. Find:
   (a) the equation of the chord of contact
   (b) the coordinates of the points of contact
   (c) the equations of the two tangents from P.

3. Show that the point $P(6, -4)$ is exterior to the parabola $x^2 = 16y$. Also find:
   (a) the equation of the chord of contact of the tangents from P
   (b) the points of intersection of this chord with the parabola
   (c) the coordinates of the mid-point of the chord
   (d) show that the chord of contact is a focal chord.

4. Tangents are drawn to the parabola $x^2 = 16y$ from the points $P(0, -4)$ and $Q(2, -2)$.
   (a) Find: (i) the equation of the chord of contact of the tangents from P
   (ii) the equation of the chord of contact of the tangents from Q.
   (b) Show that the two chords intersect at a point on the given parabola.
   (c) Show that the chord of contact of the tangents from P is bisected at the focus.
   (d) Show that the normals from the ends of this chord meet on the axis of the parabola.

5. (a) Show that the point $R(-1\frac{1}{2}, -1)$ is external to the parabola $x^2 = 4y$.
   (b) Find the equation of the chord of contact of the tangents from R.
   (c) If this chord meets the parabola at P and Q, find the coordinates of P and Q.
   (d) Find the coordinates where PQ (or PQ produced) cuts
   (i) the x axis
   (ii) the y axis
   (iii) the directrix (at the point T).
   (e) If S is the focus of the parabola show that
   $\angle TSR = \angle PRQ = 90°$

6. Tangents are drawn to the parabola $x^2 = 8y$ from the point $P(1, -3)$.
   (a) Find the equation of the chord of contact.
   (b) If the points of contact are Q and R, find their coordinates.
   (c) If M is the mid-point of the chord QR show that MP is parallel to the axis of the parabola.
   (d) (i) Find the coordinates of the mid-point of PM and
   (ii) show that it lies on the given parabola.

General Geometric Properties

EXERCISE 23.5

1. P is the point $(2ap, ap^2)$ on the parabola $x^2 = 4ay$. The tangent at P meets the axis of the parabola at T and PN is drawn perpendicular to the axis, meeting it at N. The directrix meets the axis at A.
   (a) Prove: (i) $OS = OA$ (O is vertex, S the focus)
   (ii) $ON = OT$
   (iii) the x axis bisects PT at a point, say B.
   (b) On the diagram for part (a), PM is drawn perpendicular to the directrix, meeting it at M and cutting the x axis at C.
   (i) Find the equation of the line SM.
   (ii) Show that B lies on SM.
   (iii) Show that B is the mid-point of OC.
   (iv) Show that $SB^2 = SO\,.\,SP$.
   (v) Show that triangle TOB is congruent to the triangle PBC.
   (vi) What type of figure is $TMPS$?

2. At a point $P(2ap, ap^2)$ on the parabola $x^2 = 4ay$ a tangent is drawn to cut the axis of the parabola at T and the normal from P meets the axis in G.
   If S is the focus:
   (a) prove that $ST = SG$ and
   (b) if a line PN is drawn perpendicular to the axis, meeting the axis in N, prove that the length of NG is a constant length for all positions of P.

3. Prove that if the tangent from a point P on a parabola meets the directrix at A, then the angle ASP is a right angle.

4. If the chord of contact of the tangents to a parabola $x^2 = 4ay$, from an external point $T(x_1, y_1)$ meets the directrix at R, prove that RT subtends a right angle at the focus.

5. (a) If $P(2ap, ap^2)$ and $Q(2aq, aq^2)$ are points on the parabola $x = 2at, y = at^2$ and S is the focus, prove that $PS + SQ = a(p^2 + q^2 + 2)$. Hence if PQ is a focal chord, show that the length of $PQ = a\left(p + \frac{1}{p}\right)^2$.
   (b) If the focal chord PQ is parallel to the x axis (the *latus rectum*) prove that $PQ = 4a$.
   (c) Show that the gradient of a focal chord through P is $\frac{p^2 - 1}{2p}$.

6. A straight line through $T(0, -a)$ cuts the parabola $x^2 = 4ay$ at P and Q with respective parameters p and q.
   (a) Show that the equation of TP is $2py = x(p^2 + 1) - 2ap$.
   (b) Prove that, for TP to pass through $Q, pq = 1$.
   (c) Hence prove that $\frac{1}{SP} + \frac{1}{SQ} = \frac{1}{a}$.

7. P is the point $(2ap, ap^2)$ on the parabola $x^2 = 4ay$ with focus S. The normal from P meets the axis of the parabola at G and the latus rectum produced at A. PN, perpendicular to the axis of the parabola, meets it at N.
   Prove that:
   (a) $\frac{NP}{NG} = p$ (b) $\frac{SA}{SP} = \frac{NP}{NG}$.

8. The tangent from P, at one extremity of a focal chord PQ in the parabola $x^2 = 4ay$, meets the latus rectum produced at L.
   Prove that $\frac{SP}{SL} = \frac{SL}{SQ}$.

9. Prove that the normals at the ends of a focal chord in a parabola are perpendicular to each other.

10. P is a point on the parabola $x = 2at, y = at^2$. From Q, the mid-point of OP, (O is the vertex), QA is drawn parallel to the axis of the parabola and meets the x axis at A.
    Prove:
    (a) AP is a tangent at P.
    (b) K, the mid-point of AQ, lies on the given parabola.

11. The straight line drawn from a point P on a parabola to the vertex intersects the directrix at A. Prove that AS (where S is the focus) is parallel to the tangent at P.

(5)

12. PQ is a focal chord of the parabola $x^2 = 4ay$. Perpendiculars PA, QB are drawn from P, Q to meet the directrix at A, B respectively.
Prove that PB, QA intersect at the vertex O.

13. On the parabola $x^2 = 4ay$, the normal from a point P (parameter p) meets the parabola again at R (parameter r). By using the fact that $R(2ar, ar^2)$ satisfies the equation of the normal:
(a) Prove that $p^2 + pr + 2 = 0$.
(b) If the chord PR subtends a right angle at the vertex show that:
(i) $pr = -4$ (ii) $p^2 = 2$.
(c) The normal from another distinct point Q (parameter q) on the parabola also passes through R.
(i) What relationship exists between q and r?
(ii) Show that $(p^2 - q^2) + r(p - q) = 0$ and hence that $p + q + r = 0$.
(iii) Show that $(p + q)^2 + r(p + q) + 4 = 2pq$ and hence that $pq = 2$.
(iv) Prove that PQ intersects the axis of the parabola at an independent point.

14. P is a given point with parameter p on the parabola $x^2 = 4ay$, with focus S. A line is drawn from S, perpendicular to SP and meets the normal at P in the point Q. PN, QM are drawn perpendicular to the axis of the parabola.
(a) Find the ordinate at Q.
(b) If D is the intersection of the directrix and the axis of the parabola, prove that $DM = 2DN$.

15. (a) If the normal at $P\ (2ap, ap^2)$ on the parabola $x^2 = 4ay$ meets the parabola again at Q:
(i) Show that the coordinates of Q are $\left(\dfrac{-2a(p^2 + 2)}{p}, \dfrac{a(p^2 + 2)^2}{p^2}\right)$.
(ii) Prove that the length of PQ is $\dfrac{4a(1 + p^2)^{\frac{3}{2}}}{p^2}$.
(b) If PQ subtends a right angle at the focus S and M is the foot of the perpendicular from S to the tangent at P
(i) find the coordinates of M; (ii) find the lengths SM, SP.

LOCUS PROBLEMS (EXTENSION 1)

EXERCISE 23.6

1. From a point P on the parabola $x^2 = 2y$, a tangent is drawn. From the focus S, a perpendicular is drawn to meet the tangent at R.
(a) Find the equation of SR.
(b) Find the locus of R.

2. Show that the locus of the mid-points of chords in the parabola $x^2 = 4ay$, and which pass through the vertex, is another parabola, $x^2 = 2ay$.

3. Two points P, Q move on the parabola $x^2 = 4ay$ so that the x coordinates of P and Q differ by a constant value, $2a$. What is the locus of M, the mid-point of PQ?

4. Prove that the locus of the mid-point M of focal chords in the parabola $x^2 = 4ay$ is the parabola $x^2 = 2a(y - a)$.

5. P_1, P_2 are points on the parabola $x^2 = 4ay$ with parameters p and $\frac{1}{p}$. If the tangents at P_1 and P_2 intersect at R, prove that the locus of R is the line $y = a$.

6. At a point P on the parabola $x^2 = 4ay$, a normal PK is drawn. From the vertex O a perpendicular OM is drawn to meet the normal at M. Show that the equation of the locus of M as P varies on the parabola is
$$x^4 - 2ax^2y + x^2y^2 - ay^3 = 0.$$

7. The chord PQ in the parabola $x^2 = 4ay$ subtends a right angle at the vertex of the parabola. The normals to the parabola at P and Q meet at R.
(a) Prove that $pq = -4$, where p, q are the parameters at P and Q.
(b) Show that as P, Q take various positions on the parabola, the locus of R is the parabola, $x^2 = 16a(y - 6a)$.

8. (a) P and Q are points on the parabola $x = 2at, y = at^2$. The tangents at P and Q intersect at R so that the angle PRQ equals 90°. Show that the locus of R is the directrix.
(b) If PQ is a focal chord and the tangents at P and Q meet at R, show that the locus of R is the directrix, $y = -a$.

9. If PQ is a focal chord of the parabola $x^2 = 4ay$; QR is the tangent at Q and RP is parallel to the axis of the parabola, prove that the locus of R has as its equation
$$x^2(y + 2a) = -4a^3.$$

10. PQ is a focal chord in the parabola $x^2 = 4ay$. Normals are drawn at P and Q, at the ends of the focal chord to meet at a point R. Find the locus of R for varying focal chords PQ.

11. Tangents are drawn to a parabola $x^2 = 4y$ from an external point $A(x_1, y_1)$, touching the parabola at P and Q.
   (a) Prove that the mid-point, M, of PQ is the point $(x_1, \frac{1}{2}x_1^2 - y_1)$.
   (b) If A moves along the straight line $y = x - 1$, find the equation of the locus of M.

12. P is a variable point $(6p, 3p^2)$ on the parabola $x^2 = 12y$, with focus S. SL is drawn perpendicular to the tangent at P, meeting it at L. SM is drawn perpendicular to the normal at P, meeting it at M.
   (a) Find the equation of the locus of L.
   (b) Find the equation of the locus of M.

13. If PN is a normal to the parabola $x^2 = 4ay$ at a variable point P and SN is drawn through the focus S parallel to the tangent at P to cut the normal at N.
   Prove that the locus of N is $x^2 = a(y - a)$.

14. P is a point on the parabola $x^2 = 4ay$ with vertex at the origin. A straight line through O, parallel to the tangent at P intersects the parabola again at Q. If the tangents at P and Q meet at T, show that the locus of T as P moves on the parabola is given by $x^2 = \frac{9}{2}ay$.

15. PQ is a focal chord in the parabola $x^2 = 4ay$.
   (a) PT is drawn parallel to the tangent at Q and QT is drawn parallel to the tangent at P. Show that the locus of T is $x^2 = a(y - 3a)$.
   (b) If M is the mid-point of the focal chord PQ and a line through M, parallel to the axis of the parabola, meets the normal at P in A, find the locus of A.

Parametric Parabola J & C Sol^ns.

SE 23.1 (Page 100)

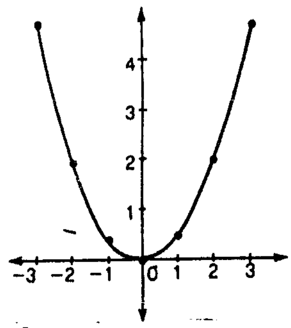
1\. (a) $a = 1; x = 2t, y = t^2$ (b) $a = 2; x = 4t, y = 2t^2$
(c) $a = \frac{1}{4}; x = \frac{1}{2}t, y = \frac{1}{4}t^2$ (d) $a = \frac{1}{2}; x = t, y = \frac{1}{2}t^2$
(e) $a = -1; x = -2t, y = -t^2$ (f) $a = -4; x = -8t, y = -4t^2$

2\. (a) $x^2 = 8y$ (b) $x^2 = 12y$ (c) $x^2 = 16y$
(d) $x^2 = y$ (e) $x^2 = -16y$ (f) $x^2 = -2y$
(g) $x^2 = -\frac{1}{2}y$ (h) $x^2 = -4ay$

3\. (a) (i)

t	-3	-2	-1	0	1	2	3
x	-3	-2	-1	0	1	2	3
y	$4\frac{1}{2}$	2	$\frac{1}{2}$	0	$\frac{1}{2}$	2	$4\frac{1}{2}$

(ii)

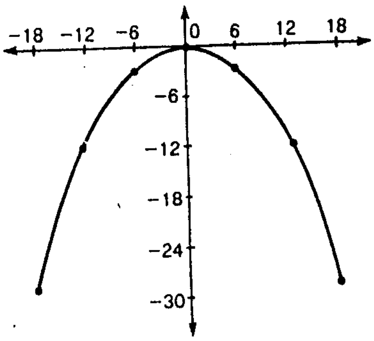


(iii) $x^2 = 2y$
(iv) $(0, 0); (0, \frac{1}{2})$
(v) $y = -\frac{1}{2}$

(b) (i)

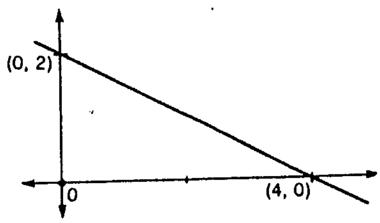
t	-3	-2	-1	0	1	2	3
x	-18	-12	-6	0	6	12	18
y	-27	-12	-3	0	-3	-12	-27

(ii)

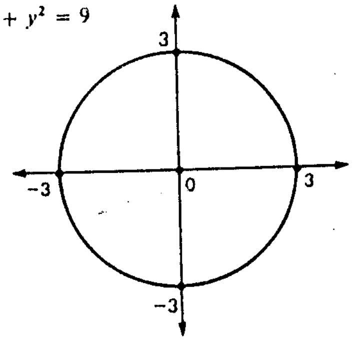


(iii) $x^2 = -12y$
(iv) $(0, 0); (0, -3)$
(v) $y = 3$

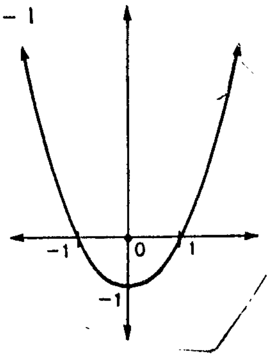
4\. (a) $x + 2y - 4 = 0$



(b) $x^2 + y^2 = 9$



(c) $y = x^2 - 1$



EXERCISE 23.2 (Page 102)

1\. (a) $x - 2y + 4 = 0$ (b) $3x - 4y + 12 = 0$
2\. (a) (i) $3x - 4y + 1 = 0$
(b) (i) $\frac{4}{3}$ (ii) $4x - 3y + 6 = 0$
(iii) $(-\frac{4}{3}, \frac{2}{3})$ (iv) $t = -\frac{1}{3}$
3\. (b) $4a$ units
4\. (b) $\frac{q}{2}$ (c) $p = -q$
5\. (a) $4x - 3y + 3a = 0$

EXERCISE 23.3 (Page 107)

1\. (a) $x + y + 1 = 0; x - y + 3 = 0$
(b) $x - 2y - 1 = 0; 4x + 2y - 9 = 0$
(c) $2x - y - 2 = 0; x + 2y - 6 = 0$
(d) $2x - y - 4 = 0; x + 2y - 12 = 0$
(e) $x + y + 2 = 0; x - y + 6 = 0$
(f) $x - y - 3 = 0; x + y - 9 = 0$
(g) $2x - y = 0; x + 2y - 5 = 0$
(h) $5x + y + 6 = 0; x - 5y + 22 = 0$
(i) $2x - y - 4a = 0; x + 2y - 12a = 0$
(j) $4px - 16y - ap^2 = 0;$
$64x + 16py - ap^3 - 32ap = 0$
2\. (a) (i) $2x + y + 2 = 0; 2x - y - 2 = 0$
(ii) $x - 2y + 6 = 0; x + 2y - 6 = 0$
(iii) $(0, -2)$ (iv) $(0, 3)$
(b) (i) $x - 2y - 2 = 0; x + y + 4 = 0$
(ii) $2x + y - 9 = 0; x - y + 12 = 0$
(iii) $(-2, -2)$ (iv) $(-1, 11)$
(c) (i) $x - y - 1 = 0; x + y + 1 = 0$
(ii) $x + y - 3 = 0; x - y + 3 = 0$
(iii) $(0, -1)$ (iv) $(0, 3)$
(d) (i) $2x - y - 12 = 0; y = 0$
(ii) $x + 2y - 36 = 0; x = 0$
(iii) $(6, 0)$ (iv) $(0, 18)$
3\. (a) $(4, 1)$ (b) $(-6, 3)$ (c) $(10, 5); t = 1.$
4\. (a) $m < -8 \cup m > 0$ (b) $m = 0, -8$
(c) $-8 < m < 0$
5\. (a) $3x - y - 27 = 0$
(b) $9x - 3y + \cancel{19}\ 109 = 0$; parallel
6\. (a) $(8, 8); (-4, 2)$
(b) $2x - y - 8 = 0; x + y + 2 = 0$
(c) $(2, -4)$
8\. (d) (i) $(\frac{1}{2}, -1)$ (ii) $(1, 2\frac{1}{2})$
9\. $y = x - a; y = -x - a; y = -x + 3a; y = x + 3a$
10\. (a) $(2, 1); (4, 4)$
(b) $x + y - 3 = 0; x + 2y - 12 = 0$
(c) $(-6, 9)$
11\. (a) (i) $2x - y - 4 = 0; 2x + 4y + 1 = 0$
(ii) $(1\frac{1}{2}, -1)$
(b) (i) $x - 2y + 12 = 0; x - y + 3 = 0$
(ii) $(6, 9)$

EXERCISE 23.4 (Page 110)

1\. (b) (i) $3x - 2y - 2 = 0$ (ii) $x - y = 0$
(iii) $6x - y + 2 = 0$ (iv) $x - 2y + 2 = 0$
(v) $2x - 3y - 3 = 0$ (vi) $y = 1$
2\. (a) $x - y = 0$ (b) $(0, 0); (8, 8)$
(c) $y = 0; 2x - y - 8 = 0$
3\. (a) $3x - 4y + 16 = 0$ (b) $(16, 16); (-4, 1)$
(c) $(6, 8\frac{1}{2})$
4\. (a) (i) $y = 4$ (ii) $x - 4y + 8 = 0$ (b) $(8, 4)$
5\. (b) $3x + 4y - 4 = 0$ (c) $(1, \frac{1}{4}); (-4, 4)$
(d) (i) $(\frac{4}{3}, 0)$ (ii) $(0, 1)$ $4\frac{1}{2}$ (iii) $(\frac{8}{3}, -1)$
6\. (a) $x - 4y + 12 = 0$ (b) $(6, 4\frac{1}{2}); (-4, 2)$
(c) $(1, 3\frac{1}{4})$ (d) (i) $(1, \frac{1}{8})$.

EXERCISE 23.5 (Page 114)

1\. (b) (i) $x + py - ap = 0$ (vi) rhombus
13\. (c) (i) $q^2 + qr + 2 = 0$
14\. (a) $(ap(1 - p^2), a(2p^2 + 1))$
15\. (b) (i) $(ap, 0)$ (ii) $a(p^2 + 1)^{\frac{1}{2}}, a(p^2 +$

EXERCISE 23.6 (Page 118)

1\. (a) $y = \dfrac{p - 2x}{2p}$ (b) $y = 0$, the x ax
3\. Parabola, $x^2 = a(4y - a)$
10\. Parabola, $x^2 = a(y - 3a)$
11\. (b) Parabola, $(x - 1)^2 = 2(y - \frac{1}{2})$
12\. (a) $y = 0$ (b) $x^2 = 3(y - 3)$
15\. (b) $x^2 = a(y - 3a)$