Exercise 6E Exam Practice

1 $$f(x) \equiv x^3 - 8x^2 + 17x - 10$$

a Show that $(x - 5)$ is a factor of $f(x)$. **(2 marks)**

b Hence, or otherwise, solve the equation $f(x) = 0$. **(5 marks)**

2 Given that

$$(x^2 + Ax + 3)^2 \equiv x^4 + Bx^3 + Cx^2 - 12x + 9,$$

find the values of the constants A, B and C. **(6 marks)**

3 $$f(x) \equiv 2x^3 + ax^2 + bx - 18.$$

Given that $(x + 3)$ and $(x - 2)$ are factors of $f(x)$,

a show that $a = 5$ and $b = {}^{-}9$, **(5 marks)**

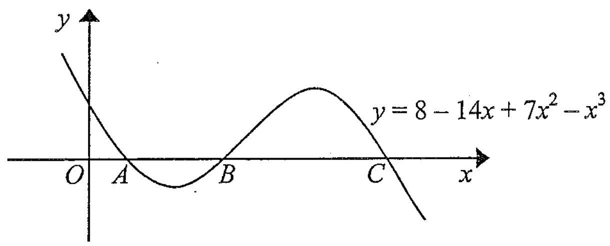
b fully factorise $f(x)$. **(5 marks)**

4 $$f(x) \equiv x^3(x + \tfrac{3}{x})(1 - \tfrac{2}{x^2})$$

a Show that $f(x)$ can be expressed in the form $(x^2 + A)(x^2 + B)$, where A and B are integers to be found. **(6 marks)**

b Hence solve the equation $f(x) = 0$. **(3 marks)**

5



The diagram shows the curve $y = 8 - 14x + 7x^2 - x^3$ which crosses the x-axis at the points A, B and C.

Given that A is the point $(1, 0)$,

a state one linear factor of the expression $8 - 14x + 7x^2 - x^3$, **(1 mark)**

b find the coordinates of the points B and C. **(6 marks)**

6 $$f(x) \equiv x^3 - x^2 + kx + 4$$

Given that $f(4) = 5f(2)$,

a find the value of k, **(4 marks)**

b show that $x = {}^{-}1$ is a solution of the equation $f(x) = 0$, **(1 mark)**

c show that the equation $f(x) = 0$ has no other real solutions. **(5 marks)**

Exercise 6E Exam Practice

1 **b** 1, 2, 5

2 $A = -2$, $B = -4$, $C = 10$

3 **b** $(x + 3)(2x + 3)(x - 2)$

4 **a** $(x^2 + 3)(x^2 - 2)$ **b** $\pm\sqrt{2}$

5 **a** $(x - 1)$ **b** $(2, 0)$, $(4, 0)$

6 **a** 2

EXERCISE 6E POLYNOMIALS

Qu 1 (a) $f(5) = 125 - 8(25) + 17(5) - 10$

$= 0$ ✓

$\therefore (x-5)$ is factor of $f(x)$ since no remainder

(b) sub $x = 1$,

$f(1) = 1 - 8 + 17 - 10$

$= 0$ ✓

$\therefore (x-1)$ is also factor.

considering coefficient of leading term & constant term,

$f(x) = (x-5)(x-1)(x-2)$

$\therefore (x-5)(x-1)(x-2) = 0$ ✓

$x = 1, 2, 5$

Qu 2 RHS: $\cancel{x^4} + \cancel{Ax^3} + \cancel{3x^2} + \cancel{Ax^3} \ldots + 3Ax$

$+ 3x^2 + 3Ax + 3$

$= x^4 + (2A)x^3 + (6 + A^2)x^2 + 6Ax + 9$

equating to left hand side,

$2A = B$ —— ①

$6 + A^2 = C$ —— ②

$6A = -12$ —— ③

$\therefore A = -2$ ✓ from ③ sub into ①

①: $-4 = B$ ✓

sub $A = -2$ into ②: $C = 10$ ✓

$\therefore A = -2$

$B = -4$ ✓

$C = 10$

Qu 3 (a) $f(-3) = 2(-27) + 9a - 3b - 18$

$0 = -54 + 9a - 3b - 18$

$72 = 9a - 3b$ ✓

$24 = 3a - b$ —— ①

$f(2) = 2(8) + 4a + 2b - 18$

$0 = -2 + 4a + 2b$

$1 = 2a + b$ ✓ —— ②

① + ②: $25 = 5a \therefore a = 5$

sub into ②: $1 = 10 + b$ ✓

$b = -9$

(b) $f(x) = 2x^3 + 5x^2 - 9x - 18$

$$
\begin{array}{r}
2x + 3 \\
x^2 + x - 6 \overline{)\, 2x^3 + 5x^2 - 9x - 18} \\
\underline{2x^3 + 2x^2 - 12x} \\
3x^2 + 3x - 18 \\
\underline{3x^2 + 3x - 18} \\
0
\end{array}
$$ ✓

$\therefore f(x) = (x+3)(x-2)(2x+3)$ ✓

Qu 4 (a) $f(x) = (x^4 + 3x^2)\left(1 - \frac{2}{x^2}\right)$

$= x^2(x^2+3)\left(1 - \frac{2}{x^2}\right)$

$= (x^2+3)(x^2-2)$ ✓

(b) $(x^2+3)(x^2-2) = 0$

$x^2 = -3$ (no real soln)

$x^2 = 2$ ✓

$\therefore x = \pm\sqrt{2}$

Qu5. (a) since A is a zero, 1 linear factor is $(x-1)$ ✓

(b)
$$\begin{array}{r} -x^2+6x-8 \\ x-1\,\overline{)\,-x^3+7x^2-14x+8} \\ \underline{-x^3+x^2} \\ 6x^2-14x \\ \underline{6x^2-6x} \\ -8x+8 \\ \underline{-8x+8} \\ 0 \end{array}$$
✓

$\therefore \quad y = (x-1)(-x+2)(x-4)$

$\therefore \quad B\ (2,0)$ ✓

$C(4,0)$ ✓

(c)
$$\begin{array}{r} x^2-2x+4 \\ x+1\,\overline{)\,x^3-x^2+2x+4} \\ \underline{x^3+x^2} \\ -2x^2+2x \\ \underline{-2x^2-2x} \\ 4x+4 \\ \underline{4x+4} \\ 0 \end{array}$$
✓

$\therefore f(x) = (x+1)(x^2-2x+4)$

for $f(x)$ to have any other zeroes,

$x^2-2x+4=0$

$\Delta = 4-4(4)$ ✓

$= -12 \quad \therefore$ no real soln

$\therefore$ no other real soln

Qu6 (a) $f(4) = 64-16+4k+4$

$= 52+4k$

$5f(2) = 5(8-4+2k+4)$

$= 40+10k$ ✓

equating the 2 eqns gives:

$52+4k = 40+10k$

$12 = 6k$ ✓

$\therefore k=2$

(b) $f(x) = x^3-x^2+2x+4$

$f(-1) = -1-1+(-2)+4$ ✓

$= -4+4 = 0$ ✓

$\therefore x=-1$ is soln of eqn $f(x)=0$

Qu5. (a) since A is a zero, 1 linear factor is $(x-1)$ ✓

(b)
$$\begin{array}{r} -x^2+6x-8 \\ x-1 \overline{)\,-x^3+7x^2-14x+8} \\ -x^3+x^2 \\ \hline 6x^2-14x \\ 6x^2-6x \\ \hline -8x+8 \\ -8x+8 \\ \hline 0 \end{array}$$
✓

$\therefore \quad y=(x-1)(-x+2)(x-4)$

$\therefore \quad B\ (2,0)$ ✓

$C(4,0)$ ✓

(c)
$$\begin{array}{r} x^2-2x+4 \\ x+1 \overline{)\,x^3-x^2+2x+4} \\ x^3+x^2 \\ \hline -2x^2+2x \\ -2x^2-2x \\ \hline 4x+4 \\ 4x+4 \\ \hline 0 \end{array}$$
✓

$\therefore f(x)=(x+1)(x^2-2x+4)$

for $f(x)$ to have any other zeroes,

$x^2-2x+4=0$

$\Delta = 4-4(4)$ ✓

$= -12 \quad \therefore$ no real soln

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$f(-1) = -1-1+(-2)+4$ ✓

$= -4+4 = 0$ ✓

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$x = 1, 2, 5$

Qu 2 RHS: $\cancel{x^4} + \cancel{Ax^3} + \cancel{3x^2} + \ldots + 3Ax$

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$= x^4 + (2A)x^3 + (6 + A^2)x^2 + 6Ax + 9$

equating to left hand side,

$2A = B$ — (1)

$6 + A^2 = C$ — (2)

$6A = -12$ — (3)

$\therefore A = -2$ from (3) sub into (1)

(1): $-4 = B$

sub $A = -2$ into (2): $C = 10$

$\therefore A = -2$

$B = -4$

$C = 10$

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$f(2) = 2(8) + 4a + 2b - 18$

$0 = -2 + 4a + 2b$

$1 = 2a + b$ — (2)

(1) + (2) $25 = 5a \therefore a = 5$

sub into (2): $1 = 10 + b$

$b = -9$

(b) $f(x) = 2x^3 + 5x^2 - 9x - 18$

$$
\begin{array}{r}
2x + 3 \\
x^2 + x - 6 \,\big)\, 2x^3 + 5x^2 - 9x - 18 \\
2x^3 + 2x^2 - 12x \\
\hline
3x^2 + 3x - 18 \\
3x^2 + 3x - 18 \\
\hline
0
\end{array}
$$

$\therefore f(x) = (x+3)(x-2)(2x+3)$

Qu 4 (a) $f(x) = (x^4 + 3x^2)\left(1 - \frac{2}{x^2}\right)$

$= x^2(x^2+3)\left(1 - \frac{2}{x^2}\right)$

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$x^2 = -3$ (no real soln)

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