ASSIGNMENT 14 : APPLICATION OF CALCULUS TO THE PHYSICAL WORLD - 2U

1 A particle moves in a straight line in such a way that its distance, x metres, from a fixed point 0 after t seconds is given by $x = -t^2 + 4t - 3$.

(a) Where is the particle initially?

(b) In which direction does it begin to move?

(c) When does the particle pass through the origin?

(d) Where and when does the particle change direction?

(e) How far does the particle travel in the first three seconds?

(f) What is the velocity of the particle each time it passes through the origin?

(g) Show that the acceleration has a constant negative value and explain the meaning of the negative.

2 A particle, initially 4 metres to the right of the origin, has a velocity in metres per second (m/s) given by:

$$v = -2\pi \sin\left(\frac{\pi}{2}t\right)$$

(a) Find the expression for the acceleration of the particle as a function of the time.

(b) Find the expression for the displacement of the particle as a function of the time and hence find when the particle first passes through the origin.

3 At a pop concert the audience enters the venue at a rate given by $\frac{dN}{dt} = 10t - \frac{t^2}{12}$ until all the audience has been admitted. N is the number of people who have been admitted t minutes after opening.

(a) How long does it take for all the audience to be admitted?

(b) Find the total number of people admitted in this time.

4 Water flows from a tank in such a way that the number of litres, L, remaining in the tank after t seconds is given by the formula:

$$L = 5000e^{-0.02t}$$

Find the rate at which the water is flowing after 30 seconds.

5 The real value v (dollars) of a certain piece of industrial equipment has been found to behave according to the function:

$$v = 100000e^{-0.1t}$$

where t measures the years since original purchase.

(a) What is the original value of the equipment?

(b) What is the expected value after five years?

(c) How long does it take for the resale value of the equipment to reach fifty per cent of its original value?

(d) Find the rate of depreciation $\frac{dv}{dt}$ after five years.

6 The number of bacteria in a culture is given by the formula:

$$N = N_0 e^{kt}$$

where t is the time in hours.

In a certain culture, the bacteria increased from 2000 to 7000 in six hours. What is the percentage rate of increase?

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(1) $x = -t^2+4t-3$

(a) At $t=0$, $x = -3$ metres

(b) $v = \frac{dx}{dt} = -2t+4$

At $t=0$, $v = 4$ m/s $\longrightarrow$ moving to the right

(c) $x=0$, $t^2-4t+3=0$

~~Hence~~ $(t-3)(t-1)=0$

$t = 3$ or 1 sec.

(d) It changes directions when $\dot{x} = 0$

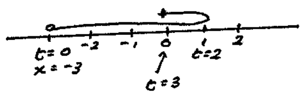
~~ie~~ $-2t+4=0$

$t=2$ at $x = -4+8-3 = 1$m

(e) At $t=0$, $x=-3$

$t=2$, $x=1$

$t=3$, $x=0$



Total distance travelled $= 4+1$

$= 5$ m

(f) At $t=1$, $\dot{x} = -2+4 = 2$ m/s

At $t=3$, $\dot{x} = -2(3)+4 = -2$ m/s

(g) $a = \frac{dv}{dt} = -2$ which is always a constant negative 2.

When the particle is in motion it is "slowing down" at 2 m/s^2.

(2) $v = -2\pi \sin\left(\frac{\pi}{2}t\right)$

(a) $a = \frac{dv}{dt} = -2\pi\cos\left(\frac{\pi}{2}t\right) \times \frac{\pi}{2}$

$= -\pi^2\cos\left(\frac{\pi}{2}t\right)$

(b) $x = \int v\,dt$

$= \int -2\pi\sin\left(\frac{\pi}{2}t\right)dt$

$= -2\pi\left(\frac{-\cos\left(\frac{\pi}{2}t\right)}{\frac{\pi}{2}}\right)+c$

$= 4\cos\left(\frac{\pi}{2}t\right)+c$

At $t=0$, $x=4$ $\Rightarrow$ $4 = 4\cos\left(\frac{\pi}{2}t\right)+c$ ($t=0$)

$\therefore c=0$ $\quad\therefore x = 4\cos\left(\frac{\pi}{2}t\right)$

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(2) (b) $x = 4\cos\left(\frac{\pi}{2}t\right)$

At $x=0$ $\quad\cos\left(\frac{\pi}{2}t\right)=0$

$\frac{\pi}{2}t = \frac{\pi}{2}$

$t = 1$ sec is the first time it passes through $x=0$

(3) $\frac{dN}{dt} = 10t - \frac{t^2}{12}$

(b) $N = \int 10t - \frac{t^2}{12}\,dt$

$= \frac{10t^2}{2} - \frac{t^3}{36}+c$

At $t=0$, $N=0$ $\Rightarrow$ $c=0$

$\therefore N = 5t^2 - \frac{t^3}{36}$

When $t=120$

$N = 5(120)^2 - \frac{120^3}{36}$

$= 24\,000$ people

(a) For all audience to be admitted

$\frac{dN}{dt} = 0$

$\therefore 10t - \frac{t^2}{12} = 0$

$t\left(10-\frac{t}{12}\right)=0$

$t = 120$ mins

(4) $L = 5000e^{-0.02t}$

$\frac{dL}{dt} = 5000 \times (-0.02)e^{-0.02t}$

When $t=30$

$\frac{dL}{dt} = -100e^{-0.02(30)}$

$= -54.88$ litres/sec (to 2d.p.)

(5) $v = 100\,000e^{-0.1t}$

(a) When $t=0$, $v = \$100\,000$

(b) When $t=5$, $v = 100\,000e^{-0.5}$

$= \$60\,653.07$

(c) $\frac{1}{2} = e^{-0.1t}$

$\therefore \ln\left(\frac{1}{2}\right) = \ln e^{-0.1t}$

$\therefore \ln\left(\frac{1}{2}\right) = -0.1t$

$t = 6.93$ years $\doteqdot 7$ years.

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(5) (d) $v = 100\,000 e^{-0.1t}$

$\therefore \frac{dv}{dt} = 100\,000 \times (-0.1) e^{-0.1t}$

At $t = 5$

$\frac{dv}{dt} = -10\,000\, e^{-0.5}$

$= -6065.31$ dollars/year