EXERCISE 6D

PROBABILITY INVOLVING PERMUTATIONS AND COMBINATIONS

1. A bag contains 4 black balls and 3 white balls. Two balls are withdrawn without replacement. Find the probability of drawing 2 black balls.

2. The letters of the word TRIPLE are arranged randomly in a row. Find the probability that the P and the L are together in the six letter word thus formed.

3. Four girls and four boys arrange themselves in a row. What is the probability that the girls and boys occupy alternate positions ?

4. Your mathematics tutorial class consists of 16 students. You and one of your friends are in this class. Three students are selected at random to write on the blackboard.

    (a) What is the probability that you will be selected ?
    b) What is the probability that you will be selected but your friend will not ?

5. Five cards are selected at random from a pack of 52 playing cards. What is the probability that they are all hearts ?

6. From a group of 7 pupils and 5 teachers, a random selection of 7 is made. What is the probability that it contains, at least, 4 pupils ?

7. A committee of 4 men and 3 women is chosen from 7 men and 6 women. What is the probability that it contains a particular man and a particular woman?

8. Eight people, of whom X and Y are two, are arranged, at random, in a straight line. What is the probability that

    (a) X and Y are next to each other;
    (b) X and Y occupy the end positions;
    (c) There are, at least, 3 people between X and Y.

9. The letters of the word potato are arranged, at random, in a row. What is the probability that

    (a) The two letters T are together;
    (b) The two letters T occupy the end positions ?

10. The numbers 1,2,3,4,5 are used to form 5-digit numbers with no repetition of digits. What is the probability that the 2 and 3 will be next to each other ?

ANSWERS

1. $\dfrac{2}{7}$

2. $\dfrac{1}{3}$

3. $\dfrac{1}{35}$

4. (a) $\dfrac{3}{16}$ (b) $\dfrac{13}{80}$

5. $\dfrac{33}{66640}$

6. $\dfrac{149}{198}$

7. $\dfrac{2}{7}$

8. (a) $\dfrac{1}{4}$ (b) $\dfrac{1}{28}$ (c) $\dfrac{5}{14}$

9. (a) $\dfrac{1}{3}$ (b) $\dfrac{1}{15}$

10. $\dfrac{2}{5}$

Probability involving m / combs

1. 4 — Black   3 — W.

$P(BB) = \dfrac{4}{7} \times \dfrac{3}{6} = \dfrac{2}{7}$ ✓

2) — — — — — — #

$P(PL) = \dfrac{2! \times 5 \times 4!}{6!}$ ✓

$= \dfrac{1}{3}$ ✓

3. 4 girls, 4 boys.

$\underline{\quad}_{G} \; \underline{\quad}_{B} \; \underline{\quad}_{G} \; \underline{\quad}_{B} \; \underline{\quad} \; \underline{\quad} \; \underline{\quad} \; \underline{\quad}$

$\Rightarrow \dfrac{(4 \times 3 \times 2 \times 1) \times \textcircled{2} \times 2}{8!}$ ✓

Boys² B.G.

$= \dfrac{1}{35}$ ✓

4) 16.

a) comb. $= 16c_3$. $\dfrac{3}{16}$ ✓

b) — P $\tilde{P}$ > No. of ways. we can be selected with friend.

$1 - \dfrac{3c_2}{16c_3} = \quad ?$

$\dfrac{1 \times {}^{14}C_2}{{}^{16}C_3} = \dfrac{14 \times 13}{2 \times 1} \times \dfrac{3 \times 2 \times 1}{16 \times 15 \times 14}_5$

$= \dfrac{13}{80}$

5. $\left(\dfrac{1}{4} \times \dfrac{12}{51} \times \dfrac{11}{50} \times \dfrac{10}{49} \times \dfrac{9}{48} \right) \times 5!$

$= \boxed{\dfrac{33}{66640}}$ ✓

OR; $\dfrac{13 c_5}{52 c_5} = \dfrac{33}{66640}$

6. 7 p, 5 t.

$P(4) + P(5) + P(6) + P(7)$.

$\dfrac{(7c_4 \times 5c_3) + (7c_5 \cdot 5c_2) + (7c_6 \cdot 5c_1) + (7c_7)}{12c_7}$ ✓

$= \dfrac{149}{198}$ ✓

7 m ; 6 w.

7. 4 men. 3 wom.

$\dfrac{1 \times {}^{6}C_3 \times 1 \times {}^{5}C_2}{{}^{7}C_4 \times {}^{6}C_3}$

$= \dfrac{4 \times 3}{2 \times 1} \times \dfrac{5 \times 4 \times 3 \times 2 \times 1}{7 \times 6 \times 5 \times 4 \times 3}$

$= \dfrac{2}{7}$

a)

_____

a) No. of ways XY can be arranged together:

$$\frac{2! \times 7 \times 6!}{8!} = \frac{1}{4} \checkmark$$

b)

$$X \quad \underline{6 \times 5 \times 4 \times 3 \times 2 \times 1} \quad Y.$$

$$\frac{2 \times 6!}{8!} = \frac{1}{28}. \checkmark$$

c) $1 - \left[P(2) \, pp! + P(1) + \left(\frac{1}{4}\right)^{\text{from a}} \right].$

$$\underbrace{X \quad \underline{\quad} \quad Y}_{1 \text{ six other } 8!} \quad \underbrace{X \quad \underline{\quad} \quad Y}_{?}$$

$(2! \times 6 \times 5!) +$

$+$

$(2! \times 6 \times 5! \times 6) + (2! 6! \times 4)$ ⁵

$+ (2! \times 7 \times 6!)$

$= 25920.$

e) $P(<3)$ between X & Y No one $\boxed{XY}$ __ __
1 person

$X \, \underline{\quad} \, Y \, \boxed{5.} \quad + \quad 2! \times 7!$

$2! \times 6c_1 \times 5!. = 1440$

2 persons.

$X \, \underline{\quad} \, \underline{\quad} \, Y. \, \boxed{4}$

$= 2! \, 6c_2 \times 4! \qquad = 720.$

$\text{Total} = 2160$

$p = \frac{2160}{8!}$

$= \frac{3}{56}.$

(c) $X \, \underline{\quad} \, Y \, \underline{\quad} = 2 \times {}^6c_3 \times 4 \times 3! \times 3!$

$+$

$X \, \underline{\quad} \, Y \, \underline{\quad} = 2 \times {}^6c_4 \times 3 \times 4! 2!$

$+$

$X \, \underline{\quad} \, Y \, \underline{\quad} = 2 \times {}^6c_5 \times 2 \times 5!$

$+ 2 \times 6!$

9. a) $P \, O \, T_1 \, A \, T_2 \, O. \approx \frac{14400}{40320}$

$\boxed{T_1 T_2} \, \underline{\quad} = \frac{5}{14}$

No. of ways TT together:

$$\frac{2! \times 5 \times 4!}{6!} = \frac{2 \times 5!}{6 \times 5!} \checkmark$$

$$= \frac{1}{3} \checkmark$$

b) $T_1 \, \underline{\quad} \, T_2$

$$= \frac{(2! \times 4!)}{6!} = \frac{1}{15} \checkmark$$

10) $1, \boxed{2, 3}, 4, 5.$

$\boxed{\underline{\quad}} \, \underline{\quad} \, \underline{\quad} \, \underline{\quad}.$

$P(2,3) = \frac{2! \times 3! \times 5}{5!}$

$= \frac{2 \times 4 \times 3!}{5!}$

$= \frac{1}{10} \times \frac{2}{5}.$

$= \frac{2}{5} \checkmark$