FORT STREET HIGH SCHOOL

Name: ____________________

Teacher: ____________________

Class: ____________________

2015

HIGHER SCHOOL CERTIFICATE PRELIMINARY COURSE

ASSESSMENT TASK 1

Mathematics Extension 1

Time allowed: 1½ Hours
(plus 5 minutes reading time)

Syllabus Outcomes	Assessment Area Description and Marking Guidelines	Questions
P3 PE3(a)	performs routine arithmetic and algebraic manipulation involving simple rational expressions solves problems involving inequalities	1
P5	understands the concept of a function and the relationship between a function and its graph	2
PE2	uses multi-step deductive reasoning in a variety of contexts	3
P4(a)	chooses and applies appropriate algebraic techniques	4
HE7	Synthesises mathematical solutions to problems and communicates them in an appropriate form	5

Total Marks 62
Attempt Questions 1-5

Question	Out of	Marks
Q1	/12	
Q2	/12	
Q3	/12	
Q4	/12	
Q5	/14	
	Percent:	

General Instructions:

- Questions 1-5 are to be started in a new booklet
- The marks allocated for each question are indicated
- In Questions 1-5, show relevant mathematical reasoning and/or calculations.
- Marks may be deducted for careless or badly arranged work.
- Board – approved calculators may be used

Question 1: [12 marks]

(a) Find $\sqrt[3]{\frac{35.65}{2.4^2}}$ correct to 3 significant figures. [2]

(b) Factorise fully $81x^8-16y^4$ [2]

(c) Solve for x: $|4x-3|=7$ [3]

(d) Rationalize $\frac{\sqrt{3}-1}{2\sqrt{3}+1}$ [2]

(e) Solve for x: $\frac{(x+3)}{x-1}\le 2$ [3]

Question 2: [12 marks]

(a) For the function $f(x)=2+\sqrt{1-x}$:

i. State the domain and range of the function. [2]

ii. Draw a neat ⅓ page sketch of the function. [2]

(b) A function is defined by $f(x)=\begin{cases}1-x, & x\ge 0\\(x+2)^2, & x<0\end{cases}$. Find the value of $f(-1)+f(0)+f(1)$. [2]

(c) Show that $f(x)=\frac{(x-1)(x+1)}{x}$ is an odd function. [2]

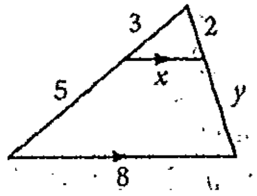
(d) For the function $y=\frac{2x-4}{x+1}$:

i. Find the horizontal and vertical asymptotes. [1]

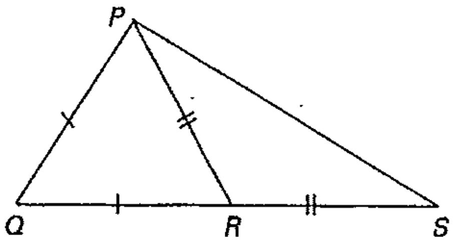
ii. Draw a neat ⅓ page sketch of the function, showing all features. [3]

Question 3: [12 marks]

(a) Find the value of x and y. [2]



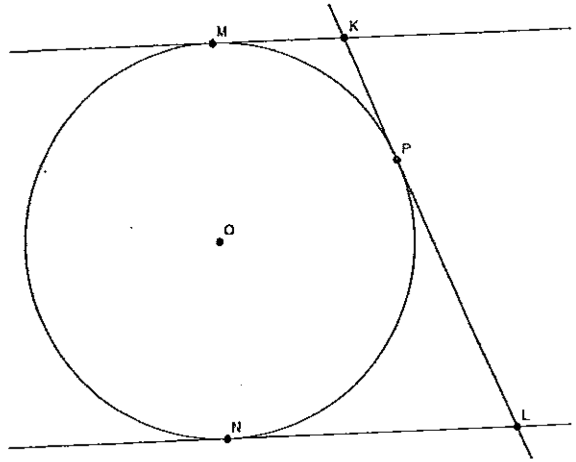
(b) QRS is a straight line, with $PQ = QR$ and $PR = RS$ as shown below. Copy the diagram to your answer booklet.



On your diagram, label $\angle PQR = \theta$ and $\angle PSR = \alpha$, and then prove, giving all reasons, that $\theta + 4\alpha = 180$. [4]

Question 3 continues over page...

(c) Two parallel tangents to the circle, centre O, meet the circle at points M and N. A third tangent to the circle, at point P, meets the other two tangents at K and L, as shown in the diagram below.



Copy the diagram into your answer booklet and use it to

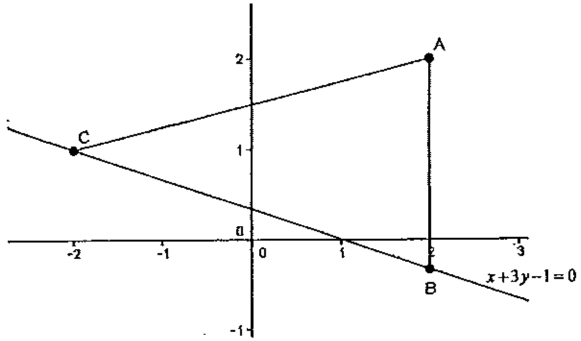
i. Prove that KO bisects $\angle MKP$. [3]

ii. Hence show that a circle with diameter KL passes though the centre O of the original circle. [3]

Question 4 Starts next page...

Question 4: [13 marks]

(a) The diagram given below shows the line $x+3y-1=0$ and the points $A(2,2)$, $B\left(2,\frac{-1}{3}\right)$ and $C(-2,1)$.



i. Find the distance BC. [1]

ii. Find the distance (in exact form) of the point $A(2,2)$ from the line $x+3y-1=0$. [1]

iii. Hence find the exact area of triangle ABC. [1]

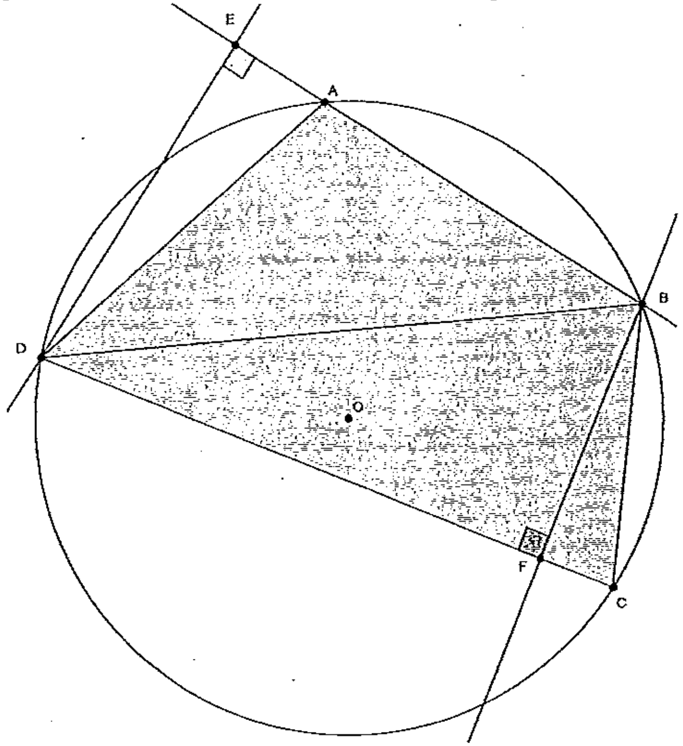
(b) Using the 'k'-Method, find the equation of the line that passes through the intersection of the lines $2x+y-3=0$ and $x+2y=2$, which also passes through the point $Q(-1,4)$. [3]

(c) Find the equation of a line with a positive gradient that is inclined at 45° to the line $x+2y-6=0$ and has the same y-intercept. [3]

(d) The point $T(2,-3)$ divides the interval between the points $A(-4,-2)$ and $B(x_1,y_1)$ in the ratio $2:3$. Find the co-ordinates of point B. [3]

Question 5: [14 marks]

(a) $ABCD$ is a cyclic quadrilateral. The perpendicular from D meets AB produced at E, and the perpendicular from B meets CD at F, as shown in the diagram below.



Copy the diagram into your answer booklet and prove $AC \parallel EF$. [4]

(b) Draw a neat ½ page sketch of $y=\dfrac{x^2-x+2}{(x-1)}$, showing all asymptotes and intercepts. [4]

(c) For the equation $y=|2x-1|+|x+1|$:

i. By determining branches or otherwise, draw a neat sketch of this graph. [3]

ii. Hence determine (algebraically) the value(s) for which $|2x-1|+|x+1|<4$. [3]

Question 1: [12 marks] *Marking* **Comments**

(a) *Find* $\sqrt[3]{\dfrac{35.65}{2.4^2}}$ *correct to 3 significant figures.* [2]

$=1.83602680308$ (by calc.)
$=1.84$ (to 3 sig.figs)

Marking: ❶ Answer ❶ Sig. figs correct.

Comments: Question 1 was generally well done overall.

(b) *Factorise fully* $81x^8-16y^4$ [2]

$=(9x^4+4y^2)(9x^4-4y^2)$
$=(9x^4+4y^2)(3x^2+2y)(3x^2-2y)$

Marking: ❶ first difference ❶ second correct

(c) *Solve for* x: $|4x-3|=7$ [3]

$4x-3=7$ *or* $4x-3=-7$
$4x=10$ $\quad$ $4x=-4$
$x=2\frac{1}{2}$ $\quad$ $x=-1$

Marking: ❶ two cases for abs val correct. ❶ for $x=2\frac{1}{2}$ ❶ for $x=-1$

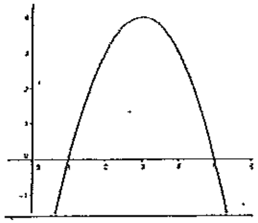
(d) *Rationalize* $\dfrac{\sqrt{3}-1}{2\sqrt{3}+1}$ [2]

$=\dfrac{\sqrt{3}-1}{2\sqrt{3}+1}\times\dfrac{2\sqrt{3}-1}{2\sqrt{3}-1}$
$=\dfrac{6-\sqrt{3}-2\sqrt{3}+1}{12-1}$
$=\dfrac{7-3\sqrt{3}}{11}$

Marking: ❶ × conjugate correct ❶ Answer

(e) *Solve for* x: $\dfrac{(x+3)}{x-1}\le 2$ [3]

$\dfrac{(x-1)^2(x+3)}{(x-1)}\le 2(x-1)^2$
$(x-1)(x+3)\le 2(x-1)^2$
$(x-1)(x+3)-2(x-1)^2\le 0$
$(x-1)[(x+3)-2(x-1)]\le 0$
$(x-1)(5-x)\le 0$
$-(x-1)(x-5)\le 0$



Hence $x<-1, x\ge 5$

Marking: ❶ correct algebra to quadratic ❶ diagram (or other equivalent) to justify answer ❶ Answer

Question 2: [12 marks]

(a) *For the function* $f(x)=2+\sqrt{1-x}$:

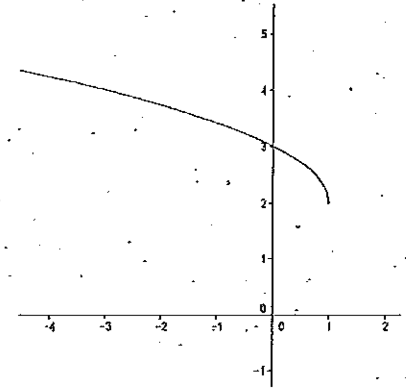
i. *State the domain and range of the function.* [2]

Domain: $x\le 1$
Range: $y\ge 2$

Marking: ❶ Domain ❶ Range

Comments: Well done.

ii. *Draw a neat ⅓ page sketch of the function.* [2]



Marking: ❶ Shape correct ❶ intercept & labels

Comments: Some curves were not smooth.

(b) *A function is defined by*

$f(x)=\begin{cases}1-x, & x\ge 0\\ (x+2)^2, & x<0\end{cases}$. *Find the value of* $f(-1)+f(0)+f(1)$. [2]

$f(-1)+f(0)+f(1)$
$=(-1+2)^2+(1-0)+(1-1)$
$=1+1+0$
$=2$

Marking: ❶ Shows which function is used ❶ Answer

Comments: Some students used the wrong function.

(c) *Show that* $f(x)=\dfrac{(x-1)(x+1)}{x}$ *is an odd function.* [2]

$f((-x))=\dfrac{((-x)-1)((-x)+1)}{(-x)}$
$=\dfrac{(-x-1)(-x+1)}{-x}$
$=\dfrac{-(x+1)(-x+1)}{-x}$

Marking: ❶ minus signs placed correctly for $f(-x)$

Comments: This wasn't well done. Need to show how the minus sign resolves.

$$= -\frac{(x+1)(x-1)}{x}$$
$$= -f(x)$$

➊ resolves for odd function correctly

(d) For the function $y=\dfrac{2x-4}{x+1}$:

i. *Find the horizontal and vertical asymptotes.* [1]

$$y=\frac{2x-4}{x+1}$$
$$=\frac{2x+2-6}{x+1}$$
$$=\frac{2(x+1)-6}{x+1}$$
$$=2-\frac{6}{x+1}$$
$$y-2=-\frac{6}{x+1}$$
$$(y-2)(x+1)=-6$$

Hence $y-2\neq 0 \quad x+1\neq 0$

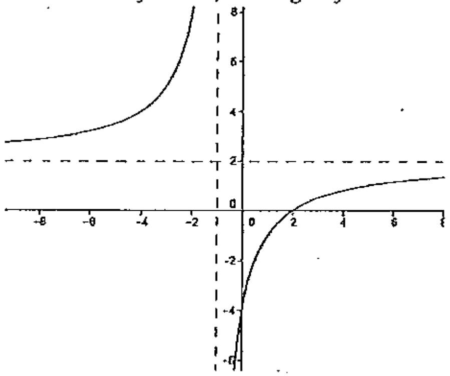
$y\neq 2 \qquad x\neq -1$

Thus horizontal asymptote is $y=2$ and vertical asymptote is $x=-1$.

➊ both asymptotes correct

Some students only found the vertical asymptote.

ii. *Draw a neat ⅓ page sketch of the function, showing all features.* [3]



➊ asymptotes stated above correctly graphed

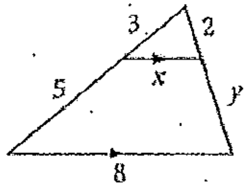
➊ hyperbola shape that matches asymptotes stated.

➊ intercepts & labels

Some students didn't find the intercepts and thus drew the incorrect shape.

Question 3: [12 marks]

(a) *Find the value of x and y.* [2]



$$\frac{x}{8}=\frac{3}{3+5}$$
$$8x=24$$
$$x=3$$

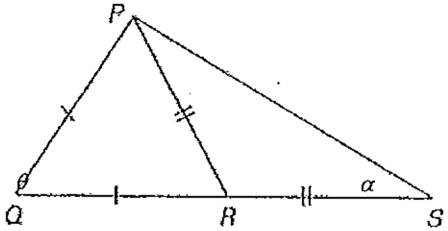
➊ x value and similarity eqn. correct

$$\frac{2}{2+y}=\frac{3}{3+5}$$
$$16=3(2+y)$$
$$16=6+3y$$
$$3y=10$$
$$y=3\frac{1}{3}$$

➊ y value and similarity eqn. Correct

Well done except for some careless errors.

(b) *QRS is a straight line, with PQ = QR and PR = RS as shown below. Copy the diagram to your answer booklet.*



➊ diagram and information correct

On your diagram, label $\angle PQR=\theta$ *and* $\angle PSR=\alpha$*, and then prove, giving all reasons, that* $\theta+4\alpha=180$. [4]

$\angle RPS=\alpha\,(base\angle's\;isos.\Delta)$

$\angle PRQ=2\alpha\,(ext\angle = sum\;opp.interior\;\angle's)$

$\angle RPQ=2\alpha\,(base\angle's\;isos.\Delta)$

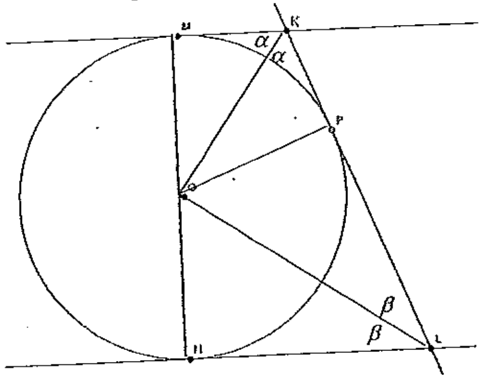
$\theta+2\alpha+\alpha=180\,(\Delta sum)$

$\theta+4\alpha=180$

➊ isos Δ's used correctly
➊ Extn. ∠'s or equivalent connecting Δ's
➊ Δ sum to result

Well done

(c) Two parallel tangents to the circle, centre O, meet the circle at points M and N. A third tangent to the circle, at point P, meets the other two tangents at K and L, as shown in the diagram below.



Copy the diagram into your answer booklet and use it to

i. *Prove that KO bisects $\angle MKP$.* **[3]**

Join OP, OP and OK.

In Δ's OMK, OPK

$i) \angle OMK = \angle OPK \left(=90^\circ, radii \perp tangent\right)$

$ii) OK \text{ is common}$

$iii) OM = OP\left(radii\ of\ circle\right)$

$\therefore \Delta OMK \equiv \Delta OPK\left(RHS\right)$

$\therefore \angle OKM = \angle OPM\left(corr. \angle's\ cong. \Delta's\right)$

Hence KO bisects $\angle MKP$.

➊ diagram copied and information added to support proof attempted

➊ proof correct

➊ conclusion from proof correct.

A simple congruence proof was required. Students used OK bisects MP to prove the result. This can be proved only after it is shown that KO bisects angle MKP not before.

ii. *Hence show that a circle with diameter KL passes though the centre O of the original circle.* **[3]**

Let $\angle MKO = \alpha$ hence $\angle PKO = \alpha$ from i).

Similarly, $\angle PLO = \angle NLO\left(=\beta\right)$.

Now

$\angle NLP + \angle PKM = 180\left(co-int. \angle's, given\ MK \parallel NL\right)$

Hence $2\alpha + 2\beta = 180$

$\alpha + \beta = 90$

As $\angle OPK = 90\left(from\ i.\right)$ and

$\angle KOP + \alpha + 90 = 180\left(\Delta sum\right)$

$\angle KOP = 90 - \alpha$

Similarly $\angle LOP = 90 - \beta$

➊ result for sum to 90

➊ connects angles across triangles

Very poorly done.

Many students did not know where to start.

Hence $\angle KOL = \angle KOP + \angle POL$

$= 90 - \alpha + 90 - \beta$

$= 180 - \left(\alpha + \beta\right)$

$= 180 - 90$

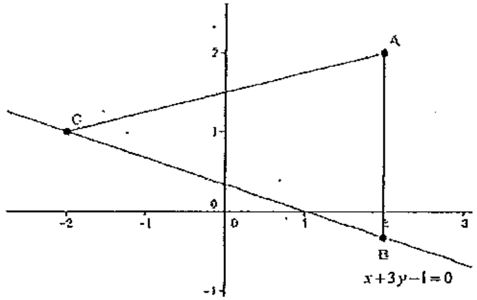
$= 90$

Thus O lies on a circle with diameter $KL\left(\angle in\ semicirc = 90^\circ\right)$.

➊ correct circle property to give circle connection

Question 4: **[12 marks]**

(a) The diagram given below shows the line $x + 3y - 1 = 0$ and the points $A(2,2)$, $B\left(2, \frac{-1}{3}\right)$ and $C(-2,1)$.



i. *Find the exact distance BC.* **[1]**

$$d = \sqrt{\left(x_2 - x_1\right)^2 + \left(y_2 - y_1\right)^2}$$

$$= \sqrt{\left(-2-2\right)^2 + \left(1 - \frac{-1}{3}\right)^2}$$

$$= \sqrt{\left(-4\right)^2 + \left(\frac{4}{3}\right)^2}$$

$$= \sqrt{16 + \frac{16}{9}}$$

$$= \sqrt{\frac{144+16}{9}}$$

$$= \frac{4\sqrt{10}}{3}$$

➊ correct distance in simplest form

Some students didn't give exact value or answer in simplest form.

ii. *Find the distance (in exact form) of the point $A(2,2)$ from the line $x + 3y - 1 = 0$.* **[1]**

$$d=\frac{|ax+by+c|}{\sqrt{a^2+b^2}}$$
$$=\frac{|1\times2+3\times2-1|}{\sqrt{1^2+3^2}}$$
$$=\frac{7}{\sqrt{10}}\left(=\frac{7\sqrt{10}}{10}\right)$$

➊ correct distance

Generally well done.

iii. *Hence find the exact area of triangle ABC.* [1]

$$A=\frac{1}{2}bh$$
$$=\frac{1}{2}\times\frac{4\sqrt{10}}{3}\times\frac{7}{\sqrt{10}}$$
$$=\frac{14}{3}$$
$$=4\frac{2}{3}\,sq.units$$

➊ Answer

Many again ignored the instruction to give an exact area, so lost the mark..

(b) *Using the 'k'-Method, find the equation of the line that passes through the intersection of the lines $2x+y-3=0$ and $x+2y=2$, which also passes through the point $Q(-1,4)$.* [3]

The equation through the intersection has the form:

$$0=2x+y-3+k(x+2y-2).$$

Through $Q(-1,4)$:

$$0=2\times(-1)+4-3+k((-1)+2\times4-2)$$
$$=-1+5k$$
$$k=\frac{1}{5}$$
$$\therefore\ 0=2x+y-3+\frac{1}{5}(x+2y-2)$$
$$=10x+5y-15+x+2y-2$$
$$=11x+7y-17$$

➊ equation using k-method correctly formed

➊ k value correct

➊ correct equation using the k-method

Some students tried to solve simultaneously and thus gained no marks for not following the instructions to us the k-method.. Otherwise generally well done.

(c) *Find the equation of a line with a positive gradient that is inclined at 45° to the line $x+2y-6=0$ and has the same y-intercept.* [3]

For $x+2y-6=0$

$$2y=6-x$$
$$y=3-\frac{1}{2}x$$

Thus $m_1=-\frac{1}{2}$. Letting the gradient of the required line be m_2 meeting at 45° gives

$$\tan 45=\left|\frac{m_2-\left(\frac{-1}{2}\right)}{1+\left(\frac{-1}{2}\right)\times m_2}\right|$$
$$1=\left|\frac{m_2+\frac{1}{2}}{1-\frac{m_2}{2}}\right|$$
$$=\left|\frac{\frac{2m_2+1}{2}}{\frac{2-m_2}{2}}\right|$$
$$=\left|\frac{2m_2+1}{2-m_2}\right|$$

i.e. $2-m_2=2m_2+1$, reject $m_2-2=2m_2+1$

$3m_2=1$ $\qquad$ $m_2=-3$

$m_2=\frac{1}{3}$ $\qquad$ <0

With a y-intercept of 3, the required equation is $y=\frac{1}{3}x+3, or$

$$0=x-3y+9$$

➊ correct relationship

➊ correct m_2

➊ correct equation

Many did not use absolute value and lost a mark.

Students who didn't use the two cases often lost the mark as they chose incorrectly (the negative gradient) and didn't consider the other alternative.

(d) *The point $T(2,-3)$ divides the interval between the points $A(-4,-2)$ and $B(x_1,y_1)$ in the ratio $2:3$. Find the co-ordinates of point B.* [3]

$A(-4,-2)\quad B(x_1,y_1)$

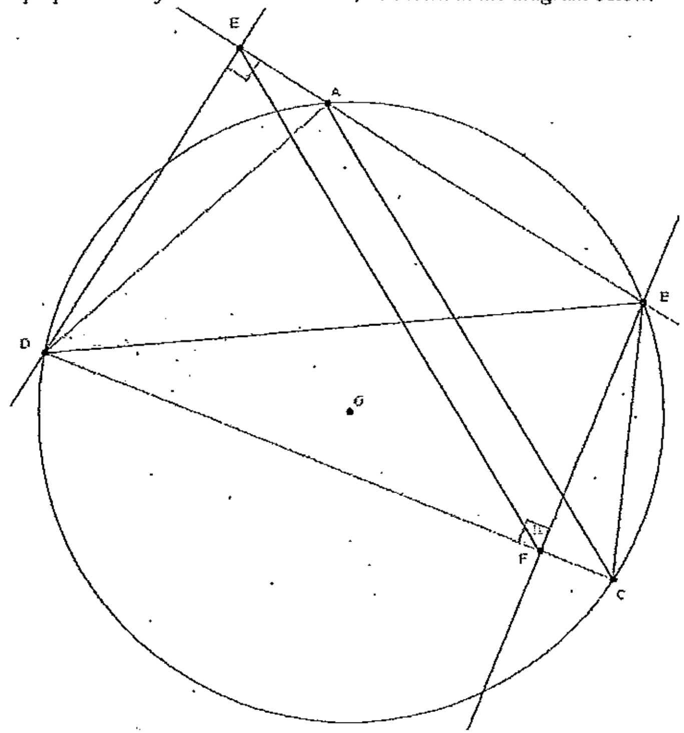
Ratio $2:3$ gives

$T(2,-3)$

Hence $T(2,-3)=\left(\dfrac{lx_0+kx_1}{k+l},\dfrac{ly_0+ky_1}{k+l}\right)$ $2=\dfrac{3\times(-4)+2x_1}{2+3}$ $\quad -3=\dfrac{3\times(-2)+2y_1}{2+3}$ $=\dfrac{-12+2x_1}{5}$ $\quad =\dfrac{-6+2y_1}{5}$ $10=-12+2x_1$ $\quad -15=-6+2y_1$ $22=2x_1$ $\quad -9=2y_1$ $x_1=11$ $\quad y_1=-4\frac{1}{2}$	❶ equations set up with correct values ❶ x value ❶ y value	Many had the formula correct but then did not substitute the values in the correct corresponding positions.

Question 5: [14 marks]

(a) *ABCD is a cyclic quadrilateral. The perpendicular from D meets AB produced at E, and the perpendicular from B meets CD at F, as shown in the diagram below.*



Copy the diagram into your answer booklet and prove $AC \parallel EF$. [4]

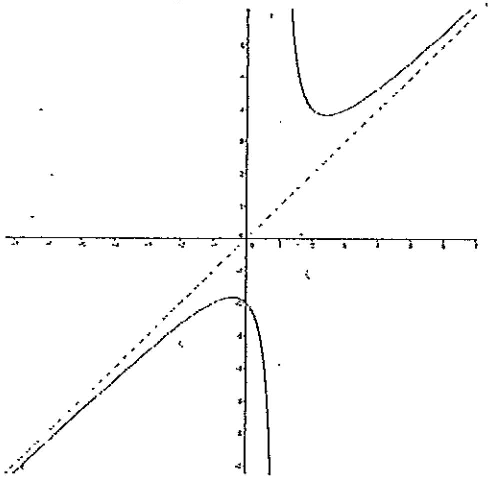
Construction: join AC and EF. (as shown above). Proof: $\angle CAB=\angle CDB$ ($\angle$'s in same seg on arc BC) Considering quadrilateral $DEBF$; $\angle DEB+\angle BFD=90+90$ $=180$ Hence $DEBF$ is a cyclic quadrilateral, thus there is a circle through $DEBF$. Now $\angle CDB=\angle FDB$ (as C, F and D are collinear) Thus $\angle FDB=\angle FEB$ ($\angle$'s in same seg on arc FB) Hence $\angle CAB=\angle FEB$ and are in a corresponding position for lines FA and FE. Thus $AC \parallel EF$ (cor. $\angle$'s eq.)	❶ correct property applied to link cor. $\angle$'s ❶ link to 2nd Cyclic quadrilateral identified ❶ identifies other linked pair of equal angles ❶ correct reason for conclusion	Many non-attempts at this part A few students recognised the angle in the same arc, fewer deduced the other cyclic quad. Only a handful managed to identify and explain the final links to complete this question.

(b) Draw a neat ½ page sketch of $y=\frac{x^2-x+2}{(x-1)}$, showing all asymptotes and intercepts. [4]

$$y=\frac{x^2-x+2}{(x-1)}$$
$$=\frac{x(x-1)+2}{(x-1)}$$
$$=x+\frac{2}{(x-1)}$$
$$y-x=\frac{2}{(x-1)}$$
$$(y-x)(x-1)=2$$

Asymptotes are $y\neq x, x\neq 1$

y- intercept: $y=0+\frac{2}{(0-1)}$
$$=-2$$



❶ asymptotes correctly found

❶ intercept correctly found and/or graphed

❶ graph towards asymptotes correctly drawn

❶ sketch – shape correct and in correct regions

Most found the $x\neq 1$ asymptote, but few derived the oblique one correctly. Dividing all terms by x only works once the polynomials are of equal or lesser degree in the numerator.

Many found the y-intercept, but a few then ignored it in their graph!

Many drew graphs that ended up going *away* from the asymptote rather than converging on it. Some even crossed an asymptote.

Many graph shapes were poorly drawn, some even becoming a relations rather than a function.

(c) *For the equation* $y=|2x-1|+|x+1|$;

i. *By determining branches or otherwise, draw a neat sketch of this graph.* [3]

Branch points when $2x-1=0$ and $x+1=0$.

$2x=1$ $\quad\quad x=-1$

$x=\frac{1}{2}$

❶ determines critical points (vertices)

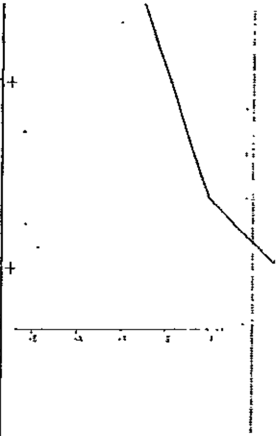
Many students could correctly identify the critical points (vertices) but not use them correctly.

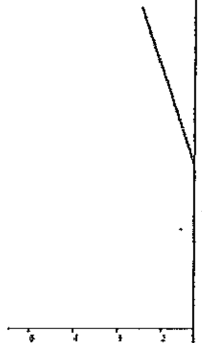
Thus when

$x<-1$:
$$y=-(2x-1)-(x+1)$$
$$=-2x+1-x-1$$
$$=-3x$$

$-1<x<\frac{1}{2}$:
$$y=-(2x-1)+(x+1)$$
$$=-2x+1+x+1$$
$$=-x+2$$

$x>\frac{1}{2}$:
$$y=(2x-1)+(x+1)$$
$$=2x-1+x+1$$
$$=3x$$





Alternative using absolute value graphs:

Domains were often poorly explained or notated.

O determines branch equations (or equivalent)

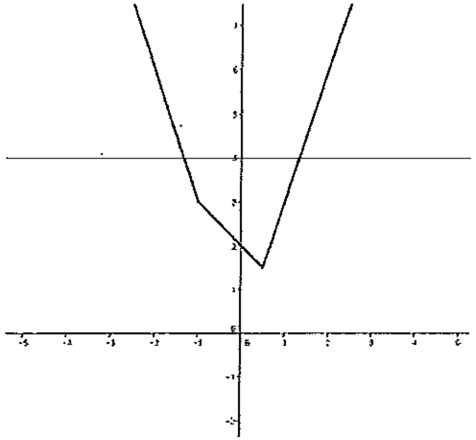
Four options were also often given, indicating a lack of understanding.

O sketch correct

Graphs often didn't even match incorrectly calculated equations from the algebra above.

Most who tried this alternative could not resolve the two absolute value graphs into the final graph required.

ii. *Hence determine (algebraically) the value(s) for which* $|2x-1|+|x+1|<4$. [3]



$|2x-1|+|x+1|<4$ when (from branches when $x>\frac{1}{2}$ and $x<-1$):

$-3x<4$ and $3x<4$

$x>-1\frac{1}{3}$ $x<1\frac{1}{3}$

Thus $-1\frac{1}{3}<x<1\frac{1}{3}$

O determines branches to use (either visually or algebraically)

O x values correct for branches

O correct domain stated

Very few used part (i) to help determine the correct branch to use – often all branches were calculated, but no attempt was made to resolve to the required domain.