Name: ____________________

Teacher: ____________________

Class: ____________________

FORT STREET HIGH SCHOOL

# 2007

## HIGHER SCHOOL CERTIFICATE COURSE
## ASSESSMENT TASK 2

# Mathematics

**Time allowed: 1 hour**

| Outcomes Assessed | Questions | Marks |
|---|---|---|
| Applies appropriate methods to calculate probabilities | 1 | |
| Uses techniques of integration to calculate area and volume | 2,3 | |
| Uses differential rules for calculus correctly and applies appropriate techniques to solve problems | 4,5 | |

| Question | 1 | 2 | 3 | 4 | 5 | Total | % |
|---|---|---|---|---|---|---|---|
| Marks | /10 | /12 | /8 | /12 | /8 | /50 | |

**Directions to candidates:**

- Attempt all questions
- The marks allocated for each question are indicated
- All necessary working should be shown in every question. Marks may be deducted for careless or badly arranged work.
- Board – approved calculators may be used
- Each new question is to be started in a new booklet

Question 1: (10 marks) MARKS

a) A factory assembles torches. Each torch requires one battery and one bulb. It is known that 6% of all batteries and 4% of all bulbs are defective.

Find the probability that, in a torch selected at random, both the battery and the bulb are NOT defective. Give your answer in exact form. 2

b) Students studying at least one of the languages, French and Japanese, attend a meeting. Of the 28 students present, 18 study French and 22 study Japanese.

i) What is the probability that a randomly chosen student studies French? 2

ii) What is the probability that two randomly chosen students both study French? 1

iii) What is the probability that a randomly chosen student studies both languages? 1

c) The die used in a new game has 20 faces. Each face has a different letter of the alphabet marked on it, however the letters *Q*, *U*, *V*, *X*, *Y*, and *Z* have not been used.

i) The die is rolled twice. What is the probability that the same letter appears on the upper face twice? 2

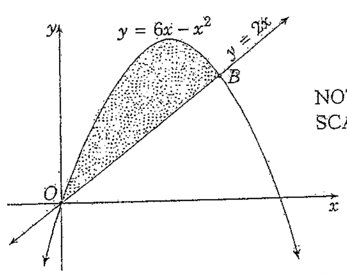
ii) The die is rolled three times. What is the probability that the letter *E* appears on the upper face exactly twice? 2

Question 2: (12 marks) MARKS

a) Find $\int (2x+3)^{10}\,dx$ 2

b) Evaluate $\int_0^1 \frac{x^3-2x^2+3x}{x}\,dx$ 2

c)

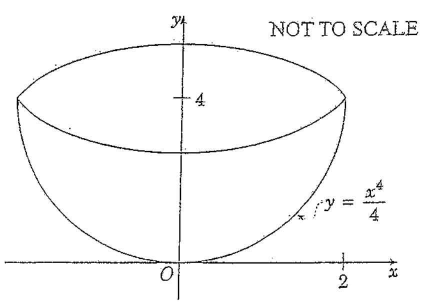



The graphs of $y=2x$ and $y=6x-x^2$ intersect at the origin and point *B*.

i) Show that the coordinates of *B* are (4, 8) 1

ii) Find the shaded area bounded by $y=6x-x^2$ and $y=2x$ 3

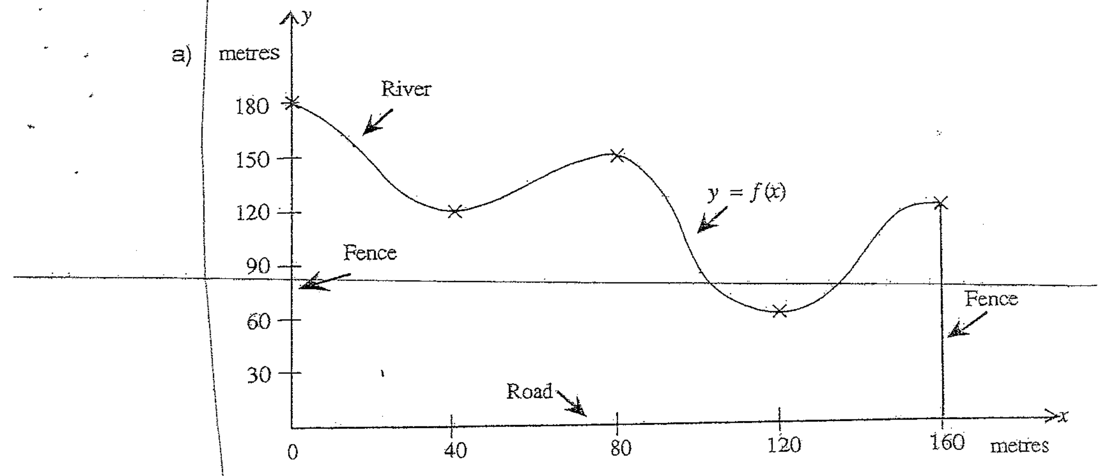
d)




A bowl is formed by rotating the part of the curve $y=\frac{x^4}{4}$ between $x=0$ and $x=2$ about the $y$ axis. Find the volume of the bowl. 4

Question 3: (8 marks) MARKS

a)




The diagram is a scale drawing of a paddock bounded by a river, a road, and two fences perpendicular to the road.

A farmer wishes to calculate the area of this paddock and has measured the perpendicular distances of the river from the road at intervals of 40 metres. These distances can be read off the diagram.

i) Take the road as the $x$ axis, the fences as the $y$ axis and the line $x=160$, and the river as $y=f(x)$.

Copy and complete the following table of values in your examination booklet: 1

| $x$ | 0 | 40 | 80 | 120 | 160 |
|---|---|---|---|---|---|
| $y=f(x)$ | | | | | |

ii) Estimate the area of the paddock using Simpson's Rule with five function values. 3

b) Find

i) The exact area under $y = x^2$ from $x = 1$ to $x = 2$ 1

ii) The approximate area to 2 decimal places using the trapezoidal rule with four sub-intervals. 3

## Question 4: (12 marks) MARKS

Consider the curve given by $y = x^3 - 12x$.

a) Find the coordinates of the stationary points and determine their nature. (leave your answers as surds) 6

b) Find the coordinates of any point of inflexion 2

c) Sketch the curve for the domain $-5 \le x \le 5$, clearly showing stationary points, points of inflexion, end points of the domain and $x$ and $y$ intercepts. 3

d) What is the maximum value of $x^3 - 12x$ in the domain $-5 \le x \le 5$? 1

## Question 5: (8 marks) MARKS

a) At all points on a certain curve, $\frac{d^2y}{dx^2} = 2x$. The point (3,6) belongs to the curve and the tangent to the curve at this point has a gradient of 1. Find the equation of the curve. 4

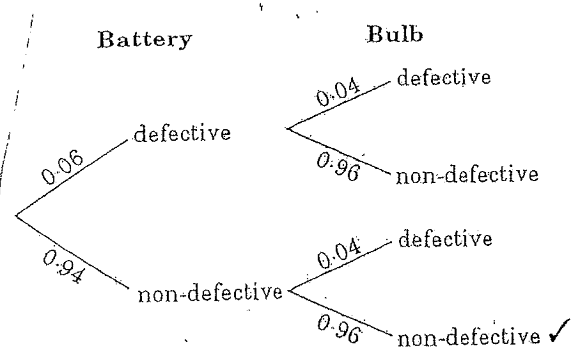
b) A woman wishes to form a rectangular enclosure using her existing fence as one side. If she has 40 metres of fencing material available to form the other three sides, find the area of the largest enclosure she can form and its dimensions. 4

# 2U - Assessment Task 2

## 2007 Solutions

## Question One

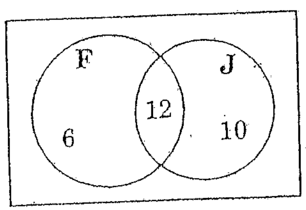
(a)




$P(\text{both non-defective}) = 0.94 \times 0.96$ ✓

$= 0.9024.$

(b) Since there are only 28 students, and 18 French students plus 22 Japanese total 40, there are $40 - 28 = 12$ students who study both languages.




✓

(i) Probability (studies French) ✓

$= \frac{18}{28} = \frac{9}{14}.$

(ii) Probability (both study French)

$= \frac{18}{28} \times \frac{17}{27} = \frac{17}{42}$ ✓

(iii) Probability of studying both languages $= \frac{12}{28} = \frac{3}{7}.$ ✓

(c) (i) Prob. of rolling a particular letter twice

$= \frac{1}{20} \times \frac{1}{20}$ ✓

$= \frac{1}{400}$

Prob. of rolling any of 20 letters twice

$= \frac{1}{400} \times 20$

$= \frac{20}{400}$

$= \frac{1}{20}.$ ✓

(ii) $P(\text{E appearing twice})$

$= P(\text{EE}\tilde{\text{E}}) + P(\text{E}\tilde{\text{E}}\text{E}) + P(\tilde{\text{E}}\text{EE})$

$= (\frac{1}{20} \times \frac{1}{20} \times \frac{19}{20}) + (\frac{1}{20} \times \frac{19}{20} \times \frac{1}{20})$ ✓

$+ (\frac{19}{20} \times \frac{1}{20} \times \frac{1}{20})$

$= \frac{19}{8000} + \frac{19}{8000} + \frac{19}{8000}$ ✓

$= \frac{57}{8000}.$

## Question Two

(a) $\int (2x+3)^{10}\,dx = \frac{(2x+3)^{11}}{11 \times 2} + C$ ✓

$= \frac{(2x+3)^{11}}{22} + C$ ✓

(b) $\int_0^1 \frac{x^3 - 2x^2 + 3x}{x}\,dx$

$= \int_0^1 (x^2 - 2x + 3)\,dx$

$= \left[\frac{x^3}{3} - x^2 + 3x\right]_0^1$ ✓

$= \frac{1}{3} - 1 + 3 = 2\frac{1}{3}$ ✓

(c) (i) Solve simultaneously:

$y = 2x$ —①

$y = 6x - x^2.$ —②

Let $6x - x^2 = 2x$

$x^2 - 4x = 0$

$x(x-4) = 0.$

$\therefore\ x = 0$ or $x = 4.$

Substitute in ①:

when $x = 0$, $y = 0$

when $x = 4$, $y = 8.$ ✓

$\therefore$ The points are $(0, 0)$ and $(4, 8)$.

(ii) Area $= \int_0^4 (6x - x^2 - 2x)\,dx$ ✓

$= \int_0^4 4x - x^2\,dx$

$= \left[2x^2 - \frac{x^3}{3}\right]_0^4$ ✓

$= 32 - \frac{64}{3} - (0 - 0)$ ✓

$= 10\frac{2}{3}$ square units.

(d)

$$y = \frac{x^4}{4}$$
$$x^4 = 4y$$
$$x^2 = 2\sqrt{y}.$$
$$V_y = \pi\int_0^4 x^2 \, dy$$
$$= \pi\int_0^4 2y^{\frac{1}{2}} \, dy$$
$$= 2\pi\int_0^4 y^{\frac{1}{2}} \, dy$$
$$= 2\pi\left[\frac{2}{3}y^{\frac{3}{2}}\right]_0^4$$
$$= \frac{4}{3}\pi\left[y^{\frac{3}{2}}\right]_0^4$$
$$= \frac{4}{3}\pi \times 4^{\frac{3}{2}}$$
$$= \frac{32}{3}\pi \text{ units}^3.$$

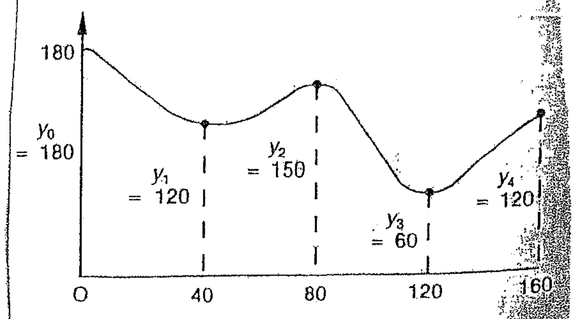
## Question Three

(a)

(i)

| $x$ | 0 | 40 | 80 | 120 | 160 |
|---|---|---|---|---|---|
| $y = f(x)$ | 180 | 120 | 150 | 60 | 120 |

(ii)




(2)

$$A \doteqdot \frac{h}{3}[y_0 + y_4 + 2y_2 + 4(y_1 + y_3)]$$
$$= \tfrac{40}{3}[180 + 120 + 2(150) + 4(120 + 60)]$$
$$= \tfrac{40}{3}[180 + 120 + 300 + 720]$$
$$= \tfrac{40}{3} \times 1320$$
$$= 17\,600 \text{ m}^2 \text{ (by calc.).}$$

(b) (i)

$$\int_1^2 x^2 \, dx = \left[\frac{x^3}{3}\right]_1^2$$
$$= \frac{8}{3} - \frac{1}{3}$$
$$= \frac{7}{3}$$

(ii)

| $x$ | 1 | $1\frac{1}{4}$ | $1\frac{1}{2}$ | $1\frac{3}{4}$ | 2 |
|---|---|---|---|---|---|
| $y$ | 1 | $\frac{25}{16}$ | $\frac{9}{4}$ | $\frac{49}{16}$ | 4 |

$$\int_1^2 x^2 \, dx \doteqdot \frac{1}{2}\,\frac{2-1}{4}\left[1 + 2\left(\frac{25}{16} + \frac{9}{4} + \frac{49}{16}\right) + 4\right]$$
$$= \frac{1}{8} \times \frac{75}{4}$$
$$= 2.34375$$
$$= 2.34 \text{ (to 2 dp)}$$

## Question Four

(a) $y = x^3 - 12x$

$y' = 3x^2 - 12$

$y'' = 6x$

For stationary points

$y' = 3x^2 - 12 = 0$

$3(x^2 - 4) = 0$

$3(x-2)(x+2) = 0$

ie $x = 2$ $y = -16$

$x = 2$ $y = 16$

At $(2, -16)$ $y'' > 0$

$\therefore$ min pt at $(2, -16)$

At $(-2, 16)$ $y'' < 0$

$\therefore$ max pt at $(-2, 16)$

(b) For possible pt of inflexion $y'' = 6x = 0$

$x = 0$

$y = 0$

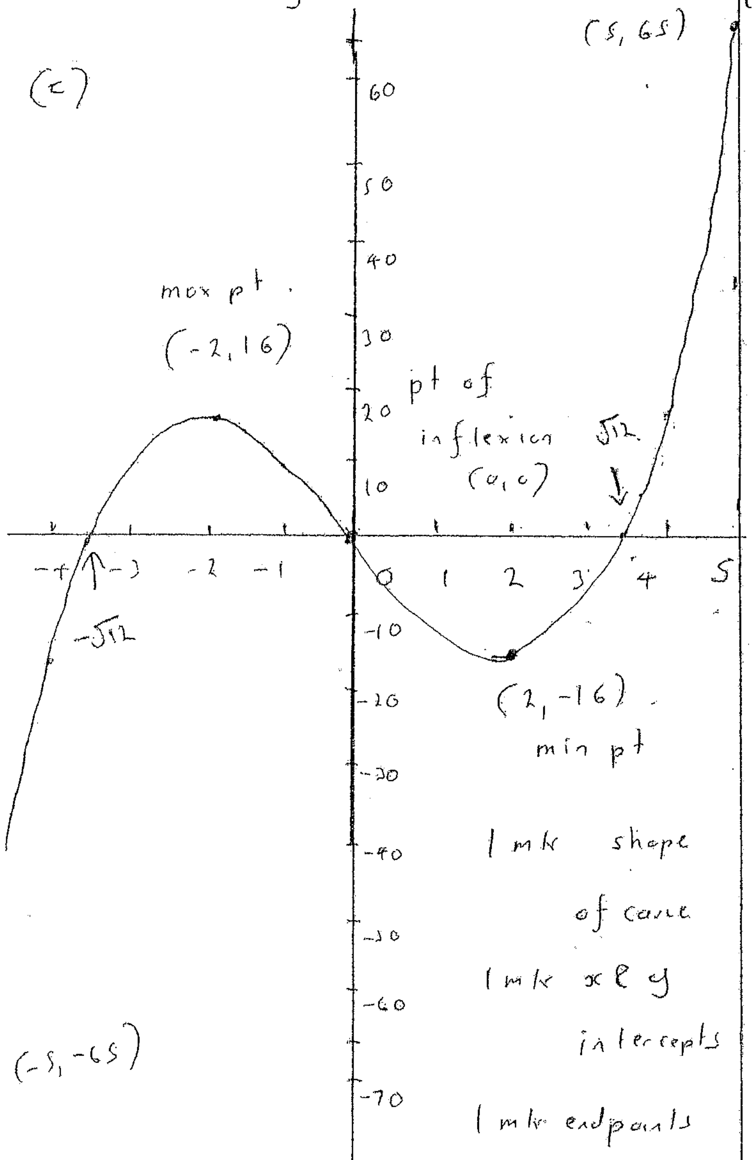
Test for change of concavity

| $x$ | $-1$ | 0 | 1 |
|---|---|---|---|
| $y''$ | $< 0$ | 0 | $> 0$ |

- ie concavity changes

$\therefore (0, 0)$ is a pt of inflexion

(c)




1 mk shape of curve

1 mk x & y intercepts

1 mk endpoints of domain

(d) 65 ✓

## Question Five

(a) $\frac{d^2y}{dx^2} = 2x$

$\frac{dy}{dx} = x^2 + c_1$ ✓

at $(3, 6)$ $\frac{dy}{dx} = 1$

$\therefore 3^2 + c_1 = 1$

$c_1 = -8$

$\therefore \frac{dy}{dx} = x^2 - 8$ ✓

$y = \frac{x^3}{3} - 8x + c_2$ ✓

$(3, 6)$ belongs to curve

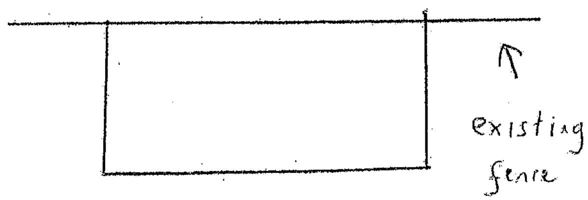
$\therefore 6 = \frac{27}{3} - 24 + c_2$

$6 = -15 + c_2$

$c_2 = 21$ ✓

$\therefore y = \frac{x^3}{3} - 8x + 21$

(b)




$2w + l = 40$

$l = 40 - 2w$

$A = wl$

$= w(40 - 2w)$ ✓

$= 40w - 2w^2$

$\frac{dA}{dw} = 40 - 4w = 0$ ✓

$w = 10 \quad l = 20$

$\frac{d^2A}{dw^2} = -4$ ✓

$\therefore$ max pt at $w = 10$

Max Area $= 10 \times 20$

$= 200\,m^2$ ✓

with width $= 10m$

length $= 20m$