Year 12 HSC 2U Mathematics

Exponential, Logarithmic and Trigonometric Functions

Term 2, 2011 | Week 6 — *Time Allowed*: 50 mins — *Marks*: 37

Show all working to gain maximum marks — Marks will be deducted for poor or illegible work

Name: _____ — *Teacher*: HRK GHW RABS CRA

PART A – Exp. and Log Functions (13 Marks) — **Marked by RABS**

1. If $\log_a 2 = 0.35$ and $\log_a 5 = 0.57$, find $\log_a 0.4$ — **1**

2. Solve $(\log_e x)^2 - \log_e x = 2$ — **3**

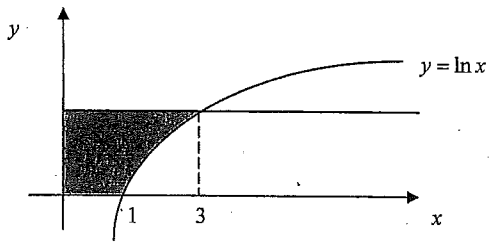
3. Differentiate $x^3 \log_e x$ with respect to x. — **2**

4.

a) Differentiate $y = e^{3x^2}$ — **1**

b) Use this result to find $\int xe^{3x^2} dx$ — **2**

5. Consider the following graph:



Determine the exact volume of the solid of revolution formed when the shaded region is rotated about the y-axis. — **4**

PART B – Trig Functions (non-Calculus) (12 Marks) — **Marked by GHW**

1. Find the exact value of the following:

a) $\sin\frac{3\pi}{4}$ — **1**

b) $\tan\frac{7\pi}{4}$ — **1**

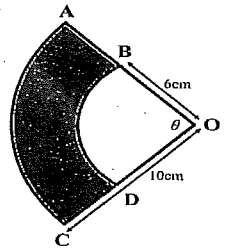
2. Solve $2\cos x = -\sqrt{3}$ for $0 \le x \le 2\pi$ — **2**

3. Consider the curve $y = 1 + 3\sin 2x$ and the line $y = 2x$

a) Sketch both the curve and the line on the same axes for $0 \le x \le 2\pi$ — **3**

b) How many solutions are there for $2x = 1 + 3\sin 2x$ in the domain $0 \le x \le 2\pi$? — **1**

4. In the following diagram AC is an arc of a circle with centre O and radius 10cm and BD is an arc of a circle with centre O and radius 6cm.



a) Find an expression for the shaded area in terms of θ. — **1**

b) Given that the shaded region is $25.6cm^2$ find the value of θ. — **1**

c) Calculate the perimeter of the shaded region. — **2**

PART C – Trig Functions (Calculus) (12 Marks) **Marked by HRK**

1. Differentiate

a) $\cos 5x$ **1**

b) $4\tan^6 2x$ **2**

c) $\dfrac{\sin x}{x^2}$ **2**

2. If $f(x)=\sin x$,

a) find $f\left(\dfrac{\pi}{3}\right)$ and $f'\left(\dfrac{\pi}{3}\right)$ **2**

b) Hence find the equation of the normal to the curve

$f(x)=\sin x$ when $x=\dfrac{\pi}{3}$ **2**

3. Find $\int 3-\sin 7x\ dx$ **2**

4. $\int 5\sec^2 x\ dx$ **1**

YEAR 12 2U ASSESS TASK 2011

LOG, EXP & TRIG F'NS

· MANY STUDENTS TRIED TO MANIPULATE USING LOG LAWS & ONLY FOUND ONE SOLUTION.

1. $\log_a 0.4 = \log_a 2/5$

$= \log_a 2 - \log_a 5$

$= 0.35 - .0.57$

$= -0.22$ ✓

· THIS Q'N SURPRISINGLY CONFUSED SOME STUDENTS.

· YOU SHOULD ALWAYS BE ABLE TO FIND THE ANSWER GIVEN THE INFO & BY USING LOG LAWS.

3. $y = x^3 \log_e x$ $u = x^3$ $v = \log_e x$

$u' = 3x^2$ $v' = \frac{1}{x}$

$\frac{dy}{dx} = \frac{x^3}{x} + 3x^2 \log_e x$ working ✓

$= x^2 + 3x^2 \log_e x$

$= x^2(1 + 3\log_e x)$ ✓

· ANSWERED QUITE WELL!

4. (a) $y = e^{3x^2}$

$\frac{dy}{dx} = 6x e^{3x^2}$ ✓

(b) $\int x e^{3x^2} dx = \frac{1}{6}\int 6x e^{3x^2} dx$ ✓

$= \frac{e^{3x^2}}{6} + c$ ✓

· SOME STUDENTS FORGOT TO INCLUDE THE CONSTANT OF INTEGRATION

2. LET $u = \log_e x$

$\therefore u^2 - u - 2 = 0$ ✓

$(u-2)(u+1) = 0$

$\therefore u = 2, -1$ ✓

$\therefore \log_e x = 2 \quad \therefore x = e^2$ ✓

$\log_e x = -1 \quad x = \frac{1}{e}$

· MANY STUDENTS DIDN'T RECOGNISE THAT THE EQ'N COULD BE REDUCIBLE TO A QUADRATIC.

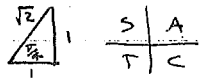
PART B - TRIG FUNCTIONS (non-calculus)

(1) (a) $\sin\left(\frac{3\pi}{4}\right)$

$= \sin\left(\frac{4\pi}{4} - \frac{\pi}{4}\right)$

$= \sin\left(\pi - \frac{\pi}{4}\right)$

$= \frac{1}{\sqrt{2}}$ ✓



(b) $\tan\left(\frac{7\pi}{4}\right)$

$= \tan\left(\frac{8\pi}{4} - \frac{\pi}{4}\right)$

$= \tan\left(2\pi - \frac{\pi}{4}\right)$

$= -1$ ✓

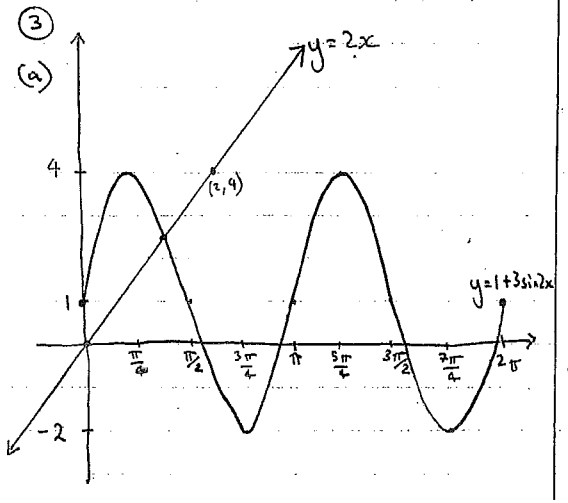
(2) $2\cos x = -\sqrt{3}$ for $0 \leq x \leq 2\pi$

$\cos x = -\frac{\sqrt{3}}{2}$

$\therefore x = \pi - \frac{\pi}{6}, \pi + \frac{\pi}{6}$

$\therefore x = \frac{5\pi}{6}, \frac{7\pi}{6}$ ✓✓

(3) (a)



✓ · 1 mark for correct $y = 2x$

✓ · 1 mark for correct range

✓ · 1 mark for correct period drawn

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5. If $y = \ln x \Rightarrow x = e^y$ ✓

VOL. ABOUT y-AXIS.

CHANGING LIMITS ... when $x = 3$, $y = \ln 3$

when $x = 1$, $y = 0$

MUST RE-ARRANGE EQN. ETC...

$\text{Vol} = \pi \int x^2\,dy$

$= \pi \int_0^{\ln 3} e^{2y}\,dy$ ✓

$= \pi \left[\frac{e^{2y}}{2}\right]_0^{\ln 3}$

$= \pi \left(\frac{e^{2\ln 3}}{2} - \frac{1}{2}\right)$ ✓

$= \pi \left(\frac{e^{\ln 9}}{2} - \frac{1}{2}\right)$

$= \pi/2\,(9 - 1) = 8\pi/2 = 4\pi$ UNITS^3 ✓

- STUDENTS TRIED TO FIND $\int_1^3 \ln x\,dx$.

READ THE QUESTION! (AND REALISE THAT WE DON'T KNOW HOW TO INTEGRATE $\log_e x$)

- SOME ANSWERS WEREN'T COMPLETELY SIMPLIFIED (BUT NO MARKS WERE DEDUCTED THOUGH)

(3)
(b) 1 solution ✓

(4) (a) Shaded area =

Area of larger sector − Area of smaller sector

$\therefore A = \frac{1}{2} \times 10^2 \times \theta - \frac{1}{2} \times 6^2 \times \theta$

$= 50\theta - 18\theta$

$\therefore A = 32\theta$ ✓

(b) $25 \cdot 6 = 32\theta$

$\therefore \theta = 0 \cdot 8$ ✓

(c) $P = 4 + 4 + \text{Small Arc} + \text{Big Arc}$

$= 4 + 4 + (6 \times 0 \cdot 8) + (10 \times 0 \cdot 8)$ ✓

$= 8 + 4 \cdot 8 + 8$

$= 20 \cdot 8 \text{ cm}$ ✓

PART C

✓ = 1 MARK

HRK

1 (a) $\frac{d}{dx} \cos 5x = -5 \sin 5x$ ✓

b) $\frac{d}{dx} 4 \tan^6(2x) = \frac{d}{dx} 4 (\tan 2x)^6$

$= 24 (\tan 2x)^5 \times 2 \sec^2 2x$ ✓ ✓

$= 48 \tan^5(2x) \sec^2(2x)$

A number of students put the 4 in the parentheses WRONG! ☹

c) $\frac{d}{dx}\left(\frac{\sin x}{x^2}\right)$ u v

$= \frac{x^2 \cos x - \sin x (2x)}{x^2}$ ✓✓

$= \frac{x(x \cos x - 2 \sin x)}{x^3}$

$= \frac{x \cos x - 2 \sin x}{x}$

A few still need to LEARN the formula!

2/a) $f\left(\frac{\pi}{3}\right) = \sin \frac{\pi}{3}$

$= \frac{\sqrt{3}}{2}$

$f'(x) = \cos x$

$f'\left(\frac{\pi}{3}\right) = \frac{1}{2}$

✓ Well done

b) $m_N = -2$ ✓

$y - \frac{\sqrt{3}}{2} = -2\left(x - \frac{\pi}{3}\right)$ ✓

$12x + 6y - 3\sqrt{3} - 4\pi = 0$

3/ $\int 3 - \sin 7x \, dx$ ✓✓

$= 3x + \frac{\cos 7x}{7} + C$

4/ $5 \int \sec^2 x \, dx = 5 \tan x + C$

} one mark deducted if BOTH C's omitted.