

**Exercise 6E Exam Practice**

- 1  $f(x) \equiv x^3 - 8x^2 + 17x - 10$   
 a Show that  $(x - 5)$  is a factor of  $f(x)$ . (2 marks)  
 b Hence, or otherwise, solve the equation  $f(x) = 0$ . (5 marks)

- 2 Given that

$$(x^2 + Ax + 3)^2 \equiv x^4 + Bx^3 + Cx^2 - 12x + 9,$$

find the values of the constants  $A$ ,  $B$  and  $C$ . (6 marks)

- 3  $f(x) \equiv 2x^3 + ax^2 + bx - 18$ .

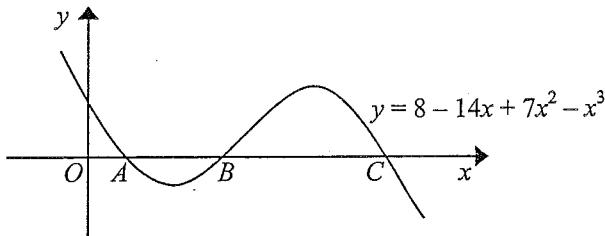
Given that  $(x + 3)$  and  $(x - 2)$  are factors of  $f(x)$ ,

- a show that  $a = 5$  and  $b = -9$ , (5 marks)  
 b fully factorise  $f(x)$ . (5 marks)

- 4  $f(x) \equiv x^3(x + \frac{3}{x})(1 - \frac{2}{x^2})$

- a Show that  $f(x)$  can be expressed in the form  $(x^2 + A)(x^2 + B)$ , where  $A$  and  $B$  are integers to be found. (6 marks)  
 b Hence solve the equation  $f(x) = 0$ . (3 marks)

- 5



The diagram shows the curve  $y = 8 - 14x + 7x^2 - x^3$  which crosses the  $x$ -axis at the points  $A$ ,  $B$  and  $C$ .

Given that  $A$  is the point  $(1, 0)$ ,

- a state one linear factor of the expression  $8 - 14x + 7x^2 - x^3$ , (1 mark)  
 b find the coordinates of the points  $B$  and  $C$ . (6 marks)

- 6  $f(x) \equiv x^3 - x^2 + kx + 4$

Given that  $f(4) = 5f(2)$ ,

- a find the value of  $k$ , (4 marks)  
 b show that  $x = -1$  is a solution of the equation  $f(x) = 0$ , (1 mark)  
 c show that the equation  $f(x) = 0$  has no other real solutions. (5 marks)

**Exercise 6E Exam Practice**

- 1 b 1, 2, 5  
 2 A = -2, B = -4, C = 10  
 3 b  $(x + 3)(2x + 3)(x - 2)$   
 4 a  $(x^2 + 3)(x^2 - 2)$  b  $\pm\sqrt{2}$   
 5 a  $(x - 1)$  b  $(2, 0), (4, 0)$   
 6 a 2

# EXERCISE 6 POLYNOMIALS



TOPIC

Ques 1 (a)  $f(x) = 12x^3 - 8x^2 + 7x - 10$

$$= 0 \quad \checkmark$$

$\therefore (x-5)$  is factor of  $f(x)$  since  $x=5$  is a root.

Ques 3 (a)  $f(x) = 2(x-3)^2 + 9x - 36 - 18$

$$0 = -54 + 9x - 36 - 18$$

$$72 = 9x - 36 \quad \checkmark$$

$$24 = 3x - 6 \quad \checkmark \quad \text{---} \quad \textcircled{1}$$

(b) sub  $x=1$ ,

$$\begin{aligned} f(1) &= 1 - 8 + 17 - 10 \\ &= 0 \quad \checkmark \end{aligned}$$

$\therefore (x-1)$  is also factor.

Considering coefficient of  $x^2$ ,  $x$  term & constant term,

$$f(x) = (x-5)(x-1)(x-2)$$

$$\therefore (x-5)(x-1)(x-2) = 0$$

$$x = 1, 2, 5$$

Ques 2 RHS:  ~~$x^4 + Ax^3 + Bx^2 + Cx^1 + Dx^0$~~

$$+ 3x^2 + 3Ax + 3$$

$$= x^4 + (6A)x^3 + (B+A^2)x^2 + 6Ax + 3$$

equating to left hand side,

$$2A = B \quad \text{---} \quad \textcircled{1}$$

$$6A^2 = C \quad \text{---} \quad \textcircled{2}$$

$$6A = -12 \quad \text{---} \quad \textcircled{3}$$

$$\therefore A = -2 \quad \checkmark \text{ from } \textcircled{3} \text{ sub into } \textcircled{1}$$

$$\textcircled{1}: -A = B \quad \checkmark$$

$$\text{sub } A = -2 \text{ into } \textcircled{2}: C = 10 \quad \checkmark$$

$$\begin{aligned} \therefore A &= -2 \\ B &= -4 \quad \checkmark \\ C &= 10 \end{aligned}$$

$$(a) f(x) = 1 \frac{2}{3}x^2 + 2x + b \quad \checkmark \quad \textcircled{2}$$

$$\textcircled{1} + \textcircled{2} \quad 2x^2 = 6x \quad \therefore x = 3$$

$$\text{sub into } \textcircled{2}: 1 = 10 + b$$

$$b = -9$$

$$(b) f(x) = 2x^3 + 5x^2 - 9x - 18$$

$$\begin{array}{r} 2x^3 \\ \hline 2x^3 + 5x^2 - 9x - 18 \end{array} \quad \checkmark$$

$$\begin{array}{r} 2x^3 + 2x^2 - 12x \\ \hline 3x^2 + 3x - 18 \\ \hline 3x^2 + 3x - 18 \\ \hline 0 \end{array} \quad \checkmark$$

$$\therefore f(x) = (x+3)(x+2)(2x+3)$$

Ques 4 (a)  $f(x) = (x^4 + 3x^2)(1 - \frac{2}{x^2})$

$$= x^2(x^2 + 3)\left(1 - \frac{2}{x^2}\right)$$

$$= (x^2 + 3)(x^2 - 2)$$

$$(b) (x^2 + 3)(x^2 - 2) = 0$$

$$x^2 = -3 \quad (\text{no real soln})$$

$$x^2 = 2 \quad \checkmark$$

$$\therefore x = \pm \sqrt{2}$$

Qub5. (a) since  $\alpha$  is a zero, 1 linear factor is  $(x-1)$

(b)  $x-1 \overline{)x^3 + 7x^2 - 14x + 8}$

$$\begin{array}{r} -x^2 + 6x - 8 \\ \hline -x^3 + x^2 \\ \hline 6x^2 - 14x \\ 6x^2 - 6x \\ \hline -8x + 8 \\ -8x + 8 \\ \hline 0 \end{array}$$

(c)  $x+1 \overline{x^3 - x^2 + 2x + 4}$

$$\begin{array}{r} x^3 + x^2 \\ \hline -2x^2 + 2x \\ -2x^2 - 2x \\ \hline 4x + 4 \\ 4x + 4 \\ \hline 0 \end{array}$$

$$\therefore f(x) = (x+1)(x^2 - 2x + 4)$$

for  $f(x)$  to have any other zeroes,

$$x^2 - 2x + 4 = 0$$

$$\Delta = 4 - 4(4) \checkmark$$

$$= -12 \quad \therefore \text{no real soln}$$

$\therefore$  no other real soln

$$\therefore y = (x-1)(-x+2)(x+4)$$

$$\therefore B(2, 0) \checkmark$$

$$C(-4, 0) \checkmark$$

Qub6 (a)  $f(4) = 64 - 16 + 4k + 4$   
 $= 52 + 4k$

$$5f(2) = 5(8 - 4 + 2k + 4)$$
 $= 40 + 10k$

equating the 2 eqns gives:

$$52 + 4k = 40 + 10k$$

$$112 = 6k$$

$$\therefore k = 2$$

(b)  $f(x) = x^3 - x^2 + 2x + 4$

$$f(-1) = -1 - 1 + (-2) + 4$$
 $= -4 + 4 = 0$

$\therefore x = -1$  is soln of eqn  $f(x) = 0$

Qn5. (a) since  $A$  is a zero, 1 linear factor is  $(x-1)$

(b)  $x-1 \overline{)x^3 + 7x^2 - 14x + 8}$   
 $\underline{-x^3 + x^2}$   
 $6x^2 - 14x$   
 $\underline{6x^2 - 6x}$   
 $-8x + 8$   
 $\underline{-8x + 8}$   
 $0$

(c)  $x+1 \overline{x^3 - x^2 + 2x + 4}$   
 $\underline{x^3 + x^2}$   
 $-2x^2 + 2x$   
 $\underline{-2x^2 - 2x}$   
 $4x + 4$   
 $\underline{4x + 4}$   
 $0$

$$\therefore f(x) = (x+1)(x^2 - 2x + 4)$$

for  $f(x)$  to have any other zeroes,

$$x^2 - 2x + 4 = 0$$

$$\Delta = 4 - 4(4) \checkmark$$

$$= -12 \therefore \text{no real soln}$$

$\therefore$  no other real soln

$$\therefore y = (x-1)(-2x+2)(x+4)$$

$$\therefore B(2, 0) \checkmark$$

$$C(-4, 0) \checkmark$$

Qn6. (a)  $f(4) = 64 - 16 + 4k + 4$   
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$$5f(2) = 5(8 - 4 + 2k + 4)$$
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equating the 2 eqns gives:

$$52 + 4k = 40 + 10k$$

$$112 = 6k$$

$$\checkmark$$

$$\therefore k = 2$$

(b)  $f(x) = x^3 - x^2 + 2x + 4$

$$f(-1) = -1 - 1 + (-2) + 4$$

$$= -4 + 4 = 0$$

$\therefore x = -1$  is soln of eqn  $f(x) = 0$

# EXERCISE 6 POLYNOMIALS

TOPIC



EXCELLENT

Ques 1. (a)  $f(s) = (s-5)(s-10)$  Ques 3. (a)  $f(x) = 2x^2 + 9x - 36 - 18$

$$= 0 \quad /$$

$\therefore (x-5)$  is factor of  $f(x)$  since no remainders.

$$0 = -54 + 9a - 3b - 18$$

$$72 = 9a - 3b \quad /$$

$$24 = 3a - b \quad \text{---} \textcircled{1}$$

(b) sub  $x=1$ ,

$$f(1) = 1 - 8 + 17 - 10$$

$$= 0 \quad /$$

$\therefore (x-1)$  is also factor.

Considering coefficient of leading term & constant term,

$$f(x) = (x-5)(x-1)(x-2)$$

$$\therefore (x-5)(x-1)(x-2) = 0$$

$$x = 1, 2, 5.$$

$$f(x) = 2x^2 + 4x + 2b - 18$$

$$1 = 2 + 4a + 2b$$

$$(1) \quad 1 = 2 + 4a + 2b \quad \text{---} \textcircled{2}$$

$$(1) + (2) \quad 2x = 6a \therefore a = 3$$

$$\text{sub into } (2): \quad 1 = 10 + b$$

$$b = -9$$

$$(b) \quad f(x) = 2x^3 + 5x^2 - 9x - 18$$

$$\begin{array}{r} 2x+3 \\ \hline x^2+x-6 ) 2x^3 + 5x^2 - 9x - 18 \end{array}$$

$$\frac{2x^3 + 2x^2 - 12x}{3x^2 + 3x - 18}$$

$$\frac{3x^2 + 3x - 18}{0} \quad \checkmark$$

Ques 2 RHS:  ~~$x^4 + Ax^3 + Bx^2 + Cx + D$~~

$$+ 3x^2 + 3Ax + 3$$

$$= x^4 + (6A)x^3 + (B+A^2)x^2 + 6Px + Q.$$

equating to left hand side,

$$\therefore f(x) = (x+3)(x-2)(2x+3)$$

$$2A = B \quad \text{---} \textcircled{1}$$

$$\text{Ques 4. (a) } f(x) = (x^4 + 3x^2)\left(1 - \frac{2}{x^2}\right)$$

$$6(A^2 + C) = D \quad \text{---} \textcircled{2}$$

$$= x^2(x^2+3)\left(1 - \frac{2}{x^2}\right)$$

$$6A = -12 \quad \text{---} \textcircled{3}$$

$$= (x^2+3)(x^2-2) \quad \checkmark$$

$$\therefore A = -2 \quad \text{from } \textcircled{3} \text{ sub into } \textcircled{1}$$

$$(1): -A = B \quad \checkmark$$

$$\text{sub } A = -2 \text{ into } \textcircled{2}: C = 10 \quad \checkmark$$

$$\therefore A = -2$$

$$B = -4 \quad \checkmark$$

$$C = 10.$$

$$(b) \quad (x^2+3)(x^2-2) = 0$$

$$x^2 = -3 \quad (\text{no real soln})$$

$$x^2 = 2$$

$$\therefore x = \pm \sqrt{2}$$