Name: ______________________

Teacher: ______________________

Class: ______________________

FABER EST SUAE QUISQUE FORTUNAE

FORT STREET HIGH SCHOOL

2010

HIGHER SCHOOL CERTIFICATE COURSE

ASSESSMENT TASK 1

Mathematics Extension 1

TIME ALLOWED: 45 MINUTES

Outcomes Assessed	Questions	Marks
Applies appropriate techniques to solve problems involving parametric representations	1,2	/21
Applies appropriate techniques from the study of calculus to solve problems	3	/9

Question	1	2	3	Total	%
Marks	5 ~~4~~ 5	14 /14	8 /9	27 /~~30~~ 28	

Directions to candidates:

- Attempt all questions
- The marks allocated for each question are indicated
- All necessary working should be shown in every question. Marks may be deducted for careless or badly arranged work.
- Board – approved calculators may be used
- Each new question is to be started in a new booklet

Question 1 (7 marks)

a) Write the Cartesian equation which is described by the parametric equations $x = t + 1, y = \frac{1}{t}$ **2**

b) i) Find the equation of the chord of contact of the tangents from the point $T(3,-2)$ to the parabola $x^2 = 8y$ **3**

ii) Show that this chord of contact passes through the focus of the parabola **2**

Question 2 (14 marks)

a) $P(2ap, ap^2)$, $Q(2aq, aq^2)$, $R(2ar, ar^2)$ are points on $x^2 = 4ay$

i) If the tangent at P is parallel to the chord QR, find the gradient at P and hence show that $q + r = 2p$ **3**

ii) M lies on QR such that $RM = MQ$ prove that PM is parallel to the axis of the parabola. **3**

b) i) Show that the equation of the line from the focus S of the parabola $x^2 = 4ay$ and perpendicular to the normal through the point P $(2ap, ap^2)$ on the parabola is $px - y + a = 0$. **3**

ii) If these lines meet at N, show that the locus of N as P moves on the parabola $x^2 = 4ay$ is also a parabola. **4**

iii) State the vertex and focus of the locus. **1**

Question 3 (9 marks)

a) i) Show that there is a root for $f(x) = x^3 - 3x - 4$ in the domain $2 \le x \le 3$ **2**

ii) Use two applications for Halving the Interval method to determine an approximation for the root correct to 2 decimal places **3**

iii) Use one application of Newton's method to determine an approximation for the root correct to 2 decimal places **1**

b) Take $x = 1.3$ and use two applications of Newton's method to find the cube root of 2. Give answer to 2 decimal places. **3**

Mathematics Extension 1, Task 1 2010

Q1 a) $x = t+1$, $y = \frac{1}{t}$

$t = x-1$

$\therefore y = \frac{1}{x-1}$

b) i)

Chord of contact is $xx_1 = 2a(y+y_1)$

$x^2 = 8y \therefore a = 2$ $x_1 = 3$, $y_1 = -2$

$\therefore 3x = 4(y-2)$

$3x = 4y - 8$

$3x - 4y + 8 = 0$

ii) Focus is at $(0,2)$

i.e $0 - 8 + 8 = 0$ True

so $3x - 4y + 8 = 0$ passes through the focus.

Q2 a)

i) $\frac{x^2}{4a} = y$

$y' = \frac{x}{2a}$ and $(2ap, ap^2)$

$m = \frac{2ap}{2a}$

$= p$

Since tangent is parallel to QR

$p = m$ of QR

$m = \frac{aq^2 - ar^2}{2aq - 2ar}$

$= \frac{a(q-r)(q+r)}{2a(q-r)}$

$= \frac{q+r}{2}$

$\therefore p = \frac{q+r}{2}$

$2p = q+r.$

1 for co-ord. of focus

1 for substit. into eq.

1 for correctly obtained gradient by calculus.

Many students tried to substite $x = ..$ and did not get an expression for y.

Mainly well done. General form is the preferre form of the answe

You need to find focus & then substitut

Can't find a expression to show $a = 2$.

ii) M is midpoint of QR

$\therefore$ M is $\left(\frac{2aq+2ar}{2}, \frac{aq^2+ar^2}{2}\right)$

$\left(\frac{2a(q+r)}{2}, \frac{a(q^2+r^2)}{2}\right)$

$\left(2ap, \frac{a(q^2+r^2)}{2}\right)$, $q+r = 2p$

PM has $m = \dfrac{\frac{a(q^2+r^2)}{2} - \frac{2ap^2}{2}}{2ap - 2ap}$

i.e m is undefined and PM is a vertical line which is parallel to $x = 0$, the axis of the parabola.

b) i) $x^2 = 4ay$

$y' = \frac{2x}{4a}$

and at P m of normal $= -\frac{1}{p}$

Line $\perp$ to normal has $m = p$

Eq is $y - y_1 = m(x - x_1)$

$y - a = p(x - 0)$ at focus

$y - a - px = 0$

$px - y + a = 0$ (1)

ii) Eq of normal is

$y - ap^2 = \frac{-1}{p}(x - 2ap)$

$py - ap^3 = -x + 2ap.$

$x + py = 2ap + ap^3$ (2)

at N $px + p^2y = 2ap^2 + ap^4$ from (2)

$y - a + p^2y = 2ap^2 + ap^4$ from (1)

1 for midpoint

1 for parallel lines using gradient information

1 for gradient

1 for eq. at focus

1 for equation correctly obtained

1 for equation of normal.

Most students understood concept but gave very poor explanations

Could also say M and P have same x value $\therefore$ lie on $x = 2ap$ which is $\parallel$ to $x = 0$.

Algebraic errors cost marks here

$y(1+p^2) = ap + ap \ldots$

$\therefore = a(p^2+1)^2$

$y = a(p^2+1)$ ✓

$\therefore x = \dfrac{a(p^2+1) - a}{p}$

$= \dfrac{a(p^2+1-1)}{p}$

$\dfrac{x}{a} = p$ i.e. $N(ap, a(p^2+1))$ ✓

so $y = a\left[\left(\dfrac{x}{a}\right)^2 + 1\right]$

$y - a = \dfrac{x^2}{a}$

$x^2 = ay - a^2$

$= a(y-a)$ ✓

$\therefore$ locus is a parabola with vertex at $(0,a)$ and focal length $\dfrac{a}{4}$ ✓

$\therefore$ Focus $\left(0, \dfrac{5a}{4}\right)$

1 for both correct

Many did not factorise

Many students failed to realise that here $a = 4 \times$ focal length

$\therefore$ focal length $= \dfrac{9}{4}$

Q3

a) i) $f(x) = x^3 - 3x - 4$

$f(2) = 8 - 6 - 4 < 0$ ✓

$f(3) = 27 - 9 - 4 > 0$

$\therefore$ a root lies in $2 \le x \le 3$. ✓

since $f(x)$ is continuous and there is a change of sign.

ii) $f(2.5) = (2.5)^3 - 3(2.5) - 4$

$= 4.125$

$f(2.5) > 0$ and $f(2) < 0$ ✓

so root lies between 2 and 2.5

$f(2.25) = (2.25)^3 - 3(2.25) - 4$

$= 0.640625$ ✓

$f(2.25) > 0$ and $f(2) < 0$

$\therefore$ so root lies between 2 and 2.25

$\therefore x = 2.25$

1 for statement of the root approx. ✓

Students must learn to fully execute two applications rather than find fn values for several values of x & then come up with an answer.

Final approx^n after two applications is $x \doteqdot 2.25$ as $f(2.25)$ is closer to zero than $f(2.5)$. DO NOT HALVE A THIRD TIME.

$f'(x) = 3x^2 - 3$, $f'(2.13) = 10.6107$

$x_2 = x_1 - \dfrac{f(x_1)}{f'(x_1)}$

$= 2.13 + \dfrac{0.726403}{10.6107}$ ✓

$\doteqdot 2.19845948$

$x = 2.20$ (to 2d.p)

1 for use of Newton's method correctly.

Some chose a value right outside the range $2 < x < 2.25$ established in (ii).

b) $x^3 = 2$ ✓

$x^3 - 2 = f(x)$

$3x^2 = f'(x)$

$x = 1.3$

$x_2 = 1.3 - \dfrac{1.3^3 - 2}{3(1.3)^2}$

$\doteqdot 1.261143984$

$x_3 = 1.261143984 - \dfrac{(1.261143984^3 - 2)}{3(1.261143984)^2}$

$\doteqdot 1.259922235$

$x = 1.26$ (to 2d.p) ✓

1 establishing $f(x)$.

Some students used $x = \sqrt[3]{2}$ $\therefore f(x) = x - \sqrt[3]{2}$ & $f'(x) = 1$ which simplified process. (gave them full marks)

✓ Must use full digit display when re-using an answer