Quadratics

1 Factorise each of the following

a $x^2 + 3x + 2$	**b** $x^2 + 5x + 4$	**c** $x^2 + 4x + 4$
d $x^2 + 6x + 8$	**e** $y^2 + 13y + 36$	**f** $x^2 - 4x + 3$
g $x^2 - 7x + 10$	**h** $x^2 - 11x + 10$	**i** $a^2 + 2a - 3$
j $y^2 + 4y - 60$	**k** $x^2 - x - 6$	**l** $p^2 + 9p + 20$
m $x^2 - 3x - 18$	**n** $x^2 - 9$	**o** $36 - 12a + a^2$
p $m^2 - 49$	**q** $70 + 17x + x^2$	**r** $y^2 + y - 30$

2 Factorise to solve each of these equations.

a $x^2 + 4x + 3 = 0$	**b** $x^2 - 2x + 1 = 0$	**c** $y^2 + 2y - 35 = 0$
d $x^2 - 18x + 17 = 0$	**e** $12 + 7a + a^2 = 0$	**f** $45 + 4x - x^2 = 0$
g $x^2 - 81 = 0$	**h** $m^2 + m - 56 = 0$	**i** $36 - 15x + x^2 = 0$

3 Factorise each of the following

a $2x^2 + 3x + 1$	**b** $3y^2 - 5y - 2$	**c** $2x^2 + 11x + 12$
d $5p^2 - 12p + 4$	**e** $1 + 7x + 6x^2$	**f** $9y^2 - 1$
g $4x^2 - 7x + 3$	**h** $4a^2 + 4a + 1$	**i** $2 - 2x - 12x^2$

4 Factorise to solve each of these equations.

a $5y^2 + 6y + 1 = 0$	**b** $3x^2 - 16x + 5 = 0$	**c** $3 - x - 2x^2 = 0$
d $x^2 + 2x = 3$	**e** $3a + 4 = a^2$	**f** $x(x - 4) = 2x - 8$
g $6 - 11m - 10m^2 = 0$	**h** $(2x - 3)^2 + 3x = 7$	**i** $9x(3 - 2x) = 10(9x + 4)$

5 Express each of the following in the form $(x + a)^2 + b$.

a $x^2 + 2x + 3$	**b** $x^2 + 8x - 1$	**c** $x^2 - 4x + 5$
d $x^2 + 3x + 1$	**e** $x^2 - 7x - 2$	**f** $x^2 + 16x$
g $x^2 + x + \frac{1}{2}$	**h** $3 - 10x + x^2$	**i** $x^2 + \frac{2}{3}x + \frac{1}{6}$

6 Express each of the following in the form $a(x + b)^2 + c$.

a $2x^2 + 4x + 1$	**b** $3x^2 - 9x - 2$	**c** $^-x^2 + 6x + 4$
d $5x^2 + 30x - 19$	**e** $2x^2 - 3x + 5$	**f** $6x^2 + x - 1$
g $4 - 3x - x^2$	**h** $4x^2 + 20x$	**i** $8 + 7x - 2x^2$

7 Complete the square and solve each equation. Leave answers in surd form where appropriate.

a $x^2 + 4x + 3 = 0$	**b** $y^2 - 14y + 40 = 0$	**c** $x^2 + 2x = 1$
d $18 - 10p + p^2 = 0$	**e** $2x^2 + 12x + 17 = 0$	**f** $7 + 4x - x^2 = 0$

8 Use the formula to solve each equation. Give answers correct to 2 dp where appropriate.

a $x^2 + 11x - 152 = 0$	**b** $x^2 - 2x - 3 = 0$	**c** $3x^2 + 5x + 1 = 0$
d $x(x - 3) = 5x + 4$	**e** $7m^2 + m = 3$	**f** $(3y + 1)^2 = 2 - 5y$

9 Use the formula to solve each equation. Leave answers in surd form where appropriate.

a $x^2 - 16x + 63 = 0$	**b** $3x^2 + 5x + 1 = 0$	**c** $y(3 - y) = 1$
d $12 - (2a - 5)^2 = 3a$	**e** $2x^2 - 16x + 23 = 0$	**f** $(7 - 3x)(x + 4) = 18$

10 Solve each of the following equations.

a $x + 3 - \frac{10}{x} = 0$ b $2y - \frac{4}{y} = 7$ c $x^4 - 10x^2 + 16 = 0$

d $p^{-2} + 2p^{-1} - 15 = 0$ e $\frac{3}{x-4} + 3 = 2x$ f $\frac{3x-1}{x+2} = \frac{4}{x}$

11 By evaluating the discriminant, determine for each equation whether it would have real and distinct roots, real repeated roots, or no real roots.

a $x^2 - x + 3 = 0$ b $x^2 + 6x + 1 = 0$ c $2x^2 + 2x - 5 = 0$

d $x^2 + 10x + 25 = 0$ e $6x^2 - 7x + 3 = 0$ f $16x^2 - 88x + 121 = 0$

12 For each graph find the coordinates of any points where it crosses or touches the coordinate axes. Give values correct to 2 dp where appropriate.

a $y = x^2 + 9x + 8$ b $y = 20 + x - x^2$ c $y = 2x^2 + 3x - 21$

d $y = x^2 - 4x + 5$ e $y = 2x^2 + 5x$ f $y = 5 + 7x - 2x^2$

13 Find the coordinates of the turning point of each of the following graphs. State whether the y-coordinate is a maximum or minimum value for the curve.

a $y = x^2 - 4x + 3$ b $y = x^2 + 2x + 7$ c $y = 7 + 6x - x^2$

d $y = 4x^2 + 24x + 11$ e $y = x^2 - 9x + 15$ f $y = 5 - 3x - x^2$

14 Sketch each graph. Show the coordinates of any turning points and the points of intersection with the x or y axes. Give non-exact answers correct to 2 dp.

a $y = x^2 + 6x + 8$ b $y = x^2 + 8x - 9$ c $y = x^2 - x - 6$

d $y = x^2 - 18x + 32$ e $y = 5 - 4x - x^2$ f $y = x^2 - 6x + 12$

g $y = x^2 - 3x + 1$ h $y = x^2 + 8x + 16$ i $y = 2 - 5x - x^2$

j $y = 2x^2 - 9x + 9$ k $y = 9x^2 - 8$ l $y = 7x^2 + x + 3$

Exercise 3S   Skills Practice

1 a $(x+2)(x+1)$ b $(x+4)(x+1)$ c $(x+2)^2$
d $(x+2)(x+4)$ e $(y+4)(y+9)$ f $(x-1)(x-3)$
g $(x-5)(x-2)$ h $(x-10)(x-1)$
i $(a+3)(a-1)$ j $(y+10)(y-6)$
k $(x-3)(x+2)$ l $(p+5)(p+4)$ m $(x-6)(x+3)$
n $(x+3)(x-3)$ o $(a-6)^2$ p $(m+7)(m-7)$
q $(x+10)(x+7)$ r $(y+6)(y-5)$

2 a -3, -1 b 1 c -7, 5 d 1, 17 e -3, -4
f -5, 9 g -9, 9 h -8, 7 i 3, 12

3 a $(2x+1)(x+1)$ b $(3y+1)(y-2)$
c $(2x+3)(x+4)$ d $(5p-2)(p-2)$
e $(6x+1)(x+1)$ f $(3y+1)(3y-1)$
g $(4x-3)(x-1)$ h $(2a+1)^2$
i $2(1+2x)(1-3x)$

4 a $-1, -\frac{1}{5}$ b $\frac{1}{3}, 5$ c $-\frac{3}{2}, 1$
d -3, 1 e -1, 4 f 2, 4
g $-\frac{3}{2}, \frac{2}{5}$ h $\frac{1}{4}, 2$ i $-\frac{8}{3}, -\frac{5}{6}$

5 a $(x+1)^2+2$ b $(x+4)^2-17$ c $(x-2)^2+1$
d $(x+\frac{3}{2})^2-\frac{5}{4}$ e $(x-\frac{7}{2})^2-\frac{57}{4}$
f $(x+8)^2-64$ g $(x+\frac{1}{2})^2+\frac{1}{4}$
h $(x-5)^2-22$ i $(x+\frac{1}{3})^2+\frac{1}{18}$

6 a $2(x+1)^2-1$ b $3(x-\frac{3}{2})^2-\frac{35}{4}$
c $-(x-3)^2+13$ d $5(x+3)^2-64$
e $2(x-\frac{3}{4})^2+\frac{31}{8}$ f $6(x+\frac{1}{12})^2-\frac{25}{24}$
g $-(x+\frac{3}{2})^2+\frac{25}{4}$ h $4(x+\frac{5}{2})^2-25$
i $-2(x-\frac{7}{4})^2+\frac{113}{8}$

7 a -3, -1 b 4, 10 c $-1\pm\sqrt{2}$
d $5\pm\sqrt{7}$ e $-3\pm\frac{1}{2}\sqrt{2}$ f $2\pm\sqrt{11}$

8 a -19, 8 b -1, 3 c -1.43, -0.23
d -0.47, 8.47 e -0.73, 0.59 f -1.31, 0.08

9 a 7, 9 b $\frac{1}{6}(-5\pm\sqrt{13})$ c $\frac{1}{2}(3\pm\sqrt{5})$
d 1, $\frac{13}{4}$ e $4\pm\frac{3}{2}\sqrt{2}$ f $\frac{1}{6}(-5\pm\sqrt{145})$

10 a -5, 2 b $-\frac{1}{2}, 4$ c $\pm\sqrt{2}, \pm2\sqrt{2}$
d $-\frac{1}{5}, \frac{1}{3}$ e 1, $\frac{9}{2}$ f -1, $\frac{8}{3}$

11 a -11, no real b 32, real distinct
c 44, real distinct d 0, real repeated
e -23, no real f 0, real repeated

12 a (-8, 0), (-1, 0), (0, 8)
b (-4, 0), (5, 0), (0, 20)
c (-4.08, 0), (2.58, 0), (0, -21)
d (0, 5) e $(-\frac{5}{2}, 0)$, (0, 0)
f (-0.61, 0), (4.11, 0), (0, 5)

13 a (2, -1) min b (-1, 6) min
c (3, 16) max d (-3, -25) min
e $(\frac{9}{2}, -\frac{21}{4})$ min f $(-\frac{3}{2}, \frac{29}{4})$ max

14 a (-4, 0), (-2, 0), (0, 8), (-3, -1)
b (-9, 0), (1, 0), (0, -9), (-4, -25)
c (-2, 0), (3, 0), (0, -6), $(\frac{1}{2}, -\frac{25}{4})$
d (2, 0), (16, 0), (0, 32), (9, -49)
e (-5, 0), (1, 0), (0, 5), (-2, 9)
f (0, 12), (3, 3)
g (0.38, 0), (2.62, 0), (0, 1), $(\frac{3}{2}, -\frac{5}{4})$
h (-4, 0), (0, 16)
i (-5.37, 0), (0.37, 0), (0, 2), $(-\frac{5}{2}, \frac{33}{4})$
j $(\frac{3}{2}, 0)$, (3, 0), (0, 9), $(\frac{9}{4}, -\frac{9}{8})$
k (-0.94, 0), (0.94, 0), (0, -8)
l (0, 3), $(-\frac{1}{14}, \frac{83}{28})$