EXERCISE 8.15

Solve the following equations for the domain $0° \leqslant \theta \leqslant 360°$.

1. $\tan \theta = \sqrt{3}$
2. $\sin \theta = -0{\cdot}7295$
3. $2 \cos \theta = 1$
4. $\sin \theta = 1$
5. $\sec \theta = \sqrt{2}$
6. $\operatorname{cosec} \theta = 6$
7. $\cot \theta = -3$
8. $\cos \theta = -\frac{1}{4}$
9. $\sin^2 \theta = 1$
10. $2 \cos^2 \theta = 1$
11. $\cos \theta + 1 = 0$
12. $2 \sin \theta - 1 = 0$
13. $2 \cos \theta = \sec \theta$
14. $3 \cot \theta = \tan \theta$
15. $4 \sin^2 \theta = 1$
16. $\tan^2 \theta = \frac{1}{3}$
17. $\operatorname{cosec}^2 \theta = 2$
18. $9 \sec^2 \theta = 16$
19. $3 \sin \theta = 2 \cos \theta$
20. $\sin \theta + 4 \cos \theta = 0$
21. $\cos^2 \theta = 1 - 2 \sin^2 \theta$
22. $\cos \theta - \sqrt{3} \sin \theta = 0$
23. $2(\sec^2 \theta + \tan^2 \theta) = 11$
24. $\cot \theta + \tan \theta = 2 \operatorname{cosec} \theta$

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1. 60°, 240°
2. 226°51′, 313°9′
3. 60°, 300°
4. 90°
5. 45°, 315°
6. 9°36′, 170°24′
7. 161°34′, 341°34′
8. 104°29′, 255°31′
9. 90°, 270°
10. 45°, 135°, 225°, 315°
11. 180°
12. 30°, 150°
13. 45°, 135°, 225°, 315°
14. 60°, 120°, 240°, 300°
15. 30°, 150°, 210°, 330°
16. 30°, 150°, 210°, 330°
17. 45°, 135°, 225°, 315°
18. 41°25′, 138°35′, 221°25′, 318°35′
19. 33°41′, 213°41′
20. 104°2′, 284°2′
21. 0°, 180°, 360°
22. 30°, 210°
23. 56°19′, 123°41′, 236°19′, 303°41′
24. 60°, 300°

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1. (a) 30°, 210° (b) 30°, 210°, 390°, 570°
2. 45°, 135°
3. 67°30′, 157°30′, 247°30′, 337°30′
4. 23°48′, 66°12′, 203°48′, 246°12′
5. (a) 30°, 90°, 150°
   (b) 104°29′, 255°31′
   (c) 71°34′, 116°34′, 251°34′, 296°34′
   (d) 18°26′, 26°34′, 198°26′, 206°34′
   (e) 60°, 180°, 300°
   (f) 41°49′, 90°, 138°11′
6. (a) 50°46′, 129°14′, 230°46′, 309°14′
   (b) 30°, 150°
   (c) 60°, 300

EXERCISE 8.16

1. Solve $\tan \theta = \dfrac{1}{\sqrt{3}}$ for values of θ in the domain (a) $0° \leqslant \theta \leqslant 360°$ (b) $0° \leqslant \theta \leqslant 720°$.
2. Solve $\sin^2 \theta = \frac{1}{2}$ for values of θ between 0° and 180°.
3. Find values of θ between 0° and 360° which satisfy $\tan 2\theta = -1$.
4. If $\sin 2\theta = 0{\cdot}7385$ find θ for the domain $0° \leqslant \theta \leqslant 360°$.
5. Solve these equations for $0° \leqslant \theta \leqslant 360°$.
   (a) $(\sin \theta - 1)(2 \sin \theta - 1) = 0$
   (b) $(\cos \theta - 3)(4 \cos \theta + 1) = 0$
   (c) $(\tan \theta - 3)(\tan \theta + 2) = 0$
   (d) $6 \tan^2 \theta - 5 \tan \theta + 1 = 0$
   (e) $2 \cos^2 \theta + \cos \theta - 1 = 0$
   (f) $3 \sin^2 \theta - 5 \sin \theta + 2 = 0$
6. Solve these equations for $0° \leqslant \theta \leqslant 360°$. First make the suggested substitution.
   (a) $3 \tan^2 \theta = 2 + \sec^2 \theta$ (Substitute $1 + \tan^2 \theta$ for $\sec^2 \theta$)
   (b) $\sin^2 \theta + 2 \sin \theta = 2 - \cos^2 \theta$ (Substitute $1 - \sin^2 \theta$ for $\cos^2 \theta$)
   (c) $15 \cos \theta + 2 \sin^2 \theta - 9 = 0$ (Substitute $1 - \cos^2 \theta$ for $\sin^2 \theta$)
7. Solve the following equations for θ in the range $0° \leqslant \theta \leqslant 360°$.
   (a) $3 \sin^2 \theta + 2 \sin \theta = 0$
   (b) $5 \cos^2 \theta + 2 \sin \theta - 2 = 0$
   (c) $2 \cos^2 \theta = 1 + \sin \theta$
   (d) $3 \sec^2 \theta = 5 + 5 \tan \theta$
   (e) $6 \tan \theta = 5 \operatorname{cosec} \theta$
   (f) $\cos^2 \theta - \sin^2 \theta = \frac{1}{2}$
   (g) $\tan \theta = 2 \cos \theta$
   (h) $3 \sin \theta = 2 \cos \theta$
   (i) $\tan \theta + 2 \cot \theta = 3$
   (j) $3 \sec \theta = 2 \cot \theta$
   (k) $3 \tan^2 \theta - 4 \tan \theta = 2$
   (l) $2 \sin^2 \theta + 5 \sin \theta - 1 = 0$
   (m) $\sin^2 \theta + 2 \sin \theta \cos \theta - 3 \cos^2 \theta = 0$
   (n) $\cos^2 \theta + 7 \sin \theta \cos \theta = 3$

7. (a) 0°, 180°, 360°, 221°49′, 318°11′
   (b) 90°, 216°52′, 323°8′
   (c) 30°, 150°, 270°
   (d) 63°26′, 161°34′, 243°26′, 341°34′
   (e) 48°11′, 311°49′

EXERCISE 8.14

Answer the following questions without using tables or the trigonometric functions on a calculator.

1. If θ is an acute angle and $\cos\theta = \frac{1}{3}$, find $\sin\theta$ and $\tan\theta$.

2. If $\sin\theta = \frac{\sqrt{5}}{3}$ and θ is acute, find $\cos\theta$ and $\tan\theta$.

3. Find values for $\cos\theta$ and $\tan\theta$ if $\sin\theta = \frac{2}{\sqrt{29}}$ and θ is obtuse.

4. Find all possible values for $\tan\theta$ and $\sin\theta$ if $\cos\theta = -\frac{1}{5}$.

5. Evaluate the following;

(a) $\sin^2 15° + \cos^2 15°$
(b) $1 - \sin^2 60°$
(c) $1 + \tan^2 60°$
(d) $\sin^2 40° + \sin^2 50°$
(e) $\cos^2 10° + \cos^2 80°$
(f) $\operatorname{cosec}^2 20° - \cot^2 20°$

6. Find the simplest expression for each of the following:

(a) $1 - \sin^2\theta$
(b) $\sec^2\theta - \tan^2\theta$
(c) $\cos^2\theta - 1$
(d) $2 - 2\cos^2\theta$
(e) $1 - \sin^2\theta - \cos^2\theta$
(f) $\sqrt{1 - \cos^2\theta}$
(g) $1 + \dfrac{\cos^2\theta}{\sin^2\theta}$
(h) $3 + 3\tan^2\theta$
(i) $4\operatorname{cosec}^2\theta - 4\cot^2\theta$
(j) $\tan\theta\cos\theta$
(k) $(\sec\theta - 1)(\sec\theta + 1)$
(l) $\sin^4\theta + \sin^2\theta\cos^2\theta$
(m) $1 - \cot^2\theta + \operatorname{cosec}^2\theta$
(n) $\cot^2\theta - \dfrac{1}{\sin^2\theta}$
(o) $\sec^2\theta(1 - \cos^2\theta)$
(p) $\sin^2\theta\cos\theta + \cos^3\theta$
(q) $(1 + \cot^2\theta)\sin^2\theta$
(r) $\cos^2\theta - \sin^2\theta\cos^2\theta$
(s) $\tan\theta\sqrt{1 - \sin^2\theta}$
(t) $\sin^2\theta\operatorname{cosec}\theta + \cos^2\theta\sec\theta$
(u) $\dfrac{\sin\theta}{\cos\theta} + \dfrac{\cos\theta}{\sin\theta}$

7. (a) If $x = a\cos\theta$ simplify: (i) $a^2 - x^2$ (ii) $\dfrac{1}{\sqrt{a^2 - x^2}}$

(b) If $x = a\tan\theta$ simplify: (i) $a^2 + x^2$ (ii) $\dfrac{1}{\sqrt{a^2 + x^2}}$

8. If $x = 5\cos\theta$ and $y = 5\sin\theta$ find an equation connecting x and y by eliminating θ.

9. Prove the following identities.

(a) $1 - 2\sin^2\theta = 2\cos^2\theta - 1$
(b) $\sin^4\theta - \cos^4\theta = 2\sin^2\theta - 1$
(c) $3\cos^2\theta - 1 = 2 - 3\sin^2\theta$
(d) $(\sin\theta + \cos\theta)^2 + (\sin\theta - \cos\theta)^2 = 2$
(e) $\dfrac{1 - \tan^2\theta}{1 + \tan^2\theta} = \cos^2\theta - \sin^2\theta$
(f) $\sin\theta\tan\theta = \dfrac{1 - \cos^2\theta}{\cos\theta}$
(g) $\sin^2\theta + \tan^2\theta = \sec^2\theta - \cos^2\theta$
(h) $(\sec^2\theta - 1)\cos^2\theta = \sin^2\theta$
(i) $(2\cos\theta + 3\sin\theta)^2 + (3\cos\theta - 2\sin\theta)^2 = 13$
(j) $(1 - \cos\theta)(1 + \sec\theta) = \sin\theta\tan\theta$
(k) $\dfrac{\sin\theta}{1 + \cos\theta} = \dfrac{1 - \cos\theta}{\sin\theta}$
(l) $\dfrac{1}{\cot^2\theta} + 1 = \dfrac{1}{\cos^2\theta}$

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1. $\frac{2\sqrt{2}}{3}, 2\sqrt{2}$
2. $\frac{2}{3}, \frac{\sqrt{5}}{2}$
3. $-\frac{5}{\sqrt{29}}, -\frac{2}{5}$
4. $\tan\theta = \mp 2\sqrt{6}, \sin\theta = \pm\frac{2\sqrt{6}}{5}$
5. (a) 1 (b) $\frac{1}{4}$ (c) 4 (d) 1 (e) 1 (f) 1
6. (a) $\cos^2\theta$ (b) 1 (c) $-\sin^2\theta$ (d) $2\sin^2\theta$ (e) 0 (f) $\sin\theta$ (g) $\operatorname{cosec}^2\theta$ (h) $3\sec^2\theta$ (i) 4 (j) $\sin\theta$ (k) $\tan^2\theta$ (l) $\sin^2\theta$ (m) 2 (n) -1 (o) $\tan^2\theta$ (p) $\cos\theta$ (q) 1 (r) $\cos^4\theta$ (s) $\sin\theta$ (t) $\sin\theta + \cos\theta$ (u) $\sec\theta\operatorname{cosec}\theta$
7. (a) (i) $a^2\sin^2\theta$ (ii) $\frac{\operatorname{cosec}\theta}{a}$
   (b) (i) $a^2\sec^2\theta$ (ii) $\frac{\cos\theta}{a}$
8. $x^2 + y^2 = 25$