EXPONENTIAL AND LOGARITHMIC FUNCTIONS

1) Evaluate $e^{2.7}$ correct to two decimal places.
2) Sketch the curve $y = e^{-x}$
3) Differentiate $4e^x$
4) Differentiate xe^x
5) Find the derivative of $\frac{x}{e^x}$
6) Differentiate $(1 + e^x)^9$
7) Differentiate e^{5x}
8) Find the derivative of $e^{3x} + 2x$.
9) Find the exact gradient of the tangent to the curve $y = e^x$ at the point where $x = 3$.
10) Find the equation of the tangent to the curve $y = e^{2x}$ at the point $(1,e^2)$.
11) Find the equation of the normal to the curve $y = 2e^{4x} + 1$ at the point where $x = 0$.
12) Find the stationary point on the curve $y = (x + 1)\, e^x$ and determine its nature.
13) Find the indefinite integral (primitive function) of e^{7x}.
14) Find $\int (e^{2x} - e^{-x})dx$
15) Evaluate $\int_0^1 e^{3x}dx$ to two decimal places.
16) Find the exact value of $\int_1^5 e^{5x}dx$
17) Find the exact area bounded by the curve $y = 2e^{2x}$, the x-axis and the lines $x = 1$ and $x = 3$
18) Find the area enclosed between the curve $y = e^{3x+1}$, the x-axis and the lines $x = 0$ and $x = 2$ correct to 3 significant figures..
19) Find the exact volume of the solid formed when the curve $y = e^{2x}$ is rotated about the x-axis from $x = 1$ to $x = 3$.
20) Use Simpson's rule with five function values to find an approximation to $\int_1^3 \frac{e^x}{x}dx$ to two decimal places.
21) Evaluate $\log_{10}$ 56.3 to one decimal place.
22) Evaluate ln 6.89 to two significant figures.
23) Evaluate $\log_2 32$.
24) Evaluate $\log_5 125$.
25) Evaluate $\log_{36} 6$.
26) Evaluate $\log_7 7$.
27) Evaluate $\log_9 1$.
28) Evaluate $\log_2 \frac{1}{16}$
29) Sketch the curve $y = \ln x$ on a number plane.
30) Solve $\log_x 16 = 2$
31) Solve $\log_3 y = 4$
32) Simplify $\log_a p + \log_a q - \log_a r$
33) Simplify $2\log_n 3 + \log_n y$

34) Evaluate $\log_8 4 + \log_8 16$.

35) Show that $\log_a 8 + 2\log_a 3 - \log_a 6 = \log_a 12$.

36) Given $\log_5 3 = 0.68$ and $\log_5 4 = 0.86$, find
(a) $\log_5 12$
(b) $\log_5 0.75$
(c) $\log_5 27$
(d) $\log_5 15$

37) If $\log_p x = n$ and $\log_p y = t$, find $\log_p xy^2$ in terms of n and t.

38) Evaluate $\log_4 7$ to one decimal place.

39) Solve $2^x = 5$ to three significant figures.

40) Solve $5^{2x+1} = 3$ to two decimal places.

41) Solve $2e^x = 13$ to two significant figures.

42) Consider the equation $x = 3e^{2t}$.
(a) Find x when $t = 2$, to one decimal place.
(b) Find t when $x = 100$, to one decimal place.
(c) Show that $\frac{d^2x}{dt^2} = 4x$.

43) Differentiate $\log_e (x+1)$

44) Differentiate $(\ln x + x^2)^4$

45) Differentiate $x\ln x$.

46) Differentiate $\log_e \frac{2x-3}{x+1}$

47) Differentiate $\log_e (e^x + 1)$

48) Find the equation of the tangent to the curve $y = \log_e x$ at the point $(4, \log_e 4)$.

49) Show that the function $f(x) = \ln (2x + 3)$ has no stationary points.

50) Find the indefinite integral (primitive function) of $\frac{1}{x-3}$

51) Evaluate $\int_1^4 \frac{dx}{2x-1}$ to two decimal places.

52) Find the exact value of $\int_2^3 \frac{x}{x^2-1} dx$

53) (a) Find the exact area bounded by the curve $y = \frac{1}{x}$, the x-axis and the lines $x = 2$ and $x = 5$.
(b) This area is rotated about the x-axis. Find the volume of the solid formed.

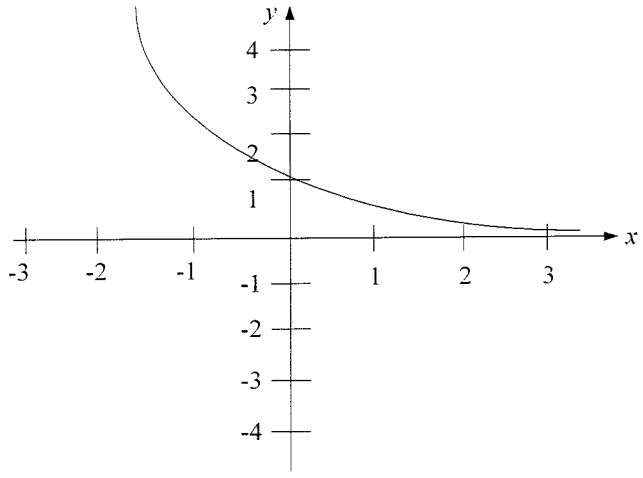
54) The curve $y = \ln x$ is rotated about the y-axis from $y = 0$ to $y = 2$. Find the exact volume of the solid of revolution.

55) (a) Find the approximate area bounded by the curve $y = \ln x$, the x-axis and the lines $x = 1$ and $x = 3$ by using Simpson's rule with three function values (to two decimal places).
(b) By considering an area involving the y-axis, find the exact area in part (a).

ANSWERS

1) 14.88

2)



3) $4e^x$

4) $xe^x + e^x = e^x(x+1)$

5) $\dfrac{e^x - xe^x}{e^{2x}} = \dfrac{1-x}{e^x}$

6) $9e^x(1+e^x)^8$

7) $5e^{5x}$

8) $3e^{3x} + 2$

9) e^3

10) $2e^2x - y - e^2 = 0$

11) $x + 8y - 24 = 0$

12) $(-2, -e^2)$ minimum

13) $\dfrac{1}{7}e^{7x} + C$

14) $\dfrac{1}{2}e^{2x} + e^{-x} + C$

15) 6.36

16) $\dfrac{1}{5}e^5(e^{20} - 1)$

17) $e^2(e^4 - 1)$ units^2

18) 365 units^2

19) $\dfrac{\pi}{4}e^4(e^8 - 1)$ units^3

20) 8.04

21) 1.8

22) 1.9

23) 5

24) 3

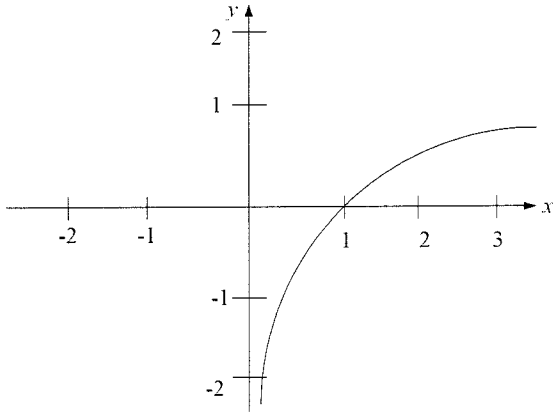
25) $\frac{1}{2}$

26) 1

27) 0

28) -4

29)



30) $x = 4$

31) $y = 81$

32) $\log_a \frac{pq}{r}$

33) $\log_n 9y$

34) 2

35) LHS $= \log_a 8 + 2\log_a 3 - \log_a 6$

$= \log_a 8 + \log_a 3^2 - \log_a 6$

$= \log_a \frac{8 \times 3^2}{6}$

$= \log_a 12$

$=$ RHS

So $\log_a 8 + 2\log_a 3 - \log_a 6 = \log_a 12$.

36) (a) 1.54 (b) -0.18 (c) 2.04 (d) 1.68

37) $n + 2t$

38) 1.4

39) $x = 2.32$

40) $x = -0.16$

41) $x = 1.9$

42) (a) $x = 163.8$ (b) $t = 1.8$

(c) $x = 3e^{2t}$

$\frac{dx}{dt} = 6e^{2t}$

$\frac{d^2x}{dt^2} = 12e^{2t}$

$= 4(3e^{2t})$

$= 4x$

43) $\dfrac{1}{x+1}$

44) $4(\dfrac{1}{x}+2x)(\ln x + x^2)^3$

45) $1 + \ln x$

46) $\dfrac{2}{2x-3} - \dfrac{1}{x+1} = \dfrac{5}{(2x-3)(x+1)}$

47) $\dfrac{e^x}{e^x+1}$

48) $x - 4y + 4\log_e 4 - 4 = 0$

49) $f^1(x) = \dfrac{2}{2x+3} \neq 0$ for any x.

50) $\ln(x - 3) + C$

51) 0.97

52) $\dfrac{1}{2}\ln\left(\dfrac{8}{3}\right) + C$

53) (a) $\ln\left(\dfrac{5}{2}\right)$ units^2 (b) $\dfrac{3\pi}{10}$ units^3

54) $\dfrac{\pi}{2}(e^4 - 1)$ units^3

55) (a) 1.29 (b) 3ln3 - 2 units^2