2.41 GENERAL LOCUS AND THE CIRCLE

1. (a) Find the equation of the locus of a point moving so that it is equidistant from the points $A(1, -3)$ and $B(2, 1)$.
   (b) Find the midpoint, P, and gradient, m, of AB.
   (c) Find the equation of the perpendicular bisector of AB.

2. (a) Find the equation of the locus of a point moving so that PA is perpendicular to PB where $A = (3, 1)$ and $B = (-5, 5)$.
   (b) Find the midpoint, C, of AB.
   (c) Find the exact length of AC.
   (d) Find the equation of the circle with centre C and radius AC.

3. Find the equation of the locus of a point moving so that $PA:PB = 3:2$ where $A = (5, 0)$ and $B = (1, -1)$.

4. Find the equation of the locus of a point moving so that it is always 3 units from the line $3x - 4y - 1 = 0$.

5. Find the equation of the circle with centre $(1, 2)$ and radius 3.

6. Find the equation of the circle with centre $(-2, 5)$ and radius $\sqrt{2}$.

7. Find the coordinates of the centre and the length of the radius of each circle:

   (a) $x^2 - 4x + y^2 - 8y - 5 = 0$
   (b) $x^2 - 6x + y^2 + 2y - 6 = 0$
   (c) $x^2 + 2x + y^2 - 10y + 1 = 0$
   (d) $x^2 + 12x + y^2 + 4y + 4 = 0$

8. Sketch the graph of $x^2 - 2x + y^2 + 4y - 4 = 0$.

GENERAL LOCUS & THE CIRCLE

1a Let $A(1,-3)$ be (x_1,y_1)
" $B(2,1)$ be (x_2,y_2)
" Pnt $M(x,y)$ lie on locus
for M to lie on locus,
$AM = BM$

ie. $\sqrt{(x-1)^2+(y+3)^2} = \sqrt{(x-2)^2+(y-1)^2}$ ✓

Squaring:
$x^2-2x+1+y^2+6y+9 = x^2-4x+4+y^2-2y+1$
ie. $2x+8y-5=0$ (general form) ✓

1b Midpnt of $A(1,-3)$ $B(2,1)$:
$P = \left(\frac{1+2}{2}, \frac{-3+1}{2}\right)$
ie. $P = (\frac{3}{2}, -1)$ ✓

∴ Gradient $= \frac{y_2-y_1}{x_2-x_1} = \frac{1+3}{2-1} = 4$ ✓
∴ $m = 4$ ✓

1c $m_{AB} = 4$
∴ $m_{\text{perp. bisector}} = -\frac{1}{4}$ ✓
∴ Equ. perp. bisector is
$y+1 = -\frac{1}{4}(x-3/2)$ ✓
ie. $4y+4 = -x+3/2$ ✓
ie. $2x+8y+5=0$ (general form) ✓

2a A is $(3,1)$, B is $(-5,5)$
Let $P(x,y)$ lie on locus where $PA \perp PB$
$m_{PA} = \frac{y-1}{x-3} = m_1$
$m_{PB} = \frac{y-5}{x+5} = m_2$
$m_1 m_2 = -1$ for $PA \perp PB$ ✓
ie. $\left(\frac{y-1}{x-3}\right)\left(\frac{y-5}{x+5}\right) = -1$
ie. $(y-1)(y-5) = -(x-3)(x+5)$
ie. $y^2-6y+5 = -x^2-2x+15$
ie. $x^2+2x+y^2-6y = 10$ ✓
completing the square:
ie. $x^2+2x+1+y^2-6y+9 = 10+1+9$
ie. $(x+1)^2+(y-3)^2 = 20$ ✓

1b Midpnt $A(3,1)$ $B(-5,5)$:
$C = \left(\frac{3-5}{2}, \frac{1+5}{2}\right)$
ie. $C = (-1,3)$ ✓

1c Distance $A(3,1)$ $C(-1,3)$:
$d = \sqrt{(3+1)^2+(1-3)^2}$
$= \sqrt{16+4} = \sqrt{20} = 2\sqrt{5}$ units. ✓

1d Equ. of circle centre $C(-1,3)$, radius $AC = 2\sqrt{5}$:
$(x+1)^2+(y-3)^2 = (2\sqrt{5})^2$
ie. $(x+1)^2+(y-3)^2 = 20$. ✓

3 A is $(5,0)$ B is $(1,-1)$
$PA:PB = 3:2$
ie. $\frac{PA}{PB} = \frac{3}{2} \Rightarrow \frac{PA^2}{PB^2} = \frac{9}{4}$
ie. $9[(x-1)^2+(y+1)^2] = 4[(x-5)^2+(y-0)^2]$ ✓
ie. $9(x^2-2x+1+y^2+2y+1) = 4(x^2-10x+25+y^2)$
ie. $9x^2-18x+9+9y^2+18y+9 = 4x^2-40x+100+4y^2$
ie. $5x^2+22x+5y^2+18y-82=0$ ✓
ie. $x^2+\frac{22x}{5}+y^2+\frac{18}{5}y = \frac{82}{5}$ ✓
ie. $\left(x^2+\frac{22}{5}x+\frac{121}{25}\right)+\left(y^2+\frac{18}{5}y+\frac{81}{25}\right) = \frac{82}{5}+\frac{121}{25}+\frac{81}{25}$
ie. $\left(x+\frac{11}{5}\right)^2+\left(y+\frac{9}{5}\right)^2 = \frac{612}{25}$ ✓

4 $3x-4y-1=0$
Let $P(x,y)$ lie on locus.
The condition that P lie on the locus
$\frac{|ax_1+by_1+c|}{\sqrt{a^2+b^2}} = 3$ [perp. dist. formula]
ie. $\frac{|3x-4y-1|}{\sqrt{3^2+4^2}} = 3$
ie. $\frac{|3x-4y-1|}{5} = 3$ ✓
ie. $|3x-4y-1| = 15$
ie. $3x-4y-1 = 15$ ✓ or $-(3x-4y-1) = 15$ ✓
ie. $3x-4y-16=0$ ie. $3x-4y+14=0$
∴ Locus is $3x-4y-16=0$ or

5] Equ. of circle centre $(1,2)$, $r=3$

$(x-1)^2+(y-2)^2=9$. ✓

6] Equ. of circ. centre $(-2,5)$, $r=\sqrt{2}$

$(x+2)^2+(y-5)^2=2$ ✓

7a] $x^2-4x+y^2-8y-5=0$

ie. $x^2-4x+4+y^2-8y+16=5+4+16$

ie. $(x-2)^2+(y-4)^2=25$ ✓

∴ centre is $(2,4)$ ✓, radius is 5 units. ✓

b] $x^2-6x+y^2+2y-6=0$

ie. $x^2-6x+9+y^2+2y+1=6+9+1$

ie. $(x-3)^2+(y+1)^2=16$ ✓

∴ centre is $(3,-1)$ ✓, radius is 4 units. ✓

c] $x^2+2x+y^2-10y+1=0$

ie. $x^2+2x+1+y^2-10y+25=-1+1+25$

ie. $(x+1)^2+(y-5)^2=25$ ✓

∴ centre is $(-1,5)$ ✓, radius is 5 units. ✓

d] $x^2+12x+y^2+4y+4=0$

ie. $x^2+12x+36+y^2+4y+4=-4+36+4$

ie. $(x+6)^2+(y+2)^2=36$ ✓

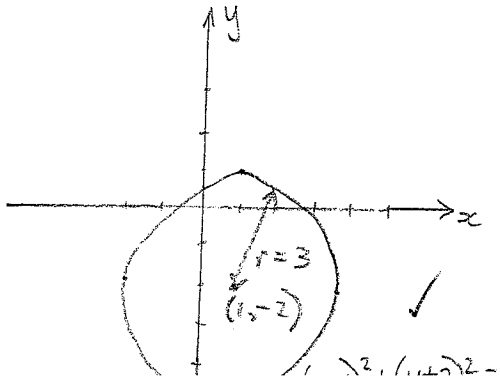
∴ centre is $(-6,-2)$ ✓, radius is 6 units ✓

9] $x^2-2x+y^2+4y-4=0$

ie. $x^2-2x+1+y^2+4y+4=4+1+4$

ie. $(x-1)^2+(y+2)^2=9$ ✓

∴ centre of circle is $(1,-2)$ ✓
radius is 3 units. ✓



✓

$(x-1)^2+(y+2)^2=9$