FORT STREET HIGH SCHOOL

Name: ____________

Teacher: ____________

Class: ____________

2015

HIGHER SCHOOL CERTIFICATE PRELIMINARY COURSE

ASSESSMENT TASK 2

Mathematics Extension 1

Time allowed: 1½ Hours
(plus 5 minutes reading time)

Syllabus Outcomes	Assessment Area Description and Marking Guidelines	Questions
P2	provides reasoning to support conclusions which are appropriate in the context of locus	1
P4	chooses and applies appropriate arithmetic and algebraic techniques to solve problems in quadratics	2
PE3	solves problems involving trigonometry	3
PE3	solves problems involving polynomials	4
HE7	Synthesises mathematical solutions to problems and communicates them in an appropriate form	5

Total Marks 70
Attempt Questions 1-5

Question	Out of	Marks
Q1	/17	
Q2	/11	
Q3	/17	
Q4	/14	
Q5	/11	
	Percent:	

General Instructions:

- Questions 1-5 are to be started in a new booklet
- The marks allocated for each question are indicated
- In Questions 1-5, show relevant mathematical reasoning and/or calculations.
- Marks may be deducted for careless or badly arranged work.
- Board – approved calculators may be used

Question 1 (17 marks) Begin a NEW booklet

a) Consider the parabola $2x = y^2 - 4y$

i) Find its vertex, focus, directrix and axis of symmetry. [5]

ii) Sketch a neat graph of this parabola, clearly showing the above features as well as any intercepts. [2]

b)

i) Find the equation of the locus of a point $P(x, y)$ which moves so that the line PA is perpendicular to the line PB, where $A = (-3,2)$ and $B = (1,4)$. [2]

ii) Describe this locus geometrically and state its features. [2]

iii) For what values of m is the line $y = mx$ a tangent to this locus? [3]

c) Find the locus of a point $P(x, y)$ which moves so that its distance from the line $x = -1$ is always twice its distance from the line $y = 2$. Describe this locus geometrically. [3]

Question 2 (11 marks) Begin a NEW booklet

a) Find the maximum value of the quadratic function $y = 16 - 3x - 2x^2$. [2]

b) The roots of the quadratic equation $x^2 + 4x + 2 = 0$ are α and β.

i) Find $\alpha + \beta$. [1]

ii) Find $\alpha\beta$. [1]

iii) Find $\alpha^2 + \beta^2$. [2]

iv) Hence, or otherwise, write down the quadratic equation whose roots are $\frac{1}{\alpha^2}$ and $\frac{1}{\beta^2}$. [2]

c) Find all real values of t for which the quadratic equation $(1-t)x^2 - (1-t)x - t = 0$ has real and different roots. [3]

Question 3 (17 marks) Begin a NEW booklet

a) i) Convert $137°45'$ in radians, correct to 3 decimal places. [1]

ii) Convert 5.63^c correct to the nearest minute. [1]

b) Simplify: $\dfrac{\sin(\frac{\pi}{2} - x)\cos(\pi - x)\tan(2\pi - x)}{\sin(\pi + x)\cos(2\pi + x)\tan(\pi + x)}$ [2]

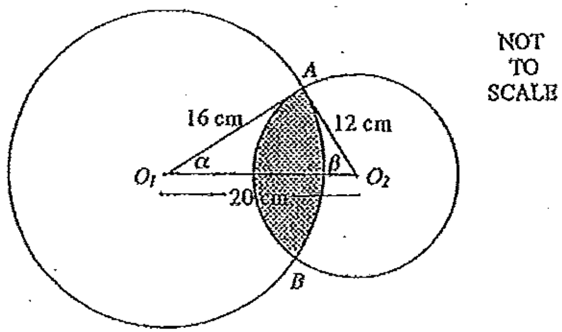
c) Evaluate exactly:

i) $\cos\left(-\dfrac{3\pi}{4}\right)$ [1]

ii) $\cot\dfrac{11\pi}{6}$ [2]

d) Solve the trigonometric equation $3\tan\theta + \sqrt{3} = 0$, for $0 \le \theta \le 2\pi$. [2]

e)



i) Show that triangle O_1AO_2 is a right angled triangle. [1]

ii) Find the size of the angles α and β in radians, correct to 2 dec. pls. [2]

iii) Find the area of sector AO_1B, correct to the nearest cm^2. [2]

iv) Find the shaded area enclosed by these circles, correct to the nearest cm^2. [3]

Question 4 (14 marks) Begin a NEW booklet

a) If α, β, γ are the roots of the equation $2x^3 - 4x^2 + 7x - 4 = 0$, find the value of:

i) $(2\alpha+1)+(2\beta+1)+(2\gamma+1)$ [2]

ii) $\alpha^2+\beta^2+\gamma^2$ [2]

iii) $\alpha^3+\beta^3+\gamma^3$ [2]

b) The polynomial $P(x)=8x^3+12x^2-18x-20=0$ has a root for $P(x)=0$ at $x=-2$.

i) Find all the roots of $P(x)=0$. [2]

ii) Draw a neat sketch of the graph of $y=P(x)$, showing the coordinates of its points of intersection with the axes. [2]

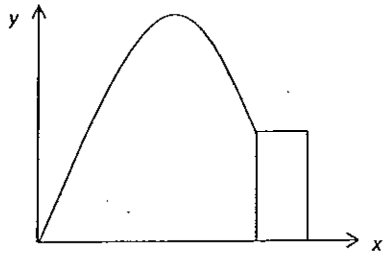
c) A monic polynomial $P(x)$ of degree 4 has zeros 2 and -2.

i) Write down $P(x)$ in factored form. [1]

ii) Find the polynomial $P(x)$ such that $P(0)=4$ and the remainder is 3 when $P(x)$ is divided by $x-1$. [3]

Question 5 (11 marks) Begin a NEW booklet

a) A water spout reaches a maximum height of 40 metres at a point 20 metres away from its source on the ground. It lands on the edge of the roof of a building 25 metres away. How high is the building? (A water spout traces out a parabola.) [3]



b) Two of the roots of the equation $x^3+ax^2+b=0$ are reciprocals of each other where a and b are real numbers.

Show that

i) The third root is equal to $-b$ [1]

ii) $a=b-\frac{1}{b}$; and [2]

iii) The two roots, which are reciprocals, will be real if $-\frac{1}{2}\le b\le\frac{1}{2}$ [2]

c) A polynomial $P(x)$ is divided by (x^2-k^2) so that a remainder $R(x)$ is obtained.

Show that the remainder is given by [3]

$$R(x)=\frac{1}{2k}[P(k)-P(-k)]x+\frac{1}{2}[P(k)+P(-k)]$$

SOLUTIONS ~ YEAR 11 EXTENSION 1, TASK 2, 2015

QUESTION 1 (17 marks)

Comments

a) $2x = y^2 - 4y$

i) $y^2 - 4y + 4 = 2x + 4$

$(y-2)^2 = 2(x+2)$ ← ① for eqn. in this form

$(y-k)^2 = 4a(x-h)$

vertex $(h,k) = (-2,2)$ ← ①

$4a = 2 \quad \therefore a = \frac{1}{2}$

focus $= (-1\frac{1}{2}, 2)$ ← ①

directrix: $x = -2\frac{1}{2}$ ← ①

axis: $y = 2$ ← ①

/5

Comments: Simple question but lots of errors

Need to learn thoroughly

ii) Intercepts: $x = 0 \rightarrow (y-2)^2 = 4$

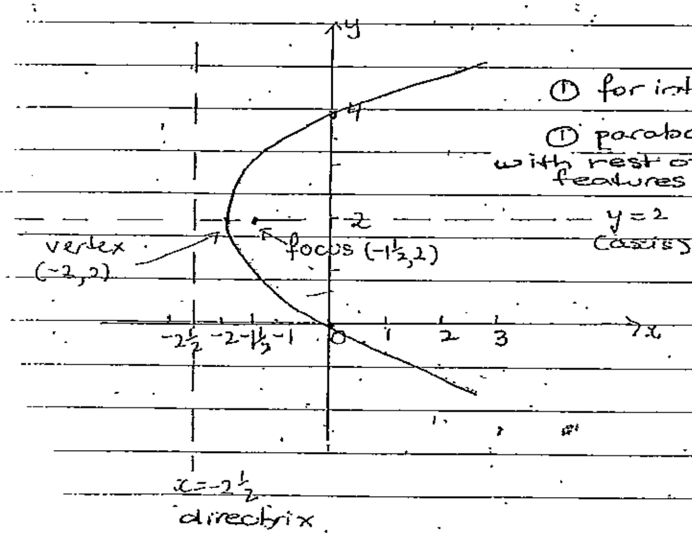
$y - 2 = \pm 2$

$\therefore y = 0$ or 4.

$y = 0 \rightarrow 4 = 2(x+2)$

$2 = x + 2$

$x = 0$



① for intercepts

① parabola with rest of features shown

/2

b) ii) $\dfrac{y-2}{x+3} \times \dfrac{y-4}{x-1} = -1$ ← ① for expressing condition of locus algebraically

$(y-2)(y-4) = -1(x+3)(x-1)$

$y^2 - 4y - 2y + 8 = -(x^2 - x + 3x - 3)$

$y^2 - 6y + 8 = -x^2 + 2x + 3$

$x^2 + y^2 + 2x - 6y + 5 = 0$ is the equation of the locus ← ① for equation

/2

ii) $x^2 + 2x + 1 + y^2 - 6y + 9 = -5 + 10$ → ① for writing eqn in this form

$(x+1)^2 + (y-3)^2 = 5$ → in form

$(x-a)^2 + (y-b)^2 = r^2 \therefore$ locus is a circle, centre $= (-1,3)$

radius $= \sqrt{5}$ units. } ① for description with main features stated.

/2

Comments: Silly error regarding centre of circle.

iii) $y = mx$ ①

$(x+1)^2 + (y-3)^2 = 5$ ②

sub ① into ②

$(x+1)^2 + (mx-3)^2 = 5$ ← ① for quad. eqn. in x

$x^2 + 2x + 1 + m^2x^2 - 6mx + 9 = 5$

$(1+m^2)x^2 + (2-6m)x + 5 = 0$

$\Delta = 0$ } ① for saying $\Delta = 0$

i.e. $(2-6m)^2 - 4(1+m^2)5 = 0$

$4 - 24m + 36m^2 - 20 - 20m^2 = 0$

$16m^2 - 24m - 16 = 0$

$2m^2 - 3m - 2 = 0$

$(2m+1)(m-2) = 0$

$\therefore m = -\frac{1}{2}$ or $m = 2$ for line $y = mx$ to be a tangent to the circle. ← ① for correct values of m.

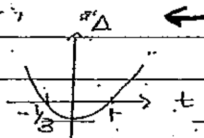
/3

Comments: Many used alternative method of ⊥ distance of point to a line which was also quite acceptable

Question 2 (11 marks)

	Comments
a) $y = 16 - 3x - 2x^2$	Poorly Done.
max. at vertex where $x = \frac{-b}{2a} = \frac{3}{-4}$ ← ① for x value	Many students were not able to state the maximum value. Instead they gave coordinates of vertex or said max. value is $x = -\frac{3}{4}$ ∴ lost one mark
sub. in for y:	
$y = 16 - 3\left(\frac{3}{-4}\right) - 2\left(\frac{3}{-4}\right)^2$	
$= 16 + \frac{9}{4} - \frac{18}{16}$	
$y = 17\frac{1}{8}$,	
∴ maximum value $= 17\frac{1}{8}$ $\left(= \frac{137}{8}\right)$ ← ① mark. $\frac{1}{2}$	
b) $x^2 + 4x + 2 = 0$	
i) $\alpha + \beta = -4$ ← ①	i) well done
ii) $\alpha\beta = 2$ ← ①	ii) well done
iii) $\alpha^2 + \beta^2 = (\alpha+\beta)^2 - 2\alpha\beta$ ← ① for knowing identity	iii) well done
$= 16 - 4$	iv) Poorly done.
$= 12$ ← ① for value	The quadratic eqn. should be written as $4x^2 - 12x + 1 = 0$.
iv) $x^2 - \left(\frac{1}{\alpha^2} + \frac{1}{\beta^2}\right)x + \frac{1}{\alpha^2\beta^2} = 0$ } ① either	Many left out '$=0$' so they were not giving a quadratic eqn.
$x^2 - \left(\frac{\beta^2+\alpha^2}{\alpha^2\beta^2}\right)x + \frac{1}{\alpha^2\beta^2} = 0$ }	In future marks will be deducted if eqn. is not given
$x^2 - 3x + \frac{1}{4} = 0$	
$4x^2 - 12x + 1 = 0$ ← ① for equation. $\frac{}{6}$	
c) $(1-t)x^2 - (1-t)x - t = 0$	
$\Delta = (1-t)^2 - 4(1-t)(-t)$	Poorly done.
$= 1 - 2t + t^2 + 4t - 4t^2$	Students need to be careful
$= 1 + 2t - 3t^2$ ← ① for finding Δ	with signs.
For real, distinct roots	Some solved for when $\Delta = 0$.
$\Delta > 0$	Students need to revise discriminant problems !!
i.e. $1 + 2t - 3t^2 > 0$ ← ① for setting up inequality to solve.	
$3t^2 - 2t - 1 < 0$	
$(3t+1)(t-1) < 0$	
∴ $-\frac{1}{3} < t < 1$ ← ① for t values $\frac{}{3}$	

$3t \quad +1$
$t \quad -1$



Question 3 (17 marks) — comments

a) i) $137°45' = 137°45' \times \frac{\pi}{180}$

$\doteqdot 2.404^c$ ← ① — mostly well done, some left answers with π

ii) $5.63^c = 5.63 \times \frac{180}{\pi}$

$\doteqdot 322.57523$

$\doteqdot 322°35'$ ← ① — well done

/2

b) $\frac{\sin(\frac{\pi}{2} - x)\cos(\pi - x)\tan(2\pi - x)}{\sin(\pi + x)\cos(2\pi + x)\tan(\pi + x)}$ — $\sin(\frac{\pi}{2} - x) \neq \sin x$, $\sin(\pi + x) \neq \sin(90 + x)$

$= \frac{(\cos x)(-\cos x)(-\tan x)}{(-\sin x)(\cos x)(+\tan x)}$ ← ① for showing understanding of ∠s of any magnitude results

$= \frac{-\cos x}{-\sin x} \times -1$

$= -\cot x$ ← ① for giving answer as $-\cot x$

/2

c) i) $\cos(-\frac{3\pi}{4})$ — students forgot the negative

$= -\cos\frac{\pi}{4}$

$= -\frac{1}{\sqrt{2}}$ ← ① for answer

ii) $\cot\frac{11\pi}{6}$ — mostly well done

$= \cot(2\pi - \frac{\pi}{6})$

$= -\cot\frac{\pi}{6}$ ← ① for some working

$= -\frac{1}{\tan\frac{\pi}{6}}$ — students forgot the negative

$= -\frac{1}{\frac{1}{\sqrt{3}}}$

$= -\sqrt{3}$ ← ① for answer

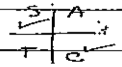
/3

Question 3 ct'd — Comments

d) $3\tan\theta = -\sqrt{3}$, for $0 \le \theta \le 2\pi$ — students wrote answers in degrees.

$\tan\theta = \frac{-\sqrt{3}}{3} = \frac{-1}{\sqrt{3}}$

acute ∠ $= \frac{\pi}{6}$



← ① for correct acute ∠

$\therefore \theta = \pi - \frac{\pi}{6}, 2\pi - \frac{\pi}{6}$

$\theta = \frac{5\pi}{6}, \frac{11\pi}{6}$ ← ① for both answers in radians

/2

e) i) $16^2 + 12^2$ — many students just wrote numbers and did not mention Pythagoras

$= 256 + 144$ ← ① for showing Pythagoras' Thm is satisfied

$= 400$

$= 20^2$

Since ΔO_1AO_2 obeys Pythagoras' Theorem, it is a right-Δ

ii) $\sin\alpha = \frac{12}{20}$ — mostly well done

$\therefore \alpha \doteqdot 0.64^c$ ← ① for α in radians

$\sin\beta = \frac{16}{20}$

$\therefore \beta \doteqdot 0.93^c$ ← ① for β in radians

iii) $A = \frac{1}{2}r^2\theta$ — students forgot to double the angle

$= \frac{1}{2}r^2 \cdot 2\alpha$

$= 16^2 \times 0.64^c$ ← ① for working

$= 163.84$

$= 164\ cm^2$ (nearest cm^2) ← ① for answer

iv) Shaded area — students forgot to double the angle.

= sum of areas of minor segments

$= \frac{1}{2} \times 16^2(2\alpha - \sin 2\alpha) + \frac{1}{2} \times 12^2(2\beta - \sin 2\beta)$

① mark $= 128(1.28 - \sin 1.28) + 72(1.86 - \sin 1.86)$

$\doteqdot 41.21396988 + 64.91006762$ ① mark

$= 106.1240375$

$= 106\ cm^2$ (nearest cm^2) ← ① for answer

/8

Question 4 — Comments

a) $2x^3-4x^2+7x-4=0$ — Well done.

i) $(2\alpha+1)+(2\beta+1)+(2\gamma+1)$

$=2(\alpha+\beta+\gamma)+3$ ← ①

$=2\left(\frac{4}{2}\right)+3$

$=7$ ← ①

ii) $\alpha^2+\beta^2+\gamma^2$ — Well done.

$=(\alpha+\beta+\gamma)^2-2(\alpha\beta+\alpha\gamma+\beta\gamma)$ ← ①

$=\left(\frac{4}{2}\right)^2-2\left(\frac{7}{2}\right)$

$=-3$ ← ①

iii) $\alpha^3+\beta^3+\gamma^3$ — Poorly done. Not many students knew how to do part (iii)

$2\alpha^3-4\alpha^2+7\alpha-4=0$ ①

$2\beta^3-4\beta^2+7\beta-4=0$ ②

$2\gamma^3-4\gamma^2+7\gamma-4=0$ ③

① + ② + ③ — ① mark

$2(\alpha^3+\beta^3+\gamma^3)-4(\alpha^2+\beta^2+\gamma^2)+7(\alpha+\beta+\gamma)-12=0$

$2(\alpha^3+\beta^3+\gamma^3)-4(-3)+7(2)-12=0$

$2(\alpha^3+\beta^3+\gamma^3)=-14$

$\alpha^3+\beta^3+\gamma^3=-7$ ← ① mark /6

OR may use: $\alpha^3+\beta^3+\gamma^3=(\alpha+\beta+\gamma)(\alpha^2+\beta^2+\gamma^2-\alpha\beta-\alpha\gamma-\beta\gamma)+3\alpha\beta\gamma$

b) $P(x)=8x^3+12x^2-18x-20$ — Well done

$$
\begin{array}{r l}
 & 8x^2-4x-10 \\
x+2 \,) & 8x^3+12x^2-18x-20 \\
 & 8x^3+16x^2 \\
 & -4x^2-18x-20 \\
 & -4x^2-8x \\
 & -10x-20 \\
 & -10x-20 \\
 & 0
\end{array}
$$

← ① for division

Question 4 ct'd

b) ct'd

$\therefore P(x)=(x+2)(8x^2-4x-10)$

$=2(x+2)(4x^2-2x-5)$

$4x^2-2x-5=0$

when $x=\dfrac{2\pm\sqrt{84}}{8}$

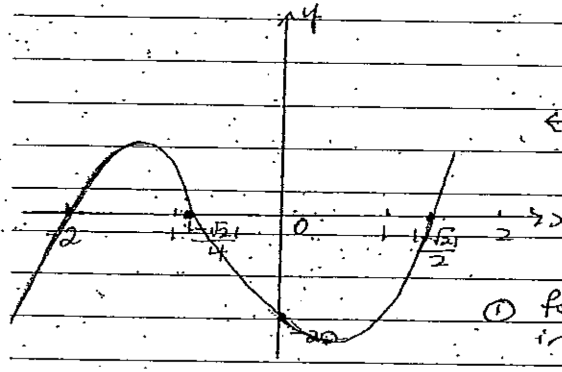
$x=\dfrac{2\pm2\sqrt{21}}{8}$

$=\dfrac{1\pm\sqrt{21}}{4}$

$\therefore$ roots are $x=-2,\ \dfrac{1\pm\sqrt{21}}{4}$ ← ① for all of roots

Comments: Some students wrote roots as $\pm\frac{\sqrt{21}}{4}$

ii) Well done once correct roots were found in (i)



← ① for shape

① for intercepts /4

c) $P(x)=x^4+\ldots$

i) $P(x)=(x-2)(x+2)(x^2+bx+c)$ ← ① mark

ii) $P(0)=4$

$4=(-2)(2)(c)$

$4=-4c$

$c=-1$ ← ① for 'c'

$P(1)=3$

$3=(-1)(3)(1+b-1)$

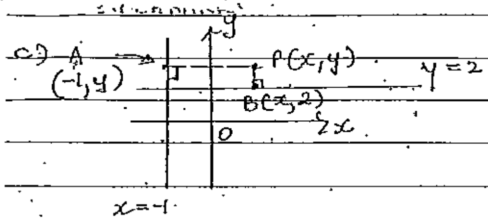
$-1=b$ ← ① for 'b'

$\therefore P(x)=(x-2)(x+2)(x^2-x-1)$ ← ① for $P(x)$ /4

Overall (c) done successfully by few students.

Comments

c)



$PA = 2PB$

$PA^2 = 4PB^2$

$(x+1)^2 = 4(y-2)^2 \quad \leftarrow ①$

$(x+1)^2 - 4(y-2)^2 = 0$

$(x+1-2(y-2))(x+1+2(y-2)) = 0$

$(x-2y+5)(x+2y-3) = 0$

$\therefore x-2y+5=0 \; , \; x+2y-3=0 \quad \leftarrow ①$

Locus consists of 2 straightlines $\leftarrow ①$

$\frac{}{3}$

Or

$|x+1| = 2|y-2|$.

and then need to consider

$(x+1) = 2(y-2)$

OR

$(x+1)' = -2(y-2)$

many arithmetical errors right throughout.

Question 5 (11 marks)

a) Eqn of parabola is in form

$(x-h)^2 = -4a(y-k)$

$\therefore (x-20)^2 = -4a(y-40)$ ← ① mark

Find a:

sub $(0,0)$

$\therefore (0-20)^2 = -4a(0-40)$

$400 = 160a$

$a = 2\cdot5$ ← ① for 'a'

$\therefore$ eqn. becomes

$(x-20)^2 = -10(y-40)$

when $x = 25$:

$(25-20)^2 = -10(y-40)$

$25 = -10y + 400$

$-375 = -10y$

$y = 37\cdot5$ ← ① for height

$\therefore$ height of building is $37\cdot5$ m

/3

Some students used $(x-h)^2 = 4a(y-k)$ form, forgetting negative. ∴ obtaining a negative focal length.

b) $x^3 + ax^2 + b = 0$

i) let roots be $\alpha, \frac{1}{\alpha}, \beta$

$\alpha + \frac{1}{\alpha} + \beta = -a$ ①

$\alpha \times \frac{1}{\alpha} + \alpha\beta + \frac{1}{\alpha} \times \beta = 0$ ②

$\alpha \times \frac{1}{\alpha} \times \beta = -b$ ③ ← ① mark for showing result by product of roots

From ③ $\beta = -b$

Mostly well done

ii) $\alpha + \frac{1}{\alpha} - b = -a$ from ①

$\alpha + \frac{1}{\alpha} = b - a$ ← ① mark for writing $\alpha + \frac{1}{\alpha}$ in terms of b & a.

$1 - \alpha b - \frac{b}{\alpha} = 0$ from ②

$1 - b\left(\alpha + \frac{1}{\alpha}\right) = 0$

$1 - b(b-a) = 0$

$\therefore b - a = \frac{1}{b}$

① mark for ← writing eqn. 2 in terms of a & b & making 'a' the subject

By remainder theorem: some subbed $\gamma = -b$ into eqn. to resolve $a = b - \frac{1}{b}$ 4

Question 5 ct'd. COMMENTS

iii) $\alpha + \frac{1}{\alpha} = b - a$

i.e. $\alpha + \frac{1}{\alpha} = b - \left(b - \frac{1}{b}\right)$

$\alpha + \frac{1}{\alpha} = \frac{1}{b}$

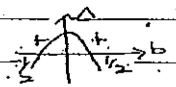
$b\alpha^2 + \alpha + b = 0$ ← ① mark

real roots: $\Delta \geq 0$

i.e. $1 - 4b^2 \geq 0$ ← ① for solving $\Delta \geq 0$ to obtain required result for 'b'

$(1+2b)(1-2b) \geq 0$

$\therefore -\frac{1}{2} \leq b \leq \frac{1}{2}$



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c) $P(x) = (x-k)(x+k)Q(x) + ax + b$, $R(x) = ax + b$

$P(k) = ak + b$ ①

$P(-k) = -ak + b$ ② ← ① mark for finding $P(k)$ & $P(-k)$

$P(k) + P(-k) = 2b$ ① + ②

$\therefore \frac{P(k) + P(-k)}{2} = b$

$P(k) - P(-k) = 2ak$ ① − ②

$\therefore \frac{P(k) - P(-k)}{2k} = a$

① mark for finding 'a' & 'b'

$\therefore R(x) = ax + b$ ← ① mark for substituting into $R(x)$.

$= \frac{P(k) - P(-k)}{2k} \cdot x + \frac{P(k) + P(-k)}{2}$

/3

many stopped at resolution $P(k)$ and $P(-k)$