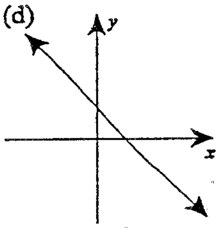
FUNCTIONS

Exercise 1 : Definitions

1. Write down the domain and range of each relation:
   (a) $\{ (1, 3), (3, 0), (5, 4) \}$ (b) $\{ (1, 1), (2, 1), (3, 1) \}$

2. State whether each relation is a FUNCTION or NOT A FUNCTION:
   (a) $\{ (1, 1), (2, 2), (3, 3) \}$ (b) $\{ (1, 2), (2, 3), (1, 4) \}$ (c) $\{ \{3, 2), (4, 3), (5, 3) \}$

(d)


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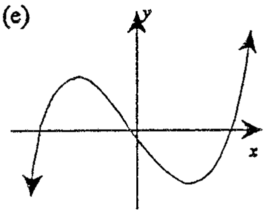

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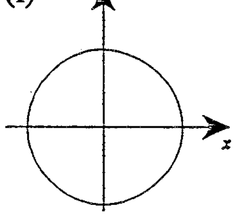

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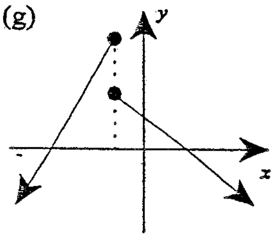


(h) $y = 2x^2$ (i) $x = y^2$ (j) $y = 2$ (k) $x = 1$

Exercise 2 : Function Notation

1. If $f(x) = 2x^2 - 1$, evaluate
   (a) $f(0)$ (b) $f(3)$ (c) $f(-2)$
   (d) $3f(1) - [\, f(2)\,]^2$ (e) $f(a)$ (f) $f(x+1)$

2. If $g(x) = 2^x + \dfrac{1}{x-1}$, (a) evaluate (i) $g(0)$
   (ii) $g(-4)$ as a fraction
   (b) For what value of x does $g(x)$ not exist ?

3. If $f(x) = x^2 + 1$ and $g(x) = 2x - 3$, evaluate (a) $f[\,g(0)\,]$ (b) $g[\,f(0)\,]$
   simplify (c) $f[\,g(x)\,]$ (d) $g[\,f(x)\,]$

4. Evaluate $h(3)$ if
   (a) $h(x) = 5 - x$ (b) $h(x) = \sqrt{25 - x^2}$ (c) $h(x) = 6x$
   (d) $h(x) = 6$ (e) $h(x) = |x|$ (f) $h(x) = |2x - 1| - |3 - 5x|$

5. If $f(x) = 3x - 2$, solve
   (a) $f(x) = 0$ (b) $f(x) = x + 1$ (c) $f(x) < 1$

6. If $P(x) = x^2 - x - 2$, solve
   (a) $P(x) = 0$ (b) $P(x) = 4$

7. If $A(x) = 2x^2 + 7x - 4$ and $B(x) = 2x - 1$, solve $A(x) = B(x)$.

8. If $f(x) = 2^x$, for what value of x does $f(x) = \frac{1}{4}$?

9. If $f(x) = \begin{cases} 1 - x & \text{if} \quad x < 3 \\ x^2 + 2 & \text{if} \quad x \geq 3 \end{cases}$ evaluate

(a) $f(-5)$ (b) $f(3)$ (c) $f(5)$

10. If $f(x) = \begin{cases} x^2 - 1 & \text{if} \quad x \leq -4 \\ 6 & \text{if} \quad -4 < x < 0 \\ 5x & \text{if} \quad x \geq 0 \end{cases}$ evaluate

(a) $f(-6)$ (b) $f(-2)$ (c) $f(-4) + f(2)$ (d) $f(a^2)$

11. If $f(x) = x^2 + x$, simplify fully $\dfrac{f(x+h) - f(x)}{h}$.

12. If $g(x) = x^2 - x - 1$, for what value(s) of x does $g(x) = g(-x)$?

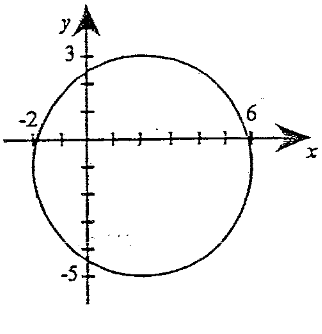
13. If $f(x) = ax + b$, $f(3) = 2$, and $f(4) = 4$, find a and b.

Exercise 3 : Graphs of Basic Functions and Relations

1. Draw neat sketches of the following functions and relations.
Show all important features of the graphs. State also the domain and range.

(a) $x = 3$
(b) $y = -1$
(c) $y = 2x - 4$
(d) $3x + 4y - 12 = 0$
(e) $y = -x^2$
(f) $y = 2x^2 + 1$
(g) $y = 9 - x^2$
(h) $y = (x - 2)^2$
(i) $y = -(x + 1)^2$
(j) $y = (x - 2)(x - 4)$
(k) $y = x^2 - x - 6$
(l) $y = 8 + 2x - x^2$
(m) $y = (x + 3)^2 + 1$
(n) $y = 4 - (x - 5)^2$
(o) $y = -x^3$
(p) $y = 3 + 2x^3$
(q) $y = x^8$
(r) $y = -x^9$
(s) $x^2 + y^2 = 16$
(t) $x^2 + (y - 3)^2 = 4$
(u) $(x - 1)^2 + (y + 2)^2 = 1$
(v) $x^2 + y^2 + 8x - 6y - 11 = 0$
(w) $x^2 + y^2 - 2x = 0$
(x) $9x^2 + 9y^2 + 9x + 6y + 1 = 0$
(y) $y = 3^x$
(z) $y = -4^{-x}$
(aa) $y = 1 - 2^{-x}$
(bb) $y = -\dfrac{3}{x}$
(cc) $xy = 8$

2. A standard hyperbola has as its asymptotes the x and y axes, and passes through the point $(3, -2)$. Find its equation.

3. 

Find the equation of this circle in general form.

(Note that the numbers given are extremities of the circle, NOT intercepts with the axes.)

Exercise 4 : Graphs With Restricted Domains, and Graphs of Piecemeal Functions

1. Sketch each function over the stated domain. State also the range of the function over the specified domain.

(a) $y = 3 - 2x, \ x \geq 1$ (b) $y = x^2, \ 0 \leq x \leq 2$

(c) $xy = 6, \ -2 < x \leq 3, \ x \neq 0$ (d) $y = (x+2)^2 - 1, \ -3 \leq x \leq 0$

2. Sketch each of the following piecemeal functions, showing the coordinates of the endpoints of each interval. State also the range of the function over the specified domain.

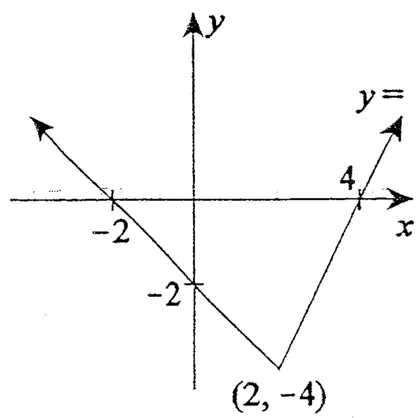
(a) $f(x) = \begin{cases} x+1 & \text{if } x < 0 \\ -x+1 & \text{if } x \geq 0 \end{cases}$ (b) $f(x) = \begin{cases} 2x+1 & \text{if } x < 1 \\ 3 & \text{if } x \geq 1 \end{cases}$

(c) $f(x) = \begin{cases} -2-x & \text{if } x \leq 2 \\ 2x-3 & \text{if } x > 2 \end{cases}$ (d) $f(x) = \begin{cases} x^2+1 & \text{if } x < 0 \\ 1-x & \text{if } x \geq 0 \end{cases}$

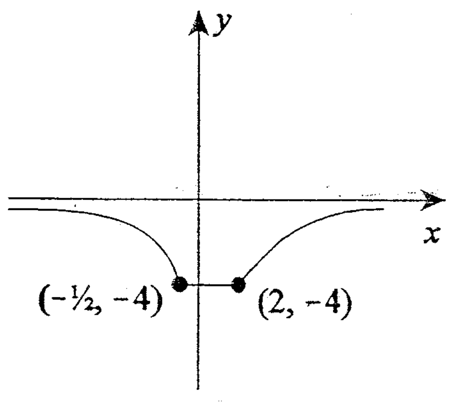
(e) $f(x) = \begin{cases} -x^2 & \text{if } x < 1 \\ 2^x & \text{if } x > 1 \end{cases}$ (f) $f(x) = \begin{cases} -2x-3 & \text{for } x < -1 \\ -1 & \text{for } -1 \leq x < 1 \\ -\dfrac{1}{x} & \text{for } x \geq 1 \end{cases}$

3. Write down piecemeal descriptions for the following functions:

(a)



(b)



(Both curved sections are hyperbolae whose asymptotes are the x and y axes)

Exercise 5 : Graphs of Polynomial Functions

1. Sketch graphs for

(a) $y = (x - 1)(x - 3)$ (b) $y = x(x + 2)(x - 3)$ (c) $y = -x(x - 1)(x - 2)$

(d) $y = (x - 2)^2$ (e) $y = (x + 1)^3$ (f) $y = x(x - 3)^2$

(g) $y = (x + 1)^2(x - 1)$ (h) $y = -(x + 2)(x - 1)^3$ (i) $y = (x - 1)^2(x - 3)^2$

(j) $y = x^2(1 - x)(x + 1)$ (k) $y = x^3(1 - x)^3$ (l) $y = 2(1 - x)^4(x + 2)^5$

2. Sketch graphs for

(a) $y = x^3 + 3x^2 + 2x$ (b) $y = x^3 - 6x^2 + 9x$ (c) $y = x^3 + 2x^2 - x - 2$

3. Solve using graphical means:

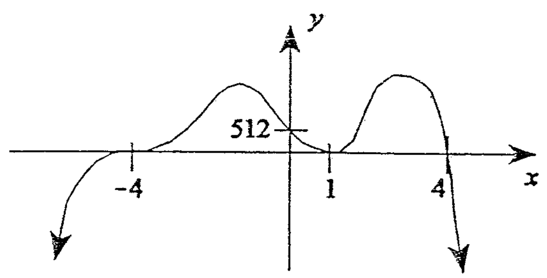
(a) $(x - 2)(x + 3) < 0$ (b) $(x - 2)(x + 3) > 0$ (c) $x^2 + 5x + 6 \le 0$

(d) $x^2 + 5x + 6 > 0$ (e) $x(x + 1)(x - 1) \le 0$ (f) $-x(x + 1)(x - 1) \le 0$

(g) $x^3 - 5x^2 + 4x \ge 0$ (h) $x(x - 2)^2 > 0$ (i) $x(x - 2)^2 \le 0$

4. Sketch the function $y = f(x)$, where $f(x) = \begin{cases} -x^2(x+2) & x < 0 \\ x - x^3 & x \ge 0 \end{cases}$

5. Write down the equation of lowest degree for the following polynomial graph:



Exercise 6 : Graphs of Rational Functions

[State the domain and range for all sketches]

1. Sketch graphs (on separate number planes) for the following functions:

(a) $y = \dfrac{1}{x}$ (b) $y = \dfrac{1}{x + 2}$ (c) $y = 1 + \dfrac{1}{x + 2}$

2. Sketch graphs (on separate number planes) for the following functions:

(a) $y = \dfrac{3}{x}$ (b) $y = \dfrac{3}{x - 3}$ (c) $y = -\dfrac{3}{x - 3}$ (d) $y = 4 - \dfrac{3}{x - 3}$

3. Sketch, stating the domain and range :

(a) $y = \dfrac{2}{x-4} + 1$ (b) $y = -2 - \dfrac{1}{x+1}$ (c) $y = \dfrac{1}{2x-3} - 3$

(d) $y = \dfrac{x+2}{x+1}$ (e) $y = \dfrac{x}{x+1}$ (f) $y = \dfrac{2x+1}{x-2}$

(g) $y = \dfrac{3x-2}{x+3}$ (h) $y = \dfrac{1-x}{1+x}$

Exercise 7 : Graphs of Surdic and Absolute Value Functions

1. Sketch each relation, stating the domain and range in each case:

(a) $y = \sqrt{25 - x^2}$ (b) $y = -\sqrt{1 - x^2}$ (c) $x = \sqrt{\frac{9}{4} - y^2}$

2. For each function, find the domain algebraicly, sketch the graph, then state the range:

(a) $y = \sqrt{x-1}$ (b) $y = -\sqrt{x+3}$ (c) $y = \sqrt{3x-2}$

(d) $y = \sqrt{2-x}$ (e) $y = -\sqrt{5-2x}$ (f) $y = 3 + \sqrt{1+2x}$

3. Calculate the domain of each of the following functions [and the range for (d) – (h)]:

(a) $y = \sqrt{x-2} + \sqrt{4-x}$ (b) $y = \sqrt{x+1} + \sqrt{x+5}$ (c) $y = \sqrt{x-3} - \sqrt{1-x}$

(d) $y = \dfrac{1}{\sqrt{x+4}}$ (e) $y = 1 - \dfrac{2}{\sqrt{1-x}}$ (f) $y = \sqrt{x^2 - 9}$

(g) $y = \dfrac{1}{\sqrt{16 - x^2}}$ (h) $y = \sqrt{1 + x^2}$

4. Sketch the following functions, stating the range in each case:

(a) $y = |x - 4|$ (b) $y = -|x + 3|$ (c) $y = |x| + 3$ (d) $y = 1 - |x|$

(e) $y = |2x - 3|$ (f) $y = |3x + 7| - 2$ (g) $y = 4 - 3|4 - x|$

Exercise 8 : Odd & Even Functions

1. State whether the following functions are EVEN, ODD, or NEITHER. Show all working.

(a) $y = 2x^2$ (b) $y = 2x^3$ (c) $y = x^4 - x^2 + 1$ (d) $y = x - x^3 + 1$

(e) $y = |x|$ (f) $y = \dfrac{x}{x^2 - 1}$ (g) $y = (x+1)^2$

2. (a) Sketch the graph of $y = x^3$, $0 \le x \le 2$. Show the coordinates of the endpoints.
   (b) The above function is part of an even function $f(x)$, defined in the domain $-2 \le x \le 2$. Draw a sketch of $y = f(x)$.
   (c) Write a piecemeal description of the function $f(x)$.

3. (a) Sketch the graphs of $y = \sin x$ and $y = \cos x$ on separate number planes, each in the domain $-180° \le x \le 180°$.
   (b) Hence decide if the sine and cosine functions are odd, even or neither.
   (c) Remembering that $\sin 45° = \cos 45° = \frac{1}{\sqrt{2}}$, write down the values of $\sin(-45°)$ and $\cos(-45°)$.

4. Show that if an odd function is defined for $x = 0$, then its graph must pass through the origin.

Exercise 9 : Curve sketching techniques

1. (i) On the **same** set of axes, sketch the graphs of $y = x$ and $y = -\frac{1}{x}$.
   (ii) By addition of ordinates, sketch the graph of $y = x - \frac{1}{x}$.

2. Sketch the following by addition of ordinates:
   (a) $y = 2x + \frac{1}{x}$ (b) $y = \frac{3}{x} - 2x$ (c) $y = x^2 + \frac{1}{x}$
   (d) $y = x^2 - \frac{1}{x}$ (e) $y = 2^x + 2^{-x}$ (f) $y = 2^x - 2^{-x}$

3. (a) Prove that $y = \sqrt{x+1} + \sqrt{1-x}$ is an even function.
   (b) Find the domain of this function.
   (c) Find the value of y when $x = 0$.
   (d) Use the information gleaned in parts (a), (b) and (c), and the method of addition of ordinates to sketch this function, stating also its range.

4. Sketch the following graphs of reciprocal functions, stating the domain and range :
   (a) $y = \operatorname{cosec} x$, $0 \le x \le 360°$ (b) $y = \cot x$, $0 \le x \le 360°$ (c) $y = \frac{2}{x^2+1}$
   (d) $y = \frac{2}{x^2-1}$ (e) $y = \frac{1}{x^3-x}$ (f) $y = \frac{1}{x^2}$
   (g) $y = \frac{1}{(x-2)^2}$ (h) $y = \frac{1}{\sqrt{4-x^2}}$

5. Sketch the following rational functions:

(a) $y = \dfrac{1}{x+1}$ (b) $y = \dfrac{x}{x+1}$ (c) $y = \dfrac{2}{(x-3)(x+2)}$

(d) $y = \dfrac{1}{x^2+x}$ (e) $y = \dfrac{x+1}{(x-1)(x+2)}$ (f) $y = \dfrac{x^2-4}{x^2+2x-3}$

Exercise 10 : Inverse of a Function

1. For each **function** in Exercise 1 Question 2, state whether or not it has an inverse function.

2. Find the inverse function of each of the following functions:

(a) $y = x + 1$ (b) $y = 3x - 2$ (c) $y = \dfrac{x+2}{3}$

(d) $y = x^3$ (e) $y = (x+1)^3$ (f) $y = \dfrac{1}{x-1}$

(g) $y = \dfrac{x}{x-1}$

3. The function $y = x$ is invariant under inversion.
That is, the equation of the function and its inverse are the same.
(i) Give examples of 2 more functions which are invariant under inversion.
(ii) What do you notice about the graphs of such functions?

4. (a) Sketch the graph of $y = \sqrt{3-x}$.
(b) On the same axes, sketch the graph of its inverse function.
(c) Find the equation of the inverse function.
(d) Find the coordinates of the point of intersection of the function and its inverse.

Exercise 11 : Revision & Extension

1. Sketch the following relations, showing main features, and stating the domain & range.

(a) $xy = 1$ (b) $y = |x|$ (c) $y = x^2 - 4$

(d) $x^2 + y^2 = 36$ (e) $y = 2^x$ (f) $y = x^3 - 4x^2$

(g) $y = -\sqrt{49 - x^2}$ (h) $y = -\sqrt{3-2x}$ (i) $y = 1 + \dfrac{3}{x+4}$

(j) $(x-1)^2 + (y+2)^2 = 1$ (k) $y = 1 + |x+1|$ (l) $y = x^3 - 4x^2 + 3x$

2. Determine the equation of the axis of symmetry and the coordinates of the vertex of the parabola $y = 3x^2 - 6x + 1$. Hence sketch the parabola and write down its range.

3. For the domain $-1 \le x \le 3$, determine the maximum value and the minimum value of each of the functions defined by $f(x) = 3x^2$, $g(x) = 4x - x^2$.
Find all values of x in the interval $-1 \le x \le 3$ for which $f(x) = g(x)$.
Sketch the curves $y = f(x)$ and $y = g(x)$, for $-1 \le x \le 3$, on the same diagram. Indicate clearly the points where maximum and minimum values are attained and the points of intersection of the two curves.

4. A function is defined by the following rule:

$$f(x) = \begin{cases} 0 & \text{if } x \le -2 \\ -1 & \text{if } -2 < x < 0 \\ x & \text{if } x \ge 0 \end{cases}$$

Find: (a) $f(-2) + f(-1) + f(0)$; (b) $f(a^2)$.

5. The function $f(x)$ is defined by the rule $f(x) = \begin{cases} 0 & \text{if } x \le 0 \\ 2x & \text{if } x > 0 \end{cases}$
Sketch the function $f(x)$, from $x = -2$ to $x = 2$.

6. If $f(x) = ax^2 + bx + c$ find a simplified expression for $f(x) - f(-x)$.

7. (a) What is the equation of the circle whose centre is at the origin and which passes through the point $(5, -7)$?
(b) A circle, of radius 5 units, has its centre at the point $(-3, 4)$. What are the coordinates of the two points at which the circle cuts the y axis?
(c) Find the equation of the parabola with vertex $(-1, -4)$ which passes through the origin and whose axis is parallel to the y-axis.

8. The parabola $y = ax^2 - c$, and the circle $x^2 + y^2 = 16$ meet on both the x and y axes. If a and c are both positive, what are their values?

9. (i) Draw the graphs of $y = |x|$ and $y = x + 4$ on the same set of axes.
(ii) Find the coordinates of the point of intersection of these two graphs.

10. State the natural (ie. the largest possible) domain of the function given by $y = \sqrt{1+x} - \sqrt{1-x}$.

11. Find the range of the function f given by $f(x) = \dfrac{1}{1+x^2}$ over the domain of all real numbers.

12. Consider $f(x) = \dfrac{1-x^2}{x^4}$, $x \ne 0$.
(i) State the zeros of this function, ie. the values of x where $f(x) = 0$.
(ii) Describe how this function behaves near $x = 0$.
(iii) Describe how this function behaves for large x.
(iv) Give a rough sketch of this function.

13. Find the maximum value of $2x(1 - x)$.

14. The function $f(x)$ is defined by the rule $f(x) = 9x(x - 2)^2$ in the domain $-1 \le x \le 3$. Draw a sketch of the graph of $y = f(x)$, showing clearly the intercepts with the x and y axes, and the values at the endpoints of the domain.

15. (a) Sketch the curve $y = x^3 + x^2 - x - 1$ over the domain $-1 \le x \le 2$.
    (b) Given that one of the turning points of this curve occurs when $x = \frac{1}{3}$, find the range of this function over the stated domain.

16. Sketch graphs for
    (a) $y = \dfrac{x^2 + x}{x}$ (b) $y = \dfrac{x^2 + x}{x + 1}$ (c) $y = \dfrac{x^3 + 2x^2 + x}{x + 1}$

17. $f(x) = \sqrt{x}$ and $g(x) = \sqrt{x + 1}$. For what values of x does
    (a) $g(x^2) = [g(x)]^2$? (b) $f(x^2) = [f(x)]^2$?

18. Find $f(x)$ if
    (a) $f(x + 1) = x + 4$ (b) $f(x + 1) = 3x - 2$
    (c) $f(x + 1) = 1 - x^2$ (d) $f(2a) = a^2 - 2a - 1$

19. $f(x) = 3x - 4$ and $g(x) = x^2 - 1$, show that
    (a) $f(a) + f(b) = f(a + b)$ (b) $g(a) + g(b) \ne g(a + b)$

20. Write down the range of the function $f(x) = \dfrac{1}{x^2 + 4x + 7}$.

21. Sketch the graph of the function $y = \sqrt{16 - (x - 3)^2} + 1$. State the domain and range of the function.

22. If $f(x) = 3x + 5$ and $g(x) = \dfrac{x - 5}{3}$, find a simplified expression for $f[g(x)] + g[f(x)]$.

23. The parabola $y = ax^2 + bx + c$ has its vertex at $(2, 1)$ and passes through the point $(0, 0)$. Find a, b and c.

24. Given that $Q(x) = ax^2 + bx + c$ for all x, and that $Q(0) = 4$, $Q(1) = 23$, $Q(-1) = 1$, determine the constants a, b, c.

25. Sketch $y = \sqrt{(x + 2)^2}$

26. (a) By first sketching $y = x^2 - 1$, draw a sketch of $y = \dfrac{1}{x^2 - 1}$.

    (b) Hence write down the domain and range of the function $y = \dfrac{1}{x^2 - 1}$.

Answers

Exercise 1

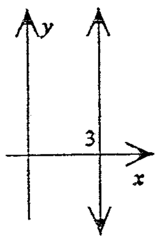
1. (a) D={1, 3, 5}, R={0, 3, 4} (b) D={1, 2, 3}, R={1}
2. (a) F (b) NF (c) F (d) F (e) F
 (f) NF (g) NF (h) F (i) NF (j) F (k) NF

Exercise 2

1. (a) −1 (b) 17 (c) 7 (d) −46
 (e) $2a^2 - 1$ (f) $2x^2 + 4x + 1$
2. (a) (i) 0 (ii) $-\frac{11}{80}$ (b) $x = 1$
3. (a) 10 (b) −1 (c) $4x^2 - 12x + 10$ (d) $2x^2 - 1$
4. (a) 2 (b) 4 (c) 18 (d) 6
 (e) 3 (f) −7
5. (a) $x = \frac{2}{3}$ (b) $x = 1\frac{1}{2}$ (c) $x < 1$
6. (a) $x = 2, -1$ (b) $x = 3, -2$
7. $x = -3, \frac{1}{2}$ 8. $x = -2$
9. (a) 6 (b) 11 (c) 27
10. (a) 35 (b) 6 (c) 25 (d) $5a^2$
11. $2x + h + 1$ 12. $x = 0$ 13. $a = 2$, $b = -4$

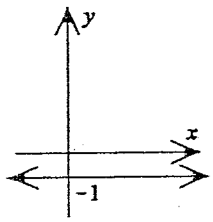
Exercise 3

1. (a)



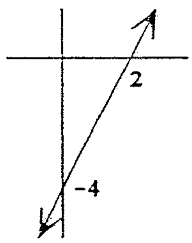
D: $x = 3$
R: all real y

(b)



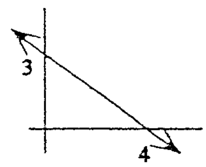
D: all real x
R: $y = -1$

(c)



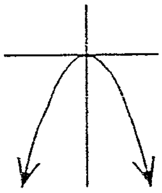
D: all real x
R: all real y

(d)



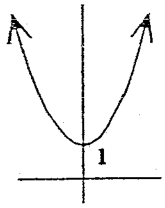
D: all real x
R: all real y

(e)



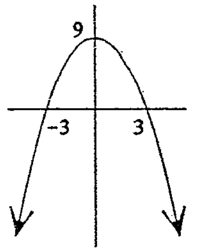
D: all real x
R: $y \le 0$

(f)



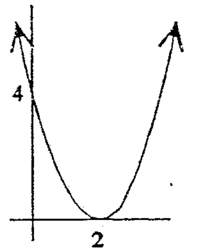
D: all real x
R: $y \ge 1$

(g)



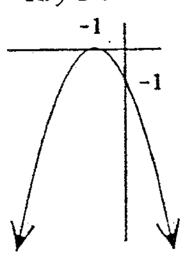
D: all real x
R: $y \le 9$

(h)



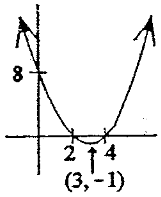
D: all real x
R: $y \ge 0$

(i)



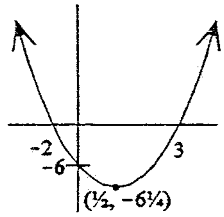
D: all real x
R: $y \le 0$

(j)



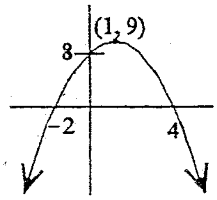
D: all real x
R: $y \ge -1$

(k)



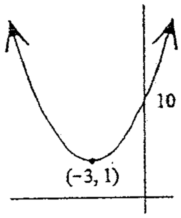
D: all real x
R: $y \ge -6\frac{1}{4}$

(l)



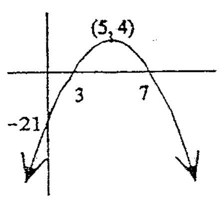
D: all real x
R: $y \le 9$

(m)



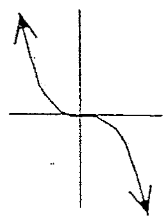
D: all real x
R: $y \ge 1$

(n)



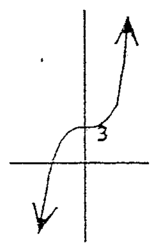
D: all real x
R: $y \le 4$

(o)

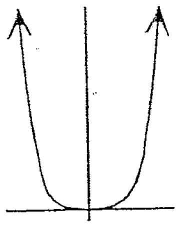


D: all real x
R: all real x

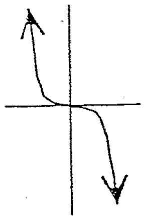
(p)



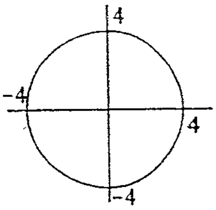
D: all real x
R: all real y

(q)


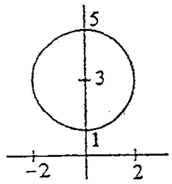
D: all real x
R: $y \geq 0$

(r)


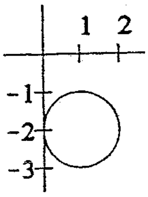
D: all real x
R: all real y

(s)


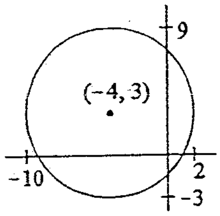
D: $-4 \leq x \leq 4$
R: $-4 \leq y \leq 4$

(t)


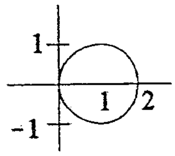
D: $-2 \leq x \leq 2$
R: $1 \leq y \leq 5$

(u)


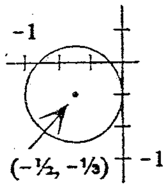
D: $0 \leq x \leq 2$
R: $-3 \leq y \leq -1$

(v)


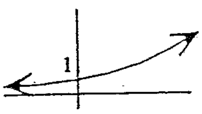
D: $-10 \leq x \leq 2$
R: $-3 \leq y \leq 9$

(w)


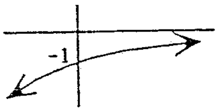
D: $0 \leq x \leq 2$
R: $-1 \leq y \leq 1$

(x)


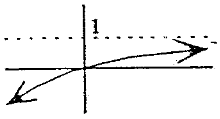
D: $-1 \leq x \leq 0$
R: $-\frac{5}{6} \leq y \leq \frac{1}{6}$

(y)


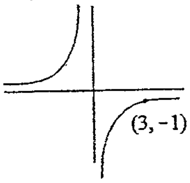
D: all real x
R: $y > 0$

(z)


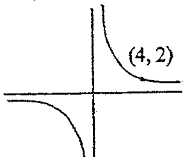
D: all real x
R: $y < 0$

(aa)


D: all real x
R: $y < 1$

(bb)


D: $x \neq 0$
R: $y \neq 0$

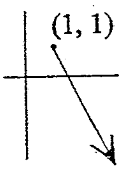
(cc)


D: $x \neq 0$
R: $y \neq 0$

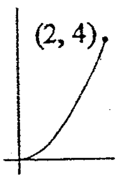
2. $xy = -6$

3. $x^2 + y^2 - 4x + 2y - 11 = 0$

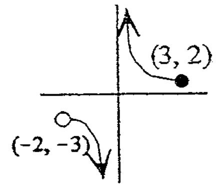
Exercise 4

1. (a)


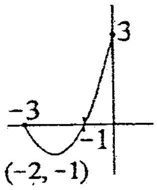
R: $y \leq 1$

(b)


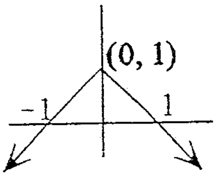
R: $0 \leq y \leq 4$

(c)


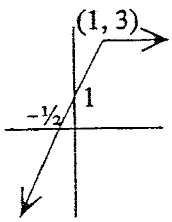
R: $y < -3,\ y \geq 2$

(d)


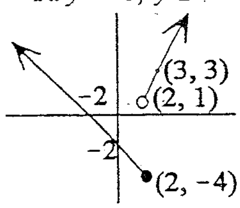
R: $-1 \leq y \leq 3$

2. (a)


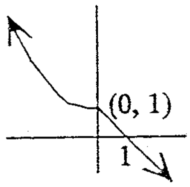
R: $y \leq 1$

(b)


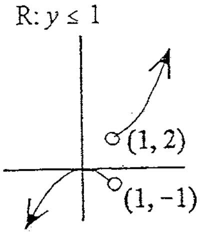
R: $y \leq 3$

(c)


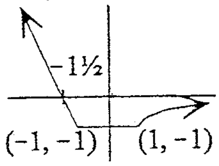
R: $y \geq -4$

(d)


R: all real y

(e)


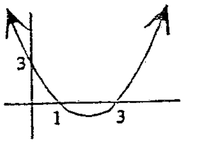
R: $y \leq 0,\ y > 2$

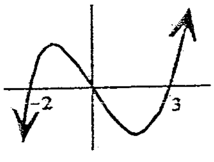
(f)


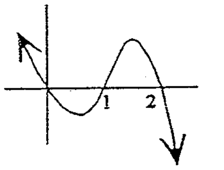
R: $y \geq -1$

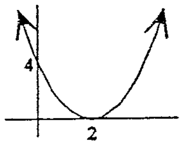
3. (a) $f(x) = \begin{cases} -x-2 & \text{if } x < 2 \\ 2x-8 & \text{if } x \geq 2 \end{cases}$ (b) $f(x) = \begin{cases} \dfrac{2}{x} & \text{if } x < -\frac{1}{2} \\ -4 & \text{if } -\frac{1}{2} \leq x < 2 \\ -\dfrac{8}{x} & \text{if } x \geq 2 \end{cases}$

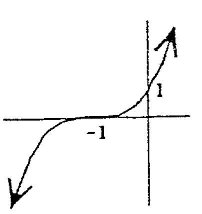
Exercise 5

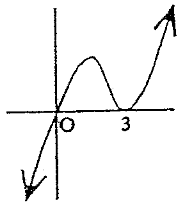
1. (a)


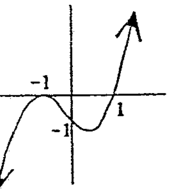
(b)


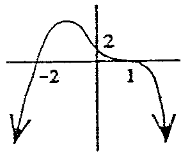
(c)


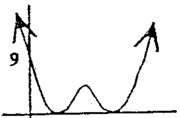
(d)


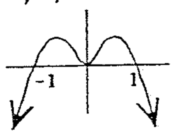
(e)


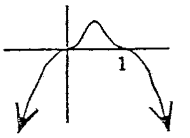
(f)


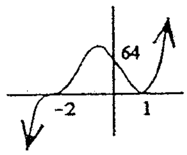
(g)


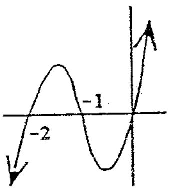
(h)


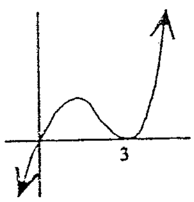
(i)


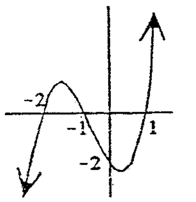
(j)


(k)


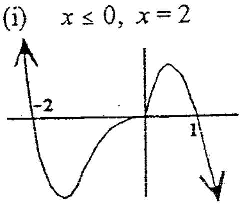
(l)


2. (a)


(b)


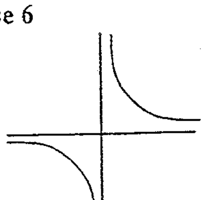
(c)


3. (a) $-3 < x < 2$ (b) $x < -3,\ x > 2$ (c) $-3 \leq x \leq -2$ (d) $x < -3,\ x > -2$
(e) $x \leq -1,\ 0 \leq x \leq 1$ (f) $-1 \leq x \leq 0,\ x \geq 1$ (g) $0 \leq x \leq 1,\ x \geq 4$ (h) $x > 0, x \neq 2$
(i) $x \leq 0,\ x = 2$

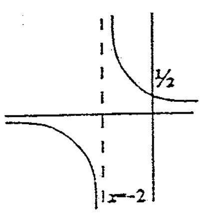
4.


5. $y = -2(x-1)^2\,(x-4)\,(x+4)^3$

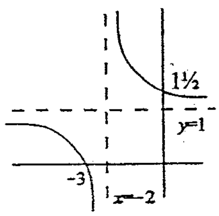
Exercise 6

1. (a)


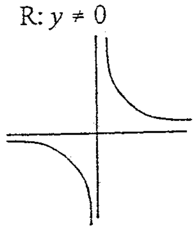
D: $x \neq 0$
R: $y \neq 0$

(b)


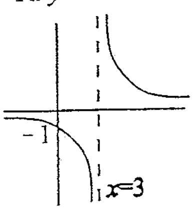
D: $x \neq -2$
R: $y \neq 0$

(c)


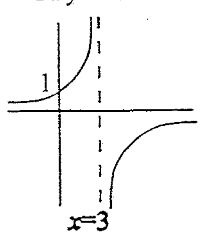
D: $x \neq -2$
R: $y \neq 1$

2. (a)


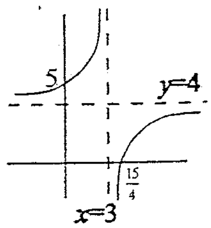
D: $x \neq 0$
R: $y \neq 0$

(b)


D: $x \neq 3$
R: $y \neq 0$

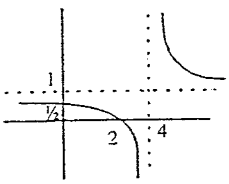
(c)


D: $x \neq 3$
R: $y \neq 0$

(d)


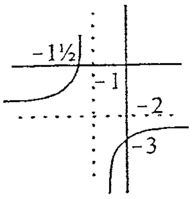
D: $x \neq 3$
R: $y \neq 4$

3. (a)



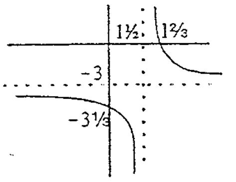
D: $x \neq 4$
R: $y \neq 1$

(b)



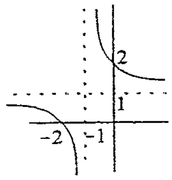
D: $x \neq -1$
R: $y \neq -2$

(c)



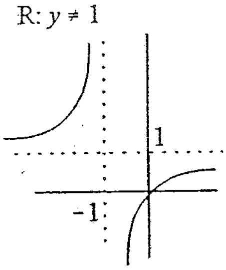
D: $x \neq 1\frac{1}{2}$
R: $y \neq -3$

(d)



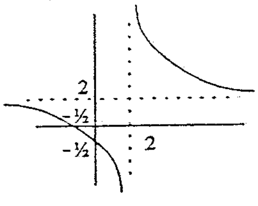
D: $x \neq -1$
R: $y \neq 1$

(e)



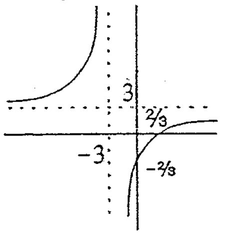
D: $x \neq -1$
R: $y \neq 1$

(f)



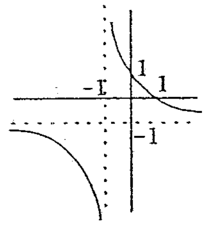
D: $x \neq 2$
R: $y \neq 2$

(g)



D: $x \neq -3$
R: $y \neq 3$

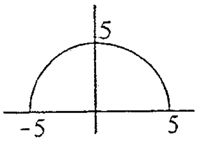
(h)



D: $x \neq -1$
R: $y \neq -1$

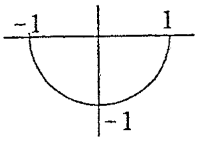
Exercise 7

1. (a)



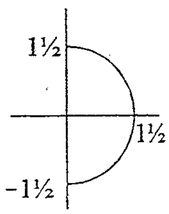
D: $-5 \leq x \leq 5$
R: $0 \leq y \leq 5$

(b)



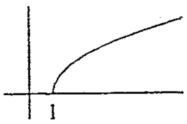
D: $-1 \leq x \leq 1$
R: $-1 \leq y \leq 0$

(c)



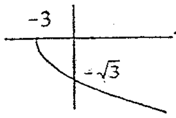
D: $0 \leq x \leq 1\frac{1}{2}$
R: $-1\frac{1}{2} \leq y \leq 1\frac{1}{2}$

2. (a)



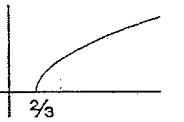
D: $x \geq 1$
R: $y \geq 0$

(b)



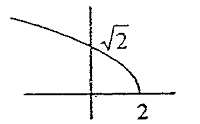
D: $x \geq -3$
R: $y \leq 0$

(c)



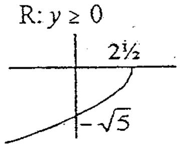
D: $x \geq \frac{2}{3}$
R: $y \geq 0$

(d)



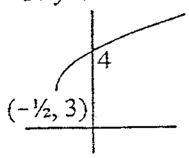
D: $x \leq 2$
R: $y \geq 0$

(e)



D: $x \leq 2\frac{1}{2}$
R: $y \leq 0$

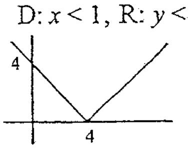
(f)



D: $x \geq -\frac{1}{2}$
R: $y \geq 3$

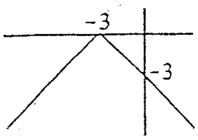
3. (a) $2 \leq x \leq 4$ (b) $x \geq -1$ (c) no real x (d) D: $x > -4$, R: $y > 0$
(e) D: $x < 1$, R: $y < 1$ (f) D: $x \leq -3, x \geq 3$, R: $y \geq 0$ (g) D: $-4 < x < 4$, R: $y \geq \frac{1}{4}$ (h) D: all real x, R: $y \geq 1$

4. (a)



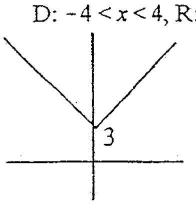
R: $y \geq 0$

(b)



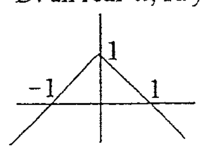
R: $y \leq 0$

(c)



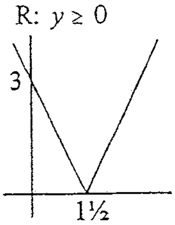
R: $y \geq 3$

(d)



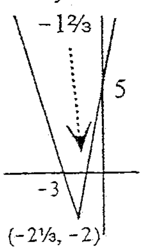
R: $y \leq 1$

(e)



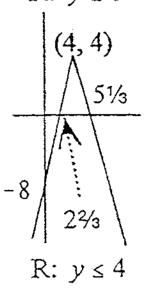
R: $y \geq 0$

(f)



R: $y \geq -2$

(g)

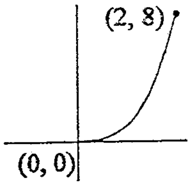


R: $y \leq 4$

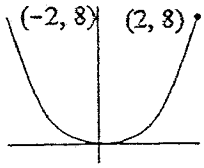
Exercise 8

1\. (a) even (b) odd (c) even (d) neither
  (e) even (f) odd (g) neither

2\. (a)

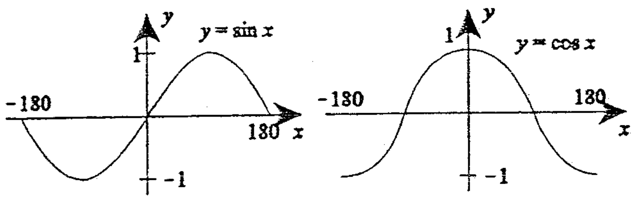


(b)



(c) $f(x) = \begin{cases} -x^3 & \text{for} \quad -2 \le x < 0 \\ x^3 & \text{for} \quad 0 \le x < 2 \end{cases}$

3\. (a)

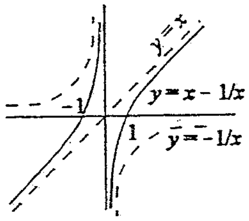


(b) sine is odd; cosine is even

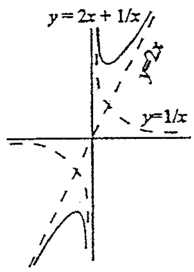
(c) $\sin(-45°) = -\dfrac{1}{\sqrt{2}}$; $\cos(-45°) = \dfrac{1}{\sqrt{2}}$

Exercise 9

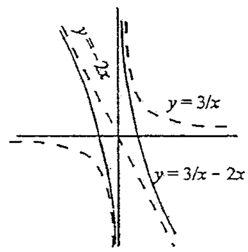
1\.



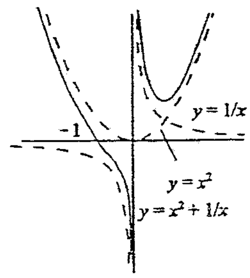
2\. (a)



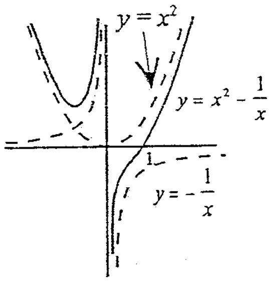
(b)



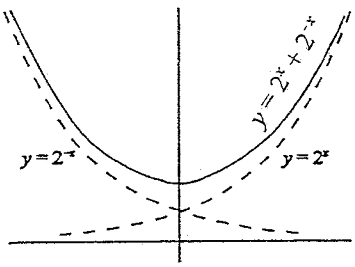
(c)



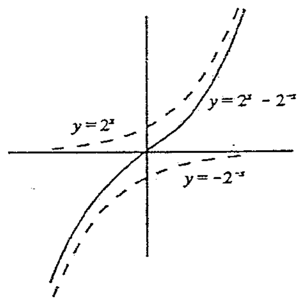
(d)



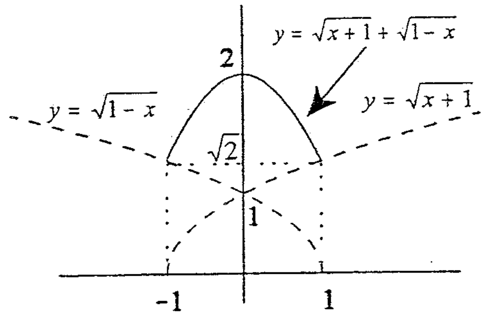
(e)



(f)

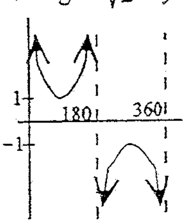


3\. (b) $-1 \le x \le 1$ (c) 2
  (d)



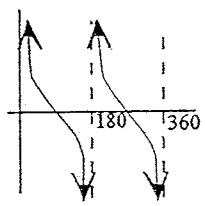
Range: $\sqrt{2} \le y \le 2$

4\. (a)



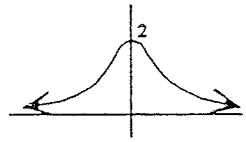
D: $0 < x < 360, x \ne 180$
R: $y \le -1,\ y \ge 1$

(b)



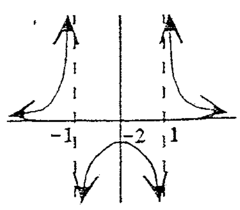
D: $0 < x < 360, x \ne 180$
R: all real y

(c)



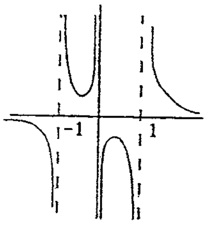
D: all real x
R: $y > 0$

(d)



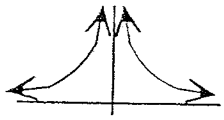
D: $x \ne \pm 1$
R: $y \le -2,\ y > 0$

(e)



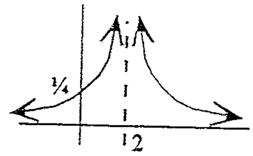
D: $x \neq 0, \pm 1$
R: $y \neq 0$

(f)



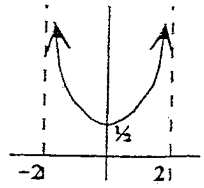
D: $x \neq 0$
R: $y > 0$

(g)



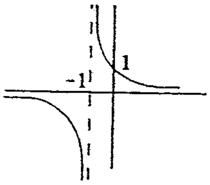
D: $x \neq 2$
R: $y > 0$

(h)

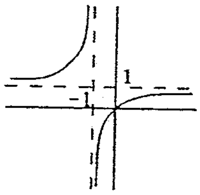


D: $-2 < x < 2$
R: $y \geq ½$

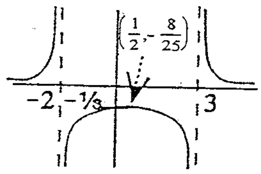
5. (a)



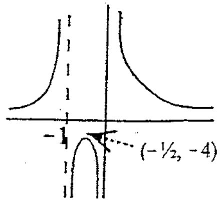
(b)



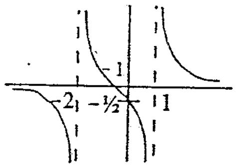
(c)



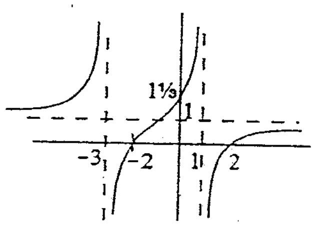
(d)



(e)



(f)



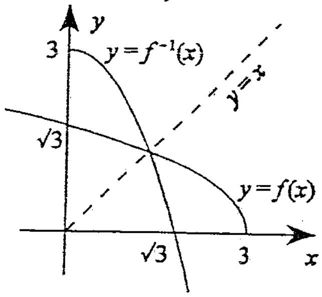
Exercise 10

1. (a) Yes (b) Not Function (c) No (d) Yes
   (e) No (f) Not Function (g) Not Function (h) No
   (i) Not Function (j) No (k) Not Function

2. (a) $y = x - 1$ (b) $y = \frac{x+2}{3}$ (c) $y = 3x - 2$ (d) $y = \sqrt[3]{x}$
   (e) $y = \sqrt[3]{x} - 1$ (f) $y = 1 + \frac{1}{x}$ (g) $y = \frac{x}{x-1}$

3. (i) $y = -x$ (or more generally, $y = c - x$, where c is any constant) ; $y = \frac{1}{x}$ (or $y = \frac{c}{x}$);
   $y = \sqrt{c^2 - x^2}$ over the domain $0 \leq x \leq c$; and an infinite number of others.
   (ii) They are all symmetrical in the line $y = x$.

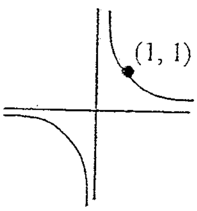
4. (a), (b)



   (c) $y = 3 - x^2,\ x \geq 0$
   (d) $\left(\frac{\sqrt{13}-1}{2}, \frac{\sqrt{13}-1}{2}\right)$

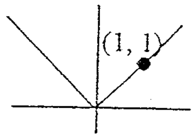
Exercise 11

1. (a)



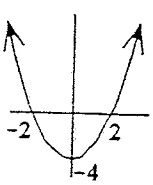
D : $x \neq 0$
R : $y \neq 0$

(b)



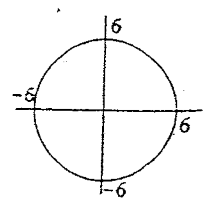
D : all real x
R : $y \geq 0$

(c)

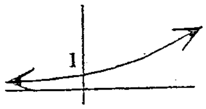


D : all real x
R : $y \geq -4$

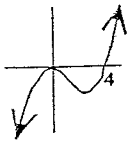
(d)



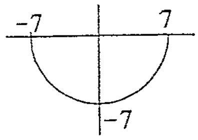
D : $-6 \leq x \leq 6$
R : $-6 \leq y \leq 6$

(e)


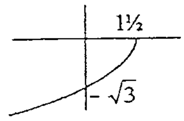
D : all real x
R : $y > 0$

(f)


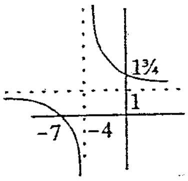
D : all real x
R : all real y

(g)


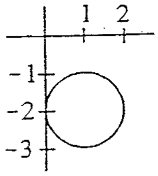
D : $-7 \le x \le 7$
R : $-7 \le y \le 0$

(h)


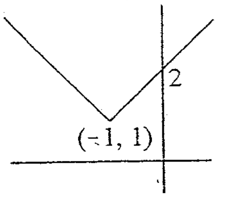
D : $x \le 1\frac{1}{2}$
R : $y \le 0$

(i)


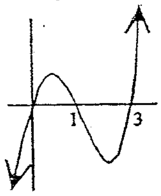
D : $x \ne -4$
R : $y \ne 1$

(j)


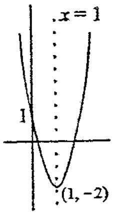
D : $0 \le x \le 2$
R : $-3 \le y \le -1$

(k)


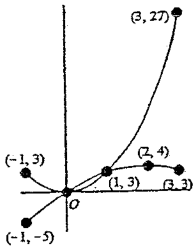
D : all real x
R : $y \ge 1$

(l)


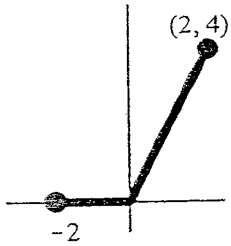
D : all real x
R : all real y

2.


Axis of Sym: $x = 1$
Range: $y \ge -2$

3.


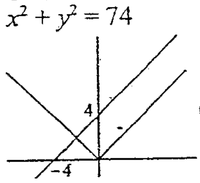
4. (a) -1 (b) a^2

5.


6. $2bx$

7. (a) $x^2 + y^2 = 74$ (b) (0, 0), (0, 8) (c) $y = 4(x+1)^2 - 4$

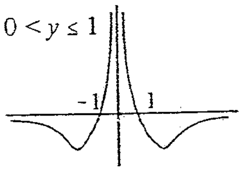
8. $a = \frac{1}{4}$, $c = 4$

9. (i)


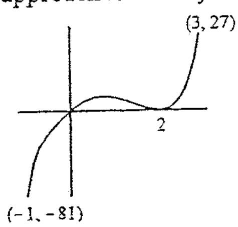
(ii) $(-2, 2)$

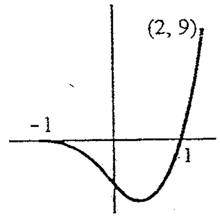
10. $-1 \le x \le 1$

11. $0 < y \le 1$

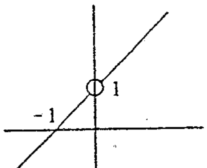
12. (i) $x = \pm 1$ (ii) approaches infinity (iii) approaches 0 from below (iv)


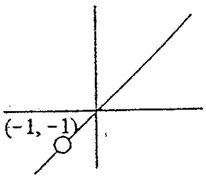
13. $\frac{1}{2}$

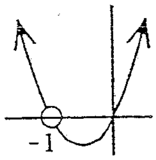
14.


15. (a)


(b) $y \ge -\frac{32}{27}$

16. (a)


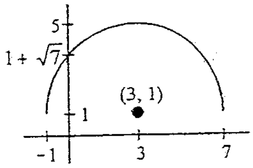
(b)


(c)


17. (a) $x = 0$ (b) $x \ge 0$

18. (a) $f(x) = x + 3$ (b) $f(x) = 3x - 5$ (c) $f(x) = 2x - x^2$ (d) $f(x) = \frac{1}{4}x^2 - x - 1$

20. $0 < f(x) \le \frac{1}{3}$

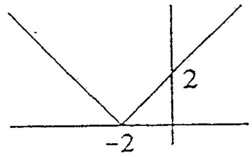
21.


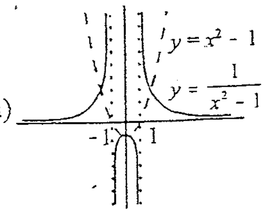
D: $-1 \le x \le 7$, R: $1 \le y \le 5$

22. $2x$

23. $a = -\frac{1}{4}$, $b = 1$, $c = 0$

24. $a = 8$, $b = 11$, $c = 4$

25.


26. (a)


(b) D: $x \ne \pm 1$
R: $y \le -1$, $y > 0$