# MORIAH COLLEGE

## Year 11

## Extension 1 MATHEMATICS

**Date:** 2nd December, 2003.

**Time Allowed:** 1 hours plus 5 minutes reading time.

**Examiners:** E.Apfelbaum, L. Bornstein, A. Joshua, G.Wagner

- Attempt ALL questions.
- Start each question on a new page
- All necessary working should be shown in every question.
- Marks maybe deducted for careless or badly arranged work.

**Question 1 (Start each question on a new page)** **(16 marks)**

a) Find $\frac{dy}{dx}$ for the following functions:

i. $y = 4e^{5x-7}$ (1)

ii. $y = \log\sqrt{\frac{1}{x+4}}$ (2)

iii. $y = \left(\frac{2x}{\log x}\right)^2$ (3)

b) Find primitive functions for the following indefinite integrals:

i. $\int \frac{1}{x^2} + e^{-x}\ dx$ (2)

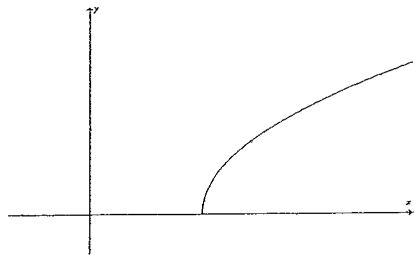
ii. $\int 2^x\ dx$ (1)

iii. $\int \frac{x^2 - 4x + 3}{4x}\, dx$ (3)

c) Find the value of $k$ if $\int_0^{\sqrt{\log k}} 5xe^{x^2}\, dx = 10$ and $k > 0$. (4)

Question 2 (Start on a new page) (20 marks)

a) The curve $y = \sqrt{5x - 7}$ is sketched below.

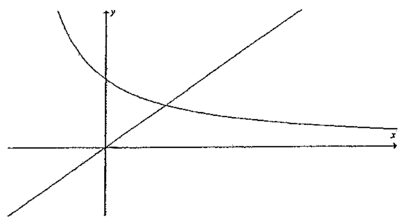


i. Find the area of the region bounded by the curve $y = \sqrt{5x-7}$, the $x$-axis and the line $x = \frac{16}{5}$. (4)

ii. Find the volume of the solid formed when the region in part i. is rotated about the $x$-axis. (Give answer in exact form) (3)

iii. Find the volume of the solid formed when the region in part i. is rotated about the $y$-axis. (Give answer correct to 2 decimal places). (5)

b) The diagram below shows part of the graphs of $y = \frac{x}{2}$ and $y = \frac{1}{x+1}$.



i. Find the area of the region bounded by the two graphs and the line $x = 3$. (Give answer correct to 3 decimal places) (4)

ii. The region above is rotated about the $x$-axis to form a solid. Find the volume of this solid, giving your answer in exact form. (4)



Question 3 (Start on a new page) (12 marks)

a) David was studying the function $y = x^2 e^{\sqrt{x}}$.

He found the gradient function to be $\frac{dy}{dx} = \frac{x\left(4+\sqrt{x}\right)e^{\sqrt{x}}}{2}$.

i. Show that he was correct with appropriate working. (2)

ii. David knew from his theory that $e^{\sqrt{x}} > 0$. He then thought that there was a stationary point at $x = 0$. Explain briefly why he was incorrect. (1)

b) Consider the definite integral $\int_1^3 \frac{x}{1+x^2}\,dx$.

i. Use the trapezoidal rule with three function values to approximate the value of the definite integral. (3)

ii. Find the exact value of the definite integral. (3)

iii. Hence, using parts i. and ii, show that $e \approx 5^{\frac{5}{8}}$. (3)

Question 4 (Start on a new page) (15 marks)

a)

i. Differentiate $y = x\log 2x - x$ (2)

ii. Hence, find $\int \log 2x \, dx$ (1)

b)

i. Sketch the graph of the function $y = \log 2x$. (1)

ii. Find the $x$-intercept of this function. (1)

iii. Find the equation of the tangent at any point $P(k, \log 2k)$ on the curve (3)

iv. Show that, if the tangent drawn through $P$ also passes through the origin, then $k = \frac{e}{2}$. (2)

v. Using part (a) or otherwise, find the area bounded by the curve $y = \log 2x$, the x-axis and the tangent when $k = \frac{e}{2}$. (5)

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#1. i). $y = 4e^{5x-7}$

$y' = 4.5.\ e^{5x-7}$

$y' = 20e^{5x-7}$

ii) $y = \log\sqrt{\frac{1}{x+4}}$

$y = \log\left(\frac{1}{x+4}\right)^{\frac{1}{2}}$

$y = \frac{1}{2}\left[\log 1 - \log(x+4)\right]$

$y' = \frac{1}{2}\left(-\frac{1}{x+4}\right)$

$y' = -\frac{1}{2(x+4)}$.

Some students did not fully simplify expression before differentiating

Many missed the negative sign

iii) $y = \left(\frac{2x}{\log x}\right)^2$

$y' = 2\left(\frac{2x}{\log x}\right)^1 \times y'$

$u = 2x \quad v = \log x$

$u' = 2 \quad v' = \frac{1}{x}$.

$y' = \frac{4x}{\log x}\left[\frac{2\log x - 2x.\frac{1}{x}}{(\log x)^2}\right]$

Many students thought that $(\log x)^2 = \log x^2 = 2\log x$

$y' = \frac{4x}{\log x}\left(\frac{2(\log x - 1)}{(\log x)^2}\right)$

$y' = \frac{8x(\log x - 1)}{(\log x)^3}$.

3m.

1b) i) $\int \frac{1}{x^2} + e^{-x}\,dx$

$\int x^{-2} + e^{-x}\,dx$

$= \frac{x^{-1}}{-1} + \frac{e^{-x}}{-1} + c$

$= -\frac{1}{x} - \frac{1}{e^x} + c.$

ii) $\int 2^x\,dx$

$= \frac{2^x}{\ln 2} + c$

iii) $\int \frac{x^2 - 4x + 3}{4x}\,dx$

$\int \frac{x^2}{4x} - \frac{4x}{4x} + \frac{3}{4x}\,dx$

$\int \frac{1}{4}x - 1 + \frac{3}{4}x^{-1}\,dx$

Many students couldn't handle $\int \frac{3}{4x}\,dx$

$= \frac{1}{4}\frac{x^2}{2} - x + \frac{3}{4}\log x + c$

$= \frac{x^2}{8} - x + \frac{3}{4}\log x + c$.

Handling the primitive was not well done.

c). $\int_0^{\sqrt{\log k}} 5xe^{x^2}\,dx = 10.$

$\frac{5}{2}\int_0^{\sqrt{\log k}} 2xe^{x^2}\,dx = 10.$

$\left[\frac{5}{2}e^{x^2}\right]_0^{\sqrt{\log k}} = 10.$

$\frac{5}{2}e^{\log k} - \frac{5}{2}e^0 = 10.$

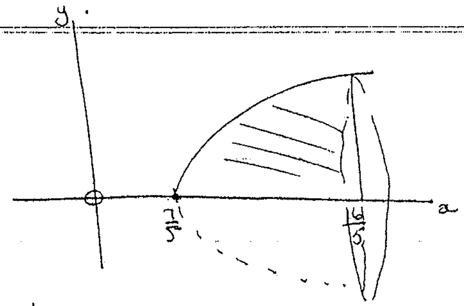
$\frac{5}{2}k - \frac{5}{2} = 10$

$\frac{5}{2}k = 12\frac{1}{2}$

$k = 12\frac{1}{2} \times \frac{2}{5}$

$k = 5.$

#2 i). $y = \sqrt{5x-7}$




Xint: $5x - 7 = 0$

$5x = 7$

$x = \frac{7}{5}$

$$\text{Area} = \int_{7/5}^{16/5} \sqrt{5x-7}\,dx$$

$$= \int_{7/5}^{16/5} (5x-7)^{1/2}\,dx$$

$$= \left[\frac{2(5x-7)^{\frac{3}{2}}}{3 \cdot 5}\right]_{7/5}^{16/5}$$

$$= \left[\frac{2}{15}(5x-7)^{3/2}\right]_{7/5}^{16/5}$$

$$= \frac{2}{15}\left[27\right]$$

$$= 3\tfrac{3}{5}\ u^2.$$

ii). $$V = \pi \int_{7/5}^{16/5} y^2\,dx$$

$$= \pi \int_{7/5}^{16/5} 5x - 7\,dx.$$

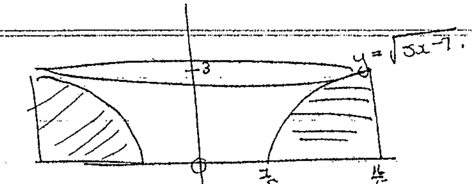
$$= \pi\left[\frac{5x^2}{2} - 7x\right]_{7/5}^{16/5}$$

$$= \pi\left[3\tfrac{1}{5} - -4\tfrac{9}{10}\right]$$

$$= \pi\left[8\tfrac{1}{10}\right]$$

$$= \frac{81}{10}\pi\ u^3.$$

#2a ii).




Many students did not realise they had to find vol. of (y).

$y^2 = 5x - 7$

$y^2 + 7 = 5x$

$\frac{y^2+7}{5} = x$

$$V = \text{Vol. of cylinder} - \pi \int_0^3 x^2\,dy.$$

$$= \pi\left(\frac{16}{5}\right)^2 \cdot 3 - \pi \int_0^3 \left(\frac{y^2+7}{5}\right)^2 dy$$

$$= \pi\ 30\tfrac{18}{25} - \frac{\pi}{25}\int_0^3 (y^4 + 14y^2 + 49)\,dy$$

$$= 30\tfrac{18}{25}\pi - \frac{\pi}{25}\left[\frac{y^5}{5} + \frac{14y^3}{3} + 49y\right]_0^3$$

$$= 30\tfrac{18}{25}\pi - \frac{\pi}{25}\left[\left(\frac{243}{5} + 126 + 147\right) - (0)\right]$$

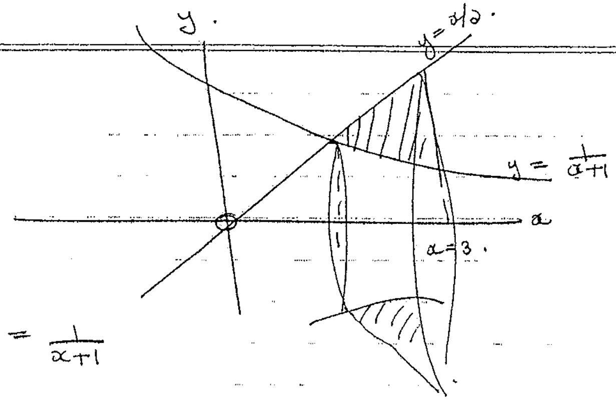
$$= 30\tfrac{18}{25}\pi - 321\tfrac{3}{5}\left(\frac{\pi}{25}\right)$$

$$= \pi\left(30\tfrac{18}{25} - 12\tfrac{108}{125}\right)$$

$$= \pi\left(17\tfrac{107}{125}\right) u^3.$$

Many students did not take their answer to 2 dec. pl.

#2b




i). $\frac{x}{2} = \frac{1}{x+1}$

$x^2 + x - 2 = 0$

$(x+2)(x-1) = 0$

$x = -2 \quad x = 1$

Common error was not to realise that the area was between the two curves. Many students added two separate areas

$$\text{Area} = \int_1^3 \frac{x}{2} - \frac{1}{x+1}\, dx$$

$$= \left[\frac{1}{2}\frac{x^2}{2} - \ln(x+1)\right]_1^3$$

$$= \left(\frac{9}{4} - \ln 4\right) - \left(\frac{1}{4} - \ln 2\right)$$

$$= \frac{9}{4} - \ln 4 - \frac{1}{4} + \ln 2$$

It is important to read the question carefull

$$= 2 - 2\ln 2 + \ln 2$$

$$= 2 - \ln 2$$

$$= 1.307 \ u^2.$$

ii). $$V = \pi \int_1^3 \left(\left(\frac{x}{2}\right)^2 - \left(\frac{1}{x+1}\right)^2\right) dx$$

$$= \pi \int_1^3 \frac{x^2}{4} - (x+1)^{-2}\, dx$$

$$= \pi \left[\frac{x^3}{12} - \frac{(x+1)^{-1}}{-1}\right]_1^3$$

$$= \pi \left[\frac{x^3}{12} + \frac{1}{x+1}\right]_1^3$$

$$= \pi \left(\frac{27}{12} + \frac{1}{4} - \frac{1}{12} - \frac{1}{2}\right) = 1\tfrac{11}{12}\pi \ u^3.$$

# 3a). $y = \underset{u}{x^2}\ \underset{v}{e^{\sqrt{x}}}$

i). $y' = e^{\sqrt{x}}.(2x) + (x^2)\left(e^{\sqrt{x}}\right)\frac{1}{2}x^{-\frac{1}{2}}$ ✓

many went wrong differentiating $e^{\sqrt{x}}$

$y' = 2x e^{\sqrt{x}} + \frac{1}{2}x^{\frac{3}{2}} e^{\sqrt{x}}$

$y' = x e^{\sqrt{x}}\left(2 + \frac{1}{2}\sqrt{x}\right)$

(2)

$y' = x e^{\sqrt{x}}\left(\frac{4 + \sqrt{x}}{2}\right)$. ✓

(1)

ii) at $x = 0$

not continuous ✓ either ✓

not differentiable at end point. ✓

needed to state either not continuous or not differentiable at end points

b). $\int_1^3 \frac{x}{1+x^2}\, dx$.

| $x$ | $f(x)$ | T | $Tf(x)$ |
|---|---|---|---|
| 1 | $\frac{1}{2}$ | 1 | $\frac{1}{2}$ |
| 2 | $\frac{2}{5}$ | 2 | $\frac{4}{5}$ |
| 3. | $\frac{3}{10}$ | 1 | $\frac{3}{10}$ |

✓

(3)

$\Sigma = 1\frac{3}{5}$ ✓

well done!

$A \doteqdot \frac{1}{2} \times 1\frac{3}{5} = \frac{4}{5}\ u^2$. ✓

ii) $\frac{1}{2}\int_1^3 \frac{2x}{1+x^2}\,dx$ ✓

$= \frac{1}{2}\left[\log(1+x^2)\right]_1^3$ ✓

$= \frac{1}{2}\left[\log 10 - \log 2\right]$

$= \frac{1}{2}\ln 5$. ✓

(3)

set out not always perfect. — show $\frac{1}{2}$ & 2.

iii) $\frac{1}{2}\ln 5 \doteqdot \frac{4}{5}$. ✓

$\ln 5 \doteqdot \frac{4}{5} \times 2$

$\log_e 5 \doteqdot \frac{8}{5}$. ✓

$5 \doteqdot e^{8/5}$

$5^{5/8} \doteqdot e$ ✓

(3)

not handled well. just equate part i) with part ii)

#4a) i) $y = x\log 2x - x$.

$y = \underset{u}{x}(\underset{v}{\log 2x - 1})$

$y' = (\log 2x - 1)(1) + (x)\left(\frac{2}{2x}\right)$

$y' = \log 2x - 1 + 1$

$y' = \log 2x$.

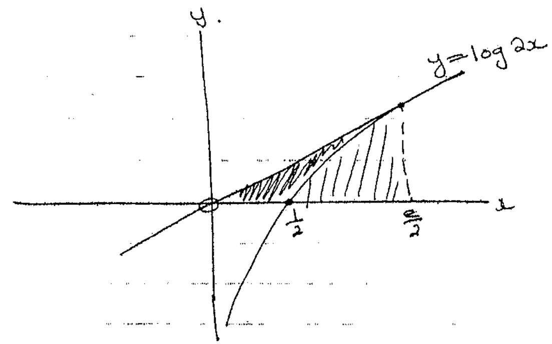
Many students had problem diff. $\log 2x$ and recognising that they had to use the product rule.

doing the hence caused difficulty for many

ii) hence $y = \int \log 2x\,dx$.

$\int \log 2x\,dx = x\log 2x - x + c$.

b) i)




$\log_e 2x = 0$.

$2x = 1$

$x = \frac{1}{2}$

iii) $m(\tan) = y' = \frac{2}{2x} = \frac{1}{x}$.

at $x = k$.

$m(\tan) = \frac{1}{k}$. $P(k, \log 2k)$

$\therefore$ eq. of tangent

$y - \log 2k = \frac{1}{k}(x - k)$

$y - \log 2k = \frac{1}{k}x - 1$.

$y = \frac{1}{k}x - 1 + \log 2k$

iv) $\overset{x}{(0}, \overset{y}{0)}$ : $0 = -1 + \log 2k$

$1 = \log_e 2k$

$e = 2k$

$\frac{e}{2} = k$.

Thus was done correctly by only a few students

v) $\text{Area} = \text{Area of } \Delta - \int_{\frac{1}{2}}^{\frac{e}{2}} \log 2x \, dx$

$= \frac{1}{2}\left(\frac{e}{2}\right)\left(\log 2.\frac{e}{2}\right) - \left[x \log 2x - x\right]_{\frac{1}{2}}^{e/2}$

$= \frac{e}{4}.\log_e e - \left[\left(\frac{e}{2}\log 2.\frac{e}{2} - \frac{e}{2}\right) - \left(\frac{1}{2}\log 2.\frac{1}{2} - \frac{1}{2}\right)\right]$

$= \frac{e}{4} - \left[\left(\frac{e}{2} - \frac{e}{2}\right) - \left(\frac{1}{2}.0 - \frac{1}{2}\right)\right]$

$= \frac{e}{4} - \left(0 - \frac{1}{2}\right)$

$= \frac{e}{4} - \frac{1}{2} \ u^2$

Incorrect understanding of area that had to be found.