

Trigonometric Ratios and Graphs

Exercise 9S Skills Practice

1 Give the exact value of

- | | | | |
|----------------------|---------------------|----------------------|----------------------|
| a $\sin 30^\circ$ | b $\cos 30^\circ$ | c $\tan 45^\circ$ | d $\sin 120^\circ$ |
| e $\tan 60^\circ$ | f $\sin 225^\circ$ | g $\cos 90^\circ$ | h $\cos 360^\circ$ |
| i $\tan 390^\circ$ | j $\cos(-60^\circ)$ | k $\sin(-45^\circ)$ | l $\tan 330^\circ$ |
| m $\cos 240^\circ$ | n $\sin 690^\circ$ | o $\tan(-120^\circ)$ | p $\sin 150^\circ$ |
| q $\sin(-300^\circ)$ | r $\tan 720^\circ$ | s $\cos 585^\circ$ | t $\tan(-405^\circ)$ |

2 Give the exact value of

- | | | | |
|--------------------------|-------------------------|---------------------------|----------------------------|
| a $\cos \frac{\pi}{4}$ | b $\tan \frac{\pi}{3}$ | c $\sin \frac{2\pi}{3}$ | d $\cos \frac{\pi}{6}$ |
| e $\sin \frac{\pi}{2}$ | f $\cos \frac{5\pi}{3}$ | g $\tan \frac{4\pi}{3}$ | h $\sin(-\frac{\pi}{4})$ |
| i $\tan(-\frac{\pi}{6})$ | j $\sin \frac{7\pi}{6}$ | k $\cos(-\frac{5\pi}{6})$ | l $\tan 3\pi$ |
| m $\sin \frac{7\pi}{4}$ | n $\tan \frac{3\pi}{4}$ | o $\cos \frac{7\pi}{2}$ | p $\tan(-\frac{17\pi}{6})$ |

In questions 3 and 4 your graphs should show the coordinates of any turning points and the equations of any asymptotes. Do not use a graphic calculator.

3 Sketch each graph in the interval $0 \leq x \leq 360^\circ$.

- | | |
|--|------------------------------|
| a $y = \sin x$ | b $y = 2 \sin x$ |
| c $y = 3 \cos x$ | d $y = \tan x$ |
| e $y = -\tan x$ | f $y = \frac{1}{4} \cos x$ |
| g $y = \sin 2x$ | h $y = \tan(\frac{1}{2}x)$ |
| i $y = 2 \cos 3x$ | j $y = \sin(-x)$ |
| k $y = \frac{1}{2} \sin(\frac{2}{3}x)$ | l $y = \tan(x - 90^\circ)$ |
| m $y = \cos(x + 30^\circ)$ | n $y = 4 \sin(x - 45^\circ)$ |
| o $y = 1 + \cos x$ | p $y = 3 - \tan x$ |

4 Sketch each graph in the interval $0 \leq x \leq 2\pi$.

- | | |
|---------------------------------|------------------------------------|
| a $y = \cos x$ | b $y = \tan 2x$ |
| c $y = \sin(x + \frac{\pi}{6})$ | d $y = \tan(x - \frac{\pi}{4})$ |
| e $y = -2 \sin 3x$ | f $y = 2 + \frac{1}{2} \cos x$ |
| g $y = 3 \tan(-x)$ | h $y = 3 \sin(x - \frac{3\pi}{4})$ |

Exercise 9S Skills Practice

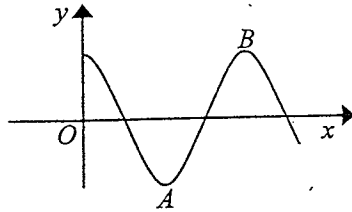
- 1 a $\frac{1}{2}$ b $\frac{\sqrt{3}}{2}$ c 1 d $\frac{\sqrt{3}}{2}$ e $\sqrt{3}$
 f $-\frac{1}{\sqrt{2}}$ g 0 h 1 i $\frac{1}{\sqrt{3}}$ j $\frac{1}{2}$
 k $-\frac{1}{\sqrt{2}}$ l $-\frac{1}{\sqrt{3}}$ m $-\frac{1}{2}$ n $-\frac{1}{2}$ o $\sqrt{3}$
 p $\frac{1}{2}$ q $\frac{\sqrt{3}}{2}$ r 0 s $-\frac{1}{\sqrt{2}}$ t -1
- 2 a $\frac{1}{\sqrt{2}}$ b $\sqrt{3}$ c $\frac{\sqrt{3}}{2}$ d $\frac{\sqrt{3}}{2}$
 e 1 f $\frac{1}{2}$ g $\sqrt{3}$ h $-\frac{1}{\sqrt{2}}$
 i $-\frac{1}{\sqrt{3}}$ j $-\frac{1}{2}$ k $-\frac{\sqrt{3}}{2}$ l 0
 m $-\frac{1}{\sqrt{2}}$ n -1 o 0 p $\frac{1}{\sqrt{3}}$
- 3 a (90, 1), (270, -1)
 b (90, 2), (270, -2)
 c (0, 3), (180, -3), (360, 3)
 d $x=90, x=270$
 e $x=90, x=270$

- f (0, $\frac{1}{4}$), (180, $-\frac{1}{4}$), (360, $\frac{1}{4}$)
 g (45, 1), (135, -1), (225, 1), (315, -1)
 h $x=180$
 i (0, 2), (60, -2), (120, 2), (180, -2),
 (240, 2), (300, -2), (360, 2)
 j (90, -1), (270, 1)
 k (135, $\frac{1}{2}$)
 l $x=0, x=180, x=360$
 m (150, -1), (330, 1)
 n (135, 4), (315, -4)
 o (0, 2), (180, 0), (360, 2)
 p $x=90, x=270$

- 4 a (0, 1), (π , -1), (2 π , 1)
 b $x=\frac{\pi}{4}, x=\frac{3\pi}{4}, x=\frac{5\pi}{4}, x=\frac{7\pi}{4}$
 c ($\frac{\pi}{3}$, 1), ($\frac{4\pi}{3}$, -1)
 d $x=\frac{3\pi}{4}, x=\frac{7\pi}{4}$
 e ($\frac{\pi}{6}$, -2), ($\frac{\pi}{2}$, 2), ($\frac{5\pi}{6}$, -2), ($\frac{7\pi}{6}$, 2),
 ($\frac{3\pi}{2}$, -2), ($\frac{11\pi}{6}$, 2)
 f (0, $\frac{5}{2}$), (π , $\frac{3}{2}$), (2 π , $\frac{5}{2}$)
 g $x=\frac{\pi}{2}, x=\frac{3\pi}{2}$
 h ($\frac{\pi}{4}$, -3), ($\frac{5\pi}{4}$, 3)

Exercise 9E Exam Practice

1



The diagram shows part of the curve $y = p \cos qx$.

The first minimum of the curve for $x > 0$ is the point $A \left(\frac{\pi}{3}, -2 \right)$.

- a Find the values of the constants p and q . (2 marks)
- b State the period of the curve. (1 mark)
- c Write down the coordinates of the point B , the first maximum of the curve for $x > 0$. (2 marks)

2 Given that

$$\tan 105^\circ = \frac{\tan 60^\circ + \tan 45^\circ}{1 - (\tan 60^\circ \times \tan 45^\circ)}$$

- a express $\tan 105^\circ$ in the form $a + b\sqrt{3}$, (5 marks)
- b express $\tan 255^\circ$ in the form $a + b\sqrt{3}$. (2 marks)

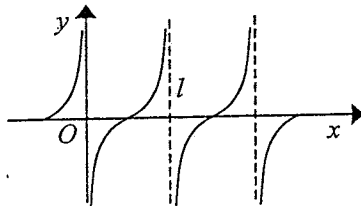
3

$$f(x) \equiv \sin x.$$

Labelling clearly the coordinates of any maximum or minimum points, sketch on separate diagrams the graphs

- a $y = f(2x)$ in the interval $0 \leq x \leq 2\pi$, (4 marks)
- b $y = 1 + f\left(x - \frac{\pi}{6}\right)$ in the interval $0 \leq x \leq 360^\circ$. (4 marks)

4 a



The diagram shows part of the curve $y = \tan(x + k^\circ)$, with x measured in degrees and $k > 0$.

- i Find the smallest possible value of the constant k . (3 marks)
- ii Write down the equation of the asymptote labelled l . (3 marks)
- b Sketch the graph of $y = \tan(-2x)$ in the interval $0 \leq x \leq 360^\circ$, labelling the values of x where the graph crosses the x -axis. (4 marks)

Exercise 9E Exam Practice

- 1 a $p = 2, q = 3$ b $2\pi/3$ c $(2\pi/3, 2)$
- 2 a $-2 - \sqrt{3}$ b $2 + \sqrt{3}$
- 3 a $(\pi/4, 1), (3\pi/4, -1), (5\pi/4, 1), (7\pi/4, -1)$
 b $(2\pi/3, 2), (5\pi/3, 0)$
- 4 a i 90 ii $x = 180$
 b 0, 90, 180, 270, 360