

EXERCISES – Polynomials

Name: _____

1. (a) Where does the parabola :
 $y = x^2 + 4x + 10$ cut the y-axis?

- (b) Using the method of completing the squares, find its vertex.

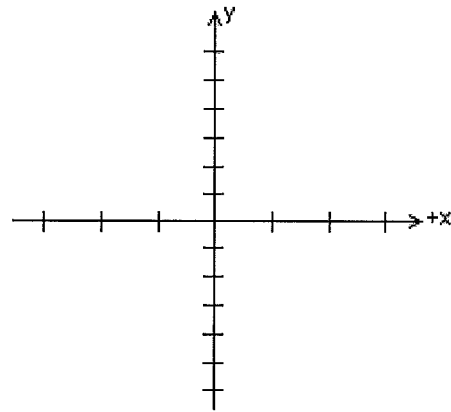
- (c) What is the minimum value of this function?

2. (a) Find the co-efficient of x^2 in the expansion of :
 $(x^2 - 3x - 1).(3x^2 - x + 2)$

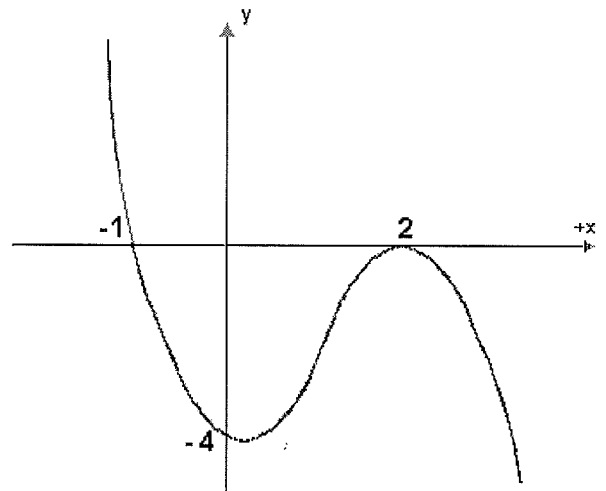
- (b) Find **a**, **b** and **c** if :-
 $ax(x+1) + b(x+1) + c = 2x^2 + x - 1$

3. (a) Where does the curve :
 $y = (x+1)(2-x)(x+3)(2x-1)$
cut the x and y axes?

- (b) Sketch the curve:



4. What is the equation of the curve below?



Equation: _____

5. (a) Show that $(x+3)$ is a factor of :

$$x^3 + x^2 - 9x - 9$$

(b) Find the other factors and fully factorise the above polynomial.

(c) Solve the equation:

$$X^3 + x^2 - 9x - 9 = 0$$

6. (a) What is the co-efficient of x^3 in the following:

(i) $(x+1)^{20}$

(ii) $(1-x)^{15}$

(iii) $(2-x)^{10}$

7. (a) Expand the binomial $(2x+3)^5$

8. (a) Simplify

(i) $\frac{8!}{6!}$ (ii) $\frac{8!}{6!2!}$ (iii) $\frac{(n+1)!}{n!}$

(b) If ${}^nC_3 = an^3 + bn^2 + cn$

Find a , b and c

8. If $x-5$ and $x+2$ are both factors of the polynomial :

$$X^3 - 5x^2 + ax + b$$

Find a and b.

EXERCISES - Polynomials

Name: Amy Owl

1. (a) Where does the parabola :

$$y = x^2 + 4x + 10 \text{ cut the y-axis?}$$

when $x=0$

$y=10$
cuts y-axis at $y=10$ ✓

- (b) Using the method of completing the squares, find its vertex.

$$\begin{aligned} x^2 + 4x + 10 &= y \\ x^2 + 4x &= y - 10 \\ x^2 + 4x + 4 &= y - 6 \\ (x+2)^2 &= y - 6 \end{aligned}$$

$$\begin{aligned} \therefore y &= (x+2)^2 + 6 \\ \therefore V \text{ is } &(-2, 6) \end{aligned}$$

- (c) What is the minimum value of this function?

concave up
 \therefore min. value is 6 ✓

2. (a) Find the co-efficient of x^2 in the expansion of :

$$\begin{aligned} &(x^2 - 3x - 1)(3x^2 - x + 2) \\ = &3x^4 - x^3 + 2x^2 - 9x^3 + 3x^2 - 6x - 3x^2 + x - 2 \\ = &3x^4 - 10x^3 + 2x^2 - 5x - 2 \\ \therefore \text{co-eff. of } x^2 &\text{ is } 2 \end{aligned}$$

- (b) Find a , b and c if :-

$$ax(x+1) + b(x+1) + c = 2x^2 + x - 1$$

$$\begin{aligned} \text{LHS} &= ax^2 + ax + bx + b + c \\ &= ax^2 + (a+b)x + b + c \end{aligned}$$

$$\therefore a = 2 \checkmark$$

$$a + b = 1$$

$$\text{sub } a = 2$$

$$\therefore b = -1 \checkmark$$

$$b + c = -1$$

$$\text{sub } b = -1$$

$$c = 0 \checkmark$$

$$\therefore a = 2, b = -1, c = 0 \checkmark$$

3. (a) Where does the curve :

$$y = (x+1)(2-x)(x+3)(2x-1)$$

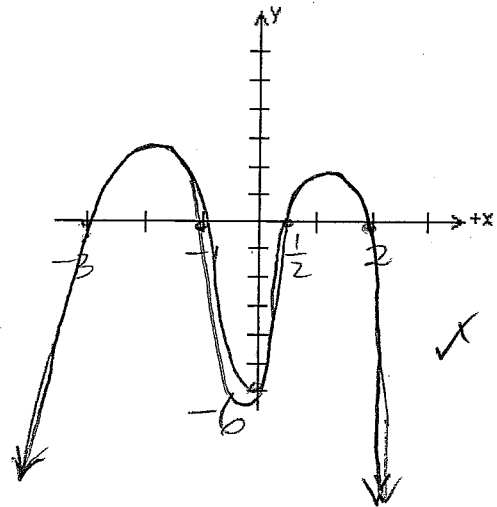
cut the x and y axes?

when $x=0, y=-6$
y-int is $(0, -6)$ ✓

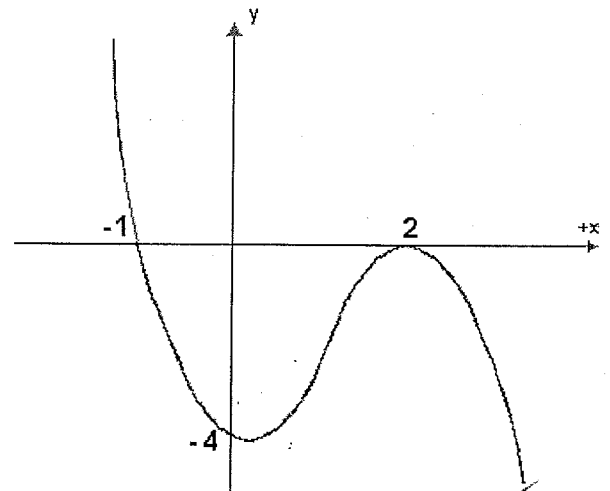
when $y=0, x = -1, 2, -3, \frac{1}{2}$

\therefore x-ints are $x = -3, -1, \frac{1}{2}, 2$

- (b) Sketch the curve:



4. What is the equation of the curve below?



Equation: $k(x-2)^2(x+1) = y \dots (i)$
 Sub $(0, -4)$ into (i)
 $k(-2)^2(1) = -4$
 $k = -\frac{4}{4} = -1$
 $\therefore y = -(x-2)^2(x+1)$

5. (a) Show that $(x+3)$ is a factor of :

$$x^3 + x^2 - 9x - 9$$

$$\text{let } P(x) = x^3 + x^2 - 9x - 9$$

$$P(-3) = (-3)^3 + (-3)^2 - 9(-3) - 9$$

$$= -27 + 9 + 27 - 9$$

$$= 0$$

$\therefore (x+3)$ is a factor

(b) Find the other factors and fully factorise the above polynomial.

$$\begin{array}{r} x^2 - 2x - 3 \\ x+3 \overline{) x^3 + x^2 - 9x - 9} \\ \underline{x^3 + 3x^2} \\ -2x^2 - 9x \\ \underline{-2x^2 - 6x} \\ -3x - 9 \\ \underline{-3x - 9} \\ 0 \end{array}$$

$$\therefore P(x) = (x+3)(x+1)(x-3)$$

(c) Solve the equation:

$$x^3 + x^2 - 9x - 9 = 0$$

$$\text{when } P(x) = 0$$

$$x = 3, -1, -3$$

6. (a) What is the co-efficient of x^3 in the following:

(i) $(x+1)^{20}$

(ii) $(1-x)^{15}$

(iii) $(2-x)^{10}$

7. (a) Expand the binomial $(2x+3)^5$

8. (a) Simplify

(i) $\frac{8!}{6!}$ (ii) $\frac{8!}{6!2!}$ (iii) $\frac{(n+1)!}{n!}$

(b) If ${}^nC_3 = an^3 + bn^2 + cn$

Find a, b and c

8. If $x-5$ and $x+2$ are both factors of the polynomial :

$$x^3 - 5x^2 + ax + b = P(x)$$

Find a and b.

$$P(5) = (5)^3 - 5(5)^2 + a(5) + b$$

$$= 125 - 125 + 5a + b = 0$$

$$P(-2) = (-2)^3 - 5(-2)^2 + a(-2) + b$$

$$= -8 - 20 - 2a + b = 0$$

$$\therefore \begin{cases} 5a + b = 0 \\ -2a + b = 28 \end{cases}$$

$$\underline{7a = -28}$$

$$a = -4 \quad b = 20$$