FORT STREET HIGH SCHOOL

Trial Higher School Certificate Examination 1992

Mathematics - 2/3 Unit

Time allowed - THREE Hours
(includes reading time)

DIRECTIONS TO CANDIDATES :

* All questions may be attempted .

*All questions are of equal value.

*All necessary working should be shown in every question. Full marks may not be awarded for careless or badly arranged work .

* Standard integrals are included .

* Each question attempted is to be started on a new page.

*If required additional paper may be obtained from the Examination Supervisor on request .

Question 1	
Question 2	
Question 3	
Question 4	
Question 5	
Question 6	
Question 7	
Question 8	
Question 9	
Question 10	
Total	

STANDARD INTEGRALS

$$\int x^n \, dx = \frac{1}{n+1} x^{n+1}, \; n \neq -1; \; x \neq 0, \text{ if } n < 0.$$

$$\int \frac{1}{x} \, dx = \log_e x, \; x > 0.$$

$$\int e^{ax} dx = \frac{1}{a} e^{ax}, \; a \neq 0.$$

$$\int \cos ax \, dx = \frac{1}{a} \sin ax, \; a \neq 0.$$

$$\int \sin ax \, dx = -\frac{1}{a} \cos ax, \; a \neq 0.$$

$$\int \sec^2 ax \, dx = \frac{1}{a} \tan ax, \; a \neq 0.$$

$$\int \sec ax \tan ax \, dx = \frac{1}{a} \sec ax, \; a \neq 0.$$

$$\int \frac{1}{a^2 + x^2} dx = \frac{1}{a} \tan^{-1} \frac{x}{a}, \; a \neq 0.$$

$$\int \frac{1}{\sqrt{(a^2 - x^2)}} dx = \sin^{-1} \frac{x}{a}, \; a > 0, \; -a < x < a.$$

$$\int \frac{1}{\sqrt{(x^2 - a^2)}} dx = \log_e \{x + \sqrt{(x^2 - a^2)}\}, \; |x| > |a|.$$

$$\int \frac{1}{\sqrt{(x^2 + a^2)}} dx = \log_e \{x + \sqrt{(x^2 + a^2)}\}.$$

QUESTION 1:

(a) Find the exact value of

(i) $\sqrt{7} + \sqrt{28}$

(ii) $36^{\frac{1}{4}} \times 36^{\frac{1}{4}}$

(b) Solve the quadratic equation $3x^2 + 3x - 1 = 0$
(Round off solutions to 2 decimal places)

(c) Factorise a) $3a^2 - 12$ b) $x^3 - 8$

(d) Find the 250th term of this sequence

5 , 5.2 , 5.4 , 5.6, ...

(e) Solve $|2x-1| \leq 5$

QUESTION 2:

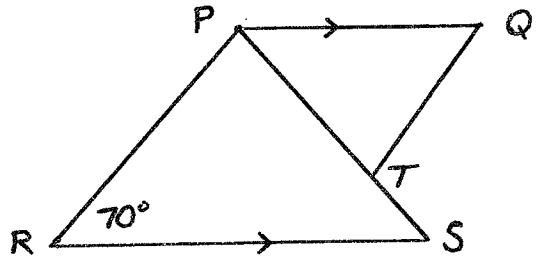
(a) Find the derivative of the following;

(i) $y = 5x^2 + 2x$
(ii) $y = e^{2x}$
(iii) $y = \log_e x^2$
(iv) $y = \sin 2x$

(b) In the diagram PQ is parallel to RS

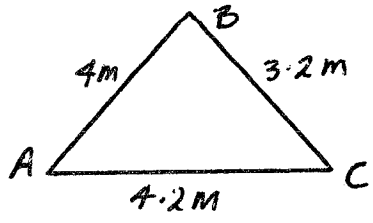
PR = PS, QP = QT and $\angle PRS = 70^0$ find $\angle PQT$

give the reasons for each step in your answer.



(c) (i) In the diagram find to the nearest minute $\angle BAC$

(ii) Find the area of $\triangle ABC$
to two decimal places.



QUESTION 3:

a) Given the points A (1,-1) B (5,1) and C (3,2)

(i) Show the equation of line AB is $x - 2y - 3 = 0$

(ii) Find the perpendicular distance C is from line AB
(to 2 decimal planes)

(iii) Find the angle, to the nearest degree, that line AB makes with the x-axis.

b) Find the exact value of $\int_1^2 \frac{1}{1+x} dx$

c) Find $\int e^{3x} dx$

QUESTION 4:

(a) For the function $y = x^3 + 3x^2 - 9x$

(i) Find all stationary points and determine their nature.

(ii) Find the point or points of inflection.

(b) A bag contains 4 black and 3 red marbles. If a marble is selected at random, its colour noted and then returned to the bag before a second marble is drawn. What is the probability that:

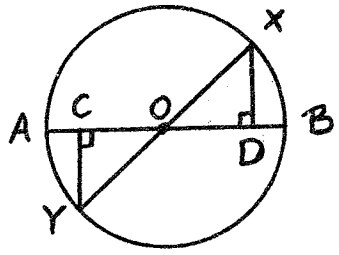
(i) both marbles are red

(ii) one is red the other black

(iii) at least one marble is red

QUESTION 5:

a) O is the centre of the circle. YC $\perp$ AB , XD $\perp$ AB
Prove $\triangle$ OYC $\equiv$ $\triangle$ OXD. State reasons for each step



b) For what values of K does the quadratic equation

$x^2 - 2x + K = 0$ have real roots

c) For the function $y = \sqrt{4 - x^2}$

(i) Find the first derivative

(ii) State its domain and range

(iii) Draw a sketch of the function.

QUESTION 6:

a) Find the volume (to two decimal places) of the solid of revolution formed by rotating the curve $y = 1/x$ about the x-axis between $x = 1$ and $x = 3$

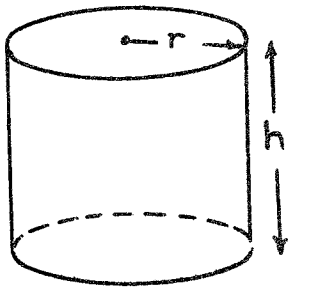
b) The sixth and thirteenth terms of an arithmetic sequence are 17 and 31 respectively.
Find the common difference and first term.

c) The roots of the quadratic equation
$px^2 - x + q = 0$ are -1 and 3
find p and q

d) Find the limiting sum of $5 + 1 + 1/5 + 1/25 + ...$

QUESTION 7:

a) A container in the shape of a cylinder is to be made so the sum of its height and radius is 100cm.



$h + r = 100$

i) Show the volume of this cylinder is given by

$$V = \pi r^2 (100 - r)$$

ii) Find the value for r so the volume is a maximum.

b) Find the equation of the tangent to the curve $y = 3 \ln x + 2$ at the point where $x = 1$

c) Draw the graph of $y = 3 \sin 2x$ for $0 \leq x \leq 2\pi$

QUESTION 8:

a) Solve $\sin^2 x = 1/4$ for $0 \leq x \leq 2\pi$ (give exact answer in radians)

b) Starting with 6, how many consecutive multiples of 6 must be added to make their sum exceed 1000.

c) Find the derivative of

i) $y = \frac{x^2 - 2x}{2x + 3}$

ii) $y = x^2 \ln x^2$

d) Evaluate $\sum_{n=1}^{12} 50 (1.01)^{n-1}$

QUESTION 9:

(a) Consider the function defined by
$f(x) = e^{2x}(1-x)$ where $-3 \leqslant x \leqslant 1$

(i) Copy and complete this table of values
Give answers correct to two decimal places.

x	-3	-2	-1	0	1
y	.01	.05			

(ii) Differentiate f(x) and hence show that the function has only one stationary point.

(iii) Sketch the curve $y = f(x)$ for $-3 \leqslant x \leqslant 1$

(iv) Use Simpsons rule with five function values (4 strips) to approximate the area under the curve $y = f(x)$ for $-3 \leqslant x \leqslant 1$

QUESTION 10:

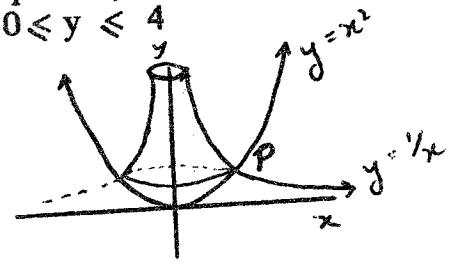
a) A wheel of radius 50cm is rolled 10cm along the ground.

(i) Find the angle this wheel has turned in radians

(ii) Give this angle in degrees to the nearest minute.

Q 10 (cont)

b) The following solid shape is made by rotating the graphs of $y = x^2$ and $y=1/x$ about the y-axis for $0 \leqslant y \leqslant 4$



(i) Show point P is (1,1)

(ii) Find the volume of this solid

c) Mr Drapiod contributes towards a pension a sum of \$2500 annually at the beginning of each year for 12 years. If he receives 5% interest per annum what is the accumulated value of his contributions at the end of the 12th year.

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Q1 (a)(i) $3\sqrt{7}$ ✓

(ii) 6 ✓

(b) $\frac{-3 \pm \sqrt{21}}{6}$ ✓

$= 0.26$ (or) -1.26 ✓

(c)(a) $3(a-2)(a+2)$ ✓

(b) $(x-2)(x^2+2x+4)$ ✓

(d) $T_{250} = 54.8$ ✓✓

(e) $-2 \leq x \leq 3$ ✓✓

Q2 (a) (i) $y' = 10x + 2$ ✓

(ii) $y' = 2e^{2x}$ ✓

(iii) $y' = \frac{2}{x}$ ✓

(iv) $y' = 2\cos 2x$ ✓

(b) $x = 40°$ ✓✓

(c) (a) $x = 45°52'$ ✓✓

(b) Area $\doteq 6.03$ unit^2 ✓

Q3 (a) (i) $x - 2y - 3 = 0$ as req'd ✓✓

(ii) 1.79 units ✓✓

(iii) 27° ✓✓

(b) $\ln 1.5$ ✓✓

(c) $\frac{1}{3}e^{3x} + C$ ✓✓

Q4 (a)(i) $(-3, 27)$ MAX T.P. ✓✓

$(1, -5)$ MIN T.P. ✓✓

(ii) $(-1, 11)$ ✓✓

(b) (i) $\frac{9}{49}$ ✓

(ii) $\frac{24}{49}$ ✓

(iii) $\frac{33}{49}$ ✓

Q5 (a) AAS ✓✓

(b) $1 \geq k$

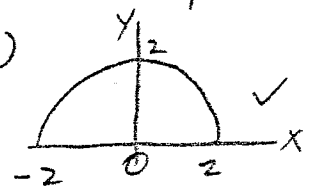
$k \leq 1$ ✓✓

(c)(i) $y' = \frac{-x}{\sqrt{4-x^2}}$ ✓✓

(ii) D: $-2 \leq x \leq 2$ ✓

R: $0 \leq y \leq 2$ ✓

(iii)



✓

Q6 (a) $\frac{2\pi}{3}$ ✓✓

(b) $a = 7, \alpha = 2$ ✓✓

(c) $p = \frac{1}{2}, q = -\frac{3}{2}$ ✓✓

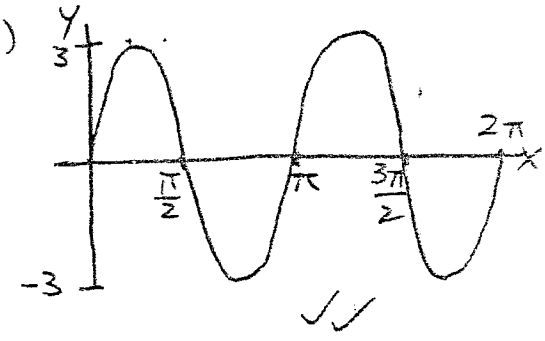
(d) $S_\infty = 6\frac{1}{4}$ ✓✓

Q7 (a)(i) $V = \pi r^2(100 - r)$ ✓✓

(ii) $r = 66\frac{2}{3}$ cm ✓✓

(b) $3x - y - 1 = 0$ ✓✓

(c)



✓✓

Q8 (a) $x = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}$ ✓✓

(b) $n = 18$ ✓✓✓

(c) (i) $\frac{2x^2 + 6x - 6}{(2x+3)^2}$ ✓✓

(ii) $2x(2\ln x + 1)$ ✓✓

(d) 634.125 ✓✓

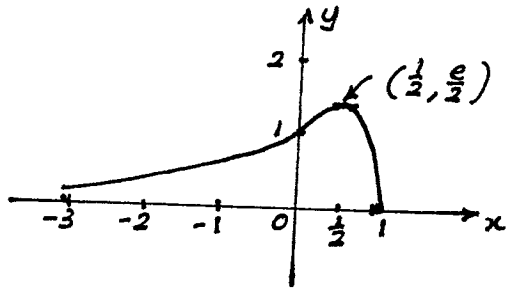
(9)

(a)(i)

x	-3	-2	-1	0	1
y	0.01	0.05	0.27	1	0

(ii) $(\frac{1}{2}, \frac{e}{2})$ max. t.p.

(iii)



(iv) $1 \cdot 58$

(10) (i) $\theta = \frac{1}{5}^c$ (ii) $11°28'$

(b)(i) Proof

(ii) $V_1 = \int_0^1 \pi y \, dy = \frac{\pi}{2}$

$V_2 = \int_1^4 \pi y^{-2} \, dy = \frac{3\pi}{4}$

$\therefore$ Total volume req^d. $= \frac{5\pi}{4}$ cm^3

(c) $A_{12} = PR^{12}$

$A_{11} = PR^{11}$

$\vdots$

$A_1 = PR^1$

$\therefore$ Total $= \frac{PR(R^n - 1)}{R-1}$

$= \frac{2500 \times 1 \cdot 05 \, (1 \cdot 05^{12} - 1)}{0 \cdot 05}$

$= \$41\,782 \cdot 46$

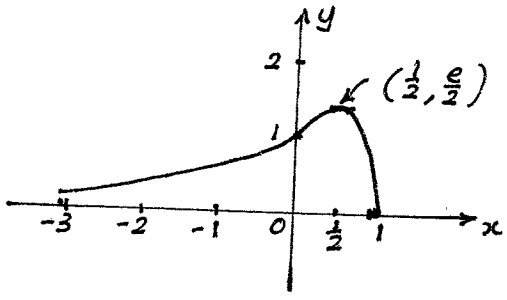
(9)

(a)(i)

x	-3	-2	-1	0	1
y	0.01	0.05	0.27	1	0

(ii) $(\frac{1}{2}, \frac{e}{2})$ max. t.p.

(iii)



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$\therefore$ Total $= \frac{PR(R^{12}-1)}{R-1}$

$= \frac{2500 \times 1.05 (1.05^{12}-1)}{0.05}$

$= \$41\,782.46$