

# FUNCTIONS

- A function  $y = f(x)$  is a relationship between  $x$  and  $y$  with the property that for each value of the independent variable  $x$  there corresponds exactly one value of the dependent variable  $y$ .
- The domain of a function  $y = f(x)$  is the set of all  $x$ -values for which the function is defined.
- The range of a function  $y = f(x)$  is the set of all  $y$ -values produced by the function.
- The interval  $a \leq x \leq b$  in  $\mathbb{R}$  can be written as  $[a, b]$
- The interval  $a < x < b$  in  $\mathbb{R}$  can be written as  $(a, b)$
- The interval  $a < x \leq b$  in  $\mathbb{R}$  can be written as  $(a, b]$

31. Express each of the following subsets of  $\mathbb{R}$  in interval notation:

a)  $1 < x < 3$       b)  $1 \leq x \leq 3$

c)  $1 < x \leq 3$       d)  $1 \leq x < 3$

e)  $x > 1$       f)  $x \leq 1$

32. For each of the following functions evaluate  $f(0)$ ,  $f(3)$ , and  $f(2a)$ :

a)  $f(x) = 4x - 3$       b)  $f(x) = x^2 + 3x - 5$

c)  $f(x) = \frac{4}{x-3}$       d)  $f(x) = |x^2 - 5x - 2|$

e)  $f(x) = \sqrt{1-x}$       f)  $f(x) = \frac{1}{\sqrt{x}}$

33. By considering the following sketch of  $y = f(x)$  draw a sketch of the graph of:

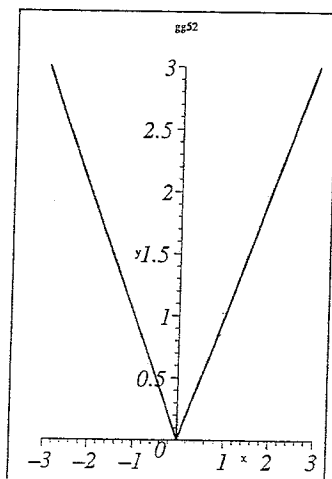
(a)  $y = f(x) + 1$

(b)  $y = f(x) - 1$

(c)  $y = f(x - 1)$

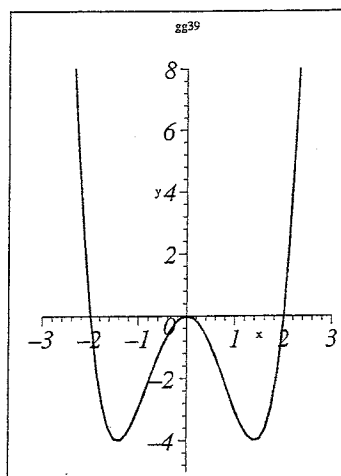
(d)  $y = f(x + 1)$

(e)  $y = -f(x)$



34. By considering the following sketch of  $y = f(x)$  draw a sketch of the graph of:

- (a)  $y = f(x) + 2$
- (b)  $y = f(x) - 2$
- (c)  $y = f(x - 1)$
- (d)  $y = f(x + 1)$
- (e)  $y = -f(x)$
- (f)  $y = f(2x)$
- (g)  $y = 2f(x)$



35. Sketch the following:

a)  $y = x^2$       b)  $y = x^2 - 1$       c)  $y = (x - 1)^2$

36. Sketch the following:

a)  $f(x) = x(x - 2)(x + 3)$       b)  $f(x) = 2(x - \frac{1}{2})(x + 1)(2x + 3)$

c)  $f(x) = x^3 - 4x$       d)  $f(x) = x^3 + 4x^2 + 4x$

e)  $y = 2x^2 - x^3$ .

37. State the domain and range of the following functions:

a)  $f(x) = \frac{1}{x}$       b)  $f(x) = 4 - x^2$

c)  $f(x) = |4 - x|$       d)  $f(x) = -\sqrt{4 - x}$

38. Sketch the following functions and state whether they are odd, even or neither:

a)  $y = 4 - x^2$       b)  $y = x^3 + 1$

c)  $y = |x|$       d)  $y = \begin{cases} -1 & x < 0 \\ 1 & x \geq 0. \end{cases}$

39. (a) (\*) Sketch the function:

$$f(x) = \begin{cases} 1 & 0 \leq x < 1 \\ 0 & 1 \leq x < 2 \\ f(x+2) & \text{for all } x \end{cases}$$

State the period of the function, and find  $f(-1\frac{1}{2})$  and  $f(10)$ .

(b) (\*) Sketch the function:

$$f(x) = \begin{cases} 3x - 2 & 0 \leq x < 1 \\ f(x+1) & \text{for all } x \end{cases}$$

Find  $f(7)$  and  $f(-2\frac{1}{4})$ .

# SOLUTIONS

## Functions

31. a) (1,3)    b) [1,3]    c) (1,3]    d) [1,3)    e) (1,  $\infty$ )    f) ( $-\infty$ , 1]

32. (a) -3, 9,  $8a - 3$ .

(b) -5, 13,  $4a^2 + 6a - 5$ .

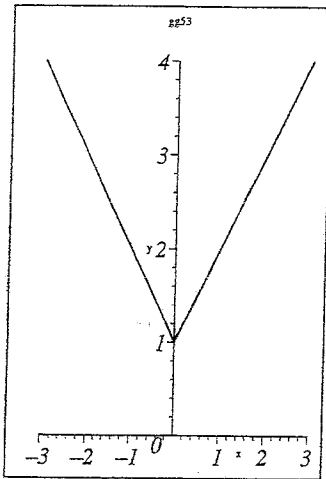
(c)  $-4/3$ , not defined,  $\frac{4}{2a-3}$ .

(d) 2, 8,  $|4a^2 - 10a - 2|$

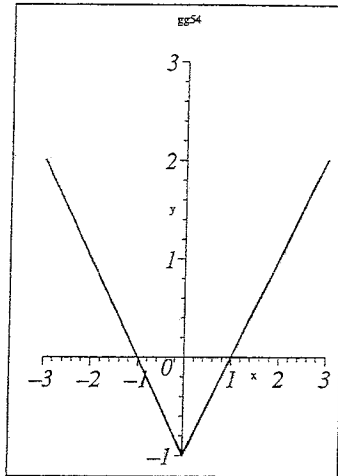
(e) 1, not defined,  $\sqrt{1-2a}$ .

(f) not defined,  $\frac{1}{\sqrt{3}}$ ,  $\frac{1}{\sqrt{2a}}$ .

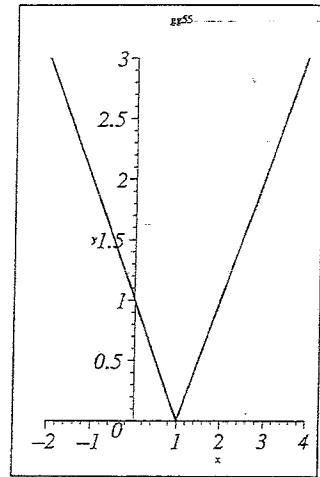
33. a)



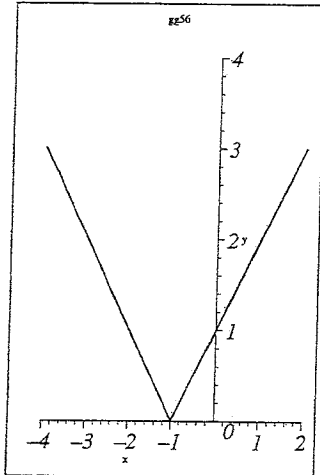
b)



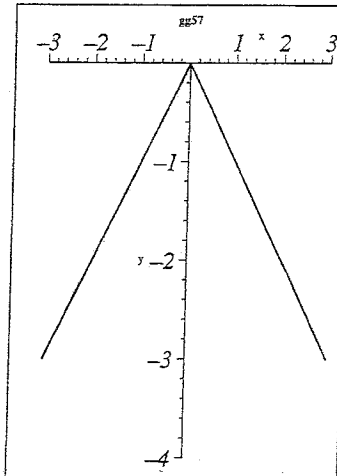
c)



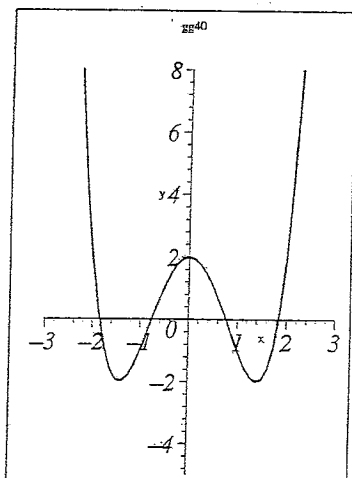
d)



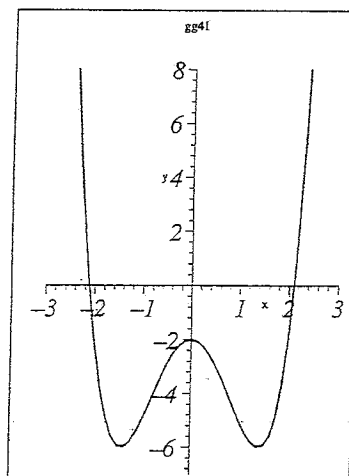
e)



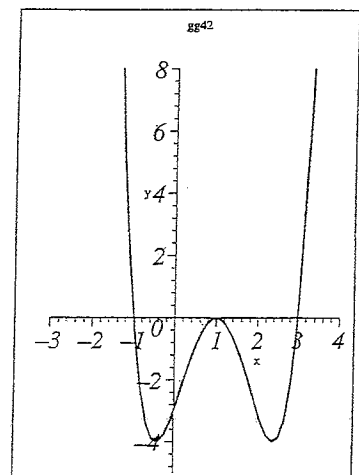
34. a)



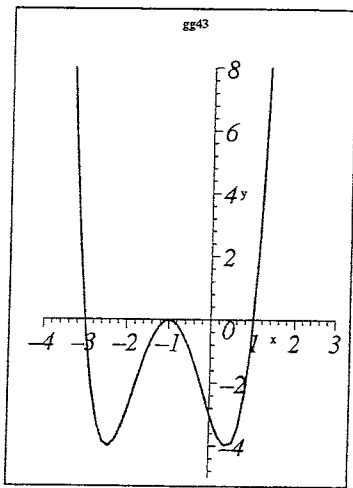
b)



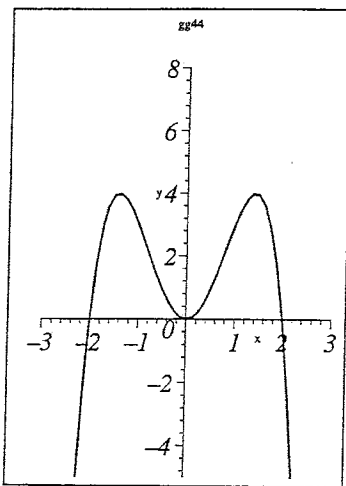
c)



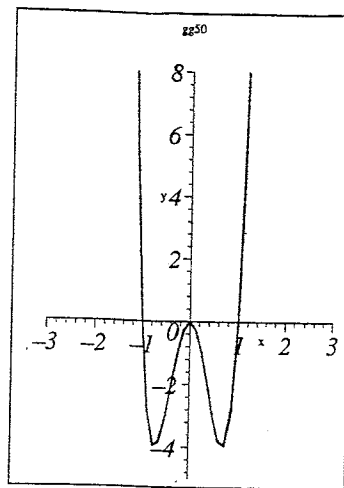
d)



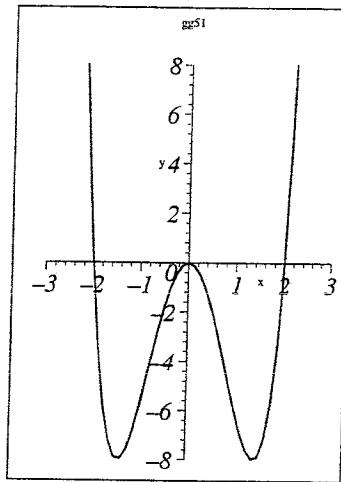
e)



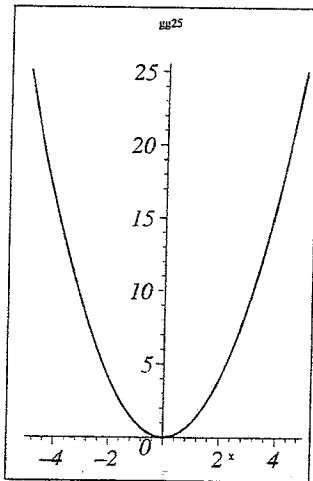
f)



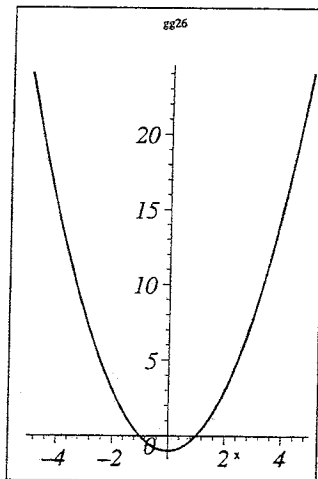
g)



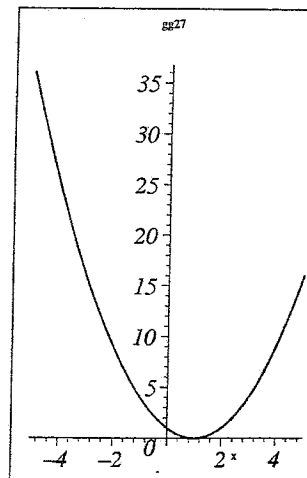
35. a)



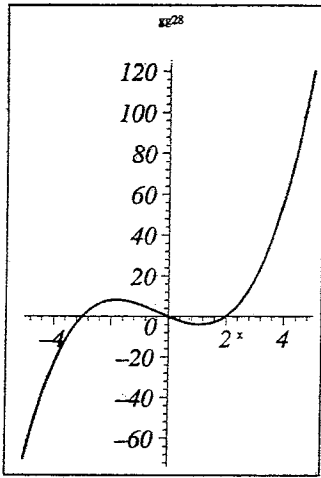
b)



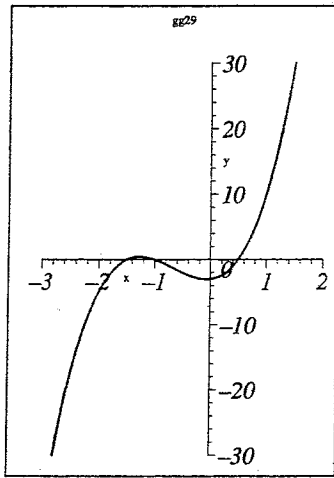
c)



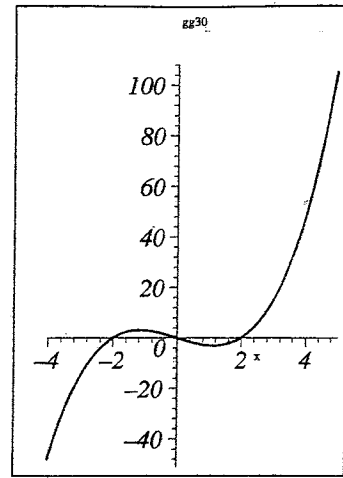
36. a)



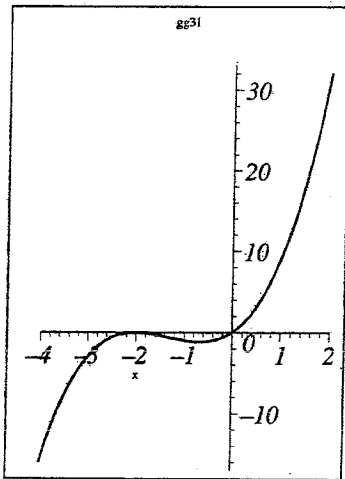
b)



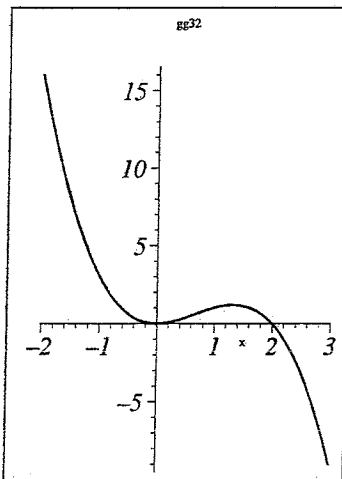
c)



d)



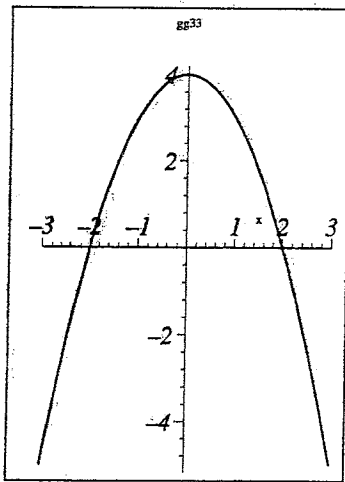
e)



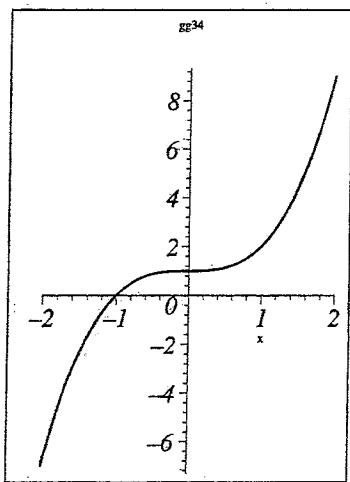
37. (a)  $x \neq 0, y \neq 0$ .  
 (b)  $x \in \mathbb{R}, y \leq 4$ .  
 (c)  $x \in \mathbb{R}, y \geq 0$ .  
 (d)  $x \leq 4, y \leq 0$ .

38. (a) even (b) neither (c) even (d) neither...f(0)???

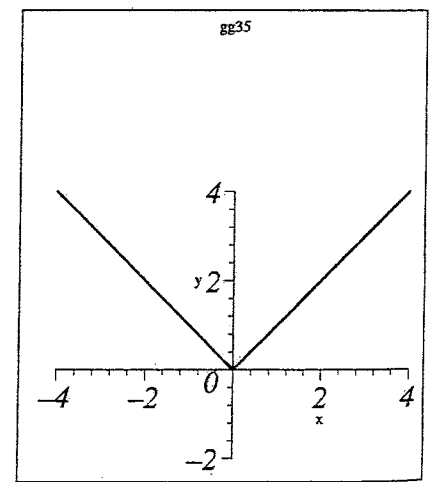
a)



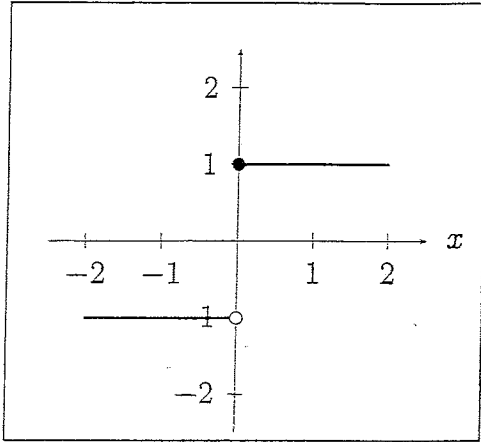
b)



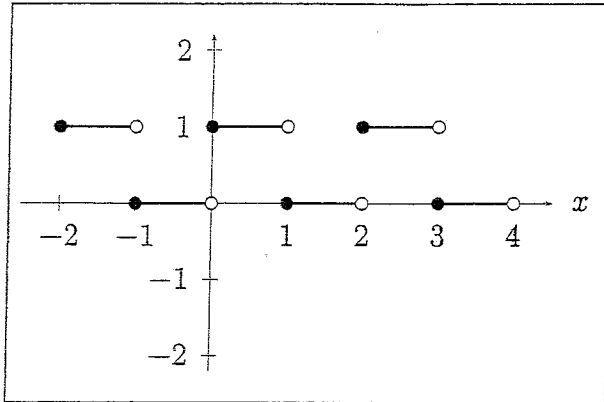
c)



d)

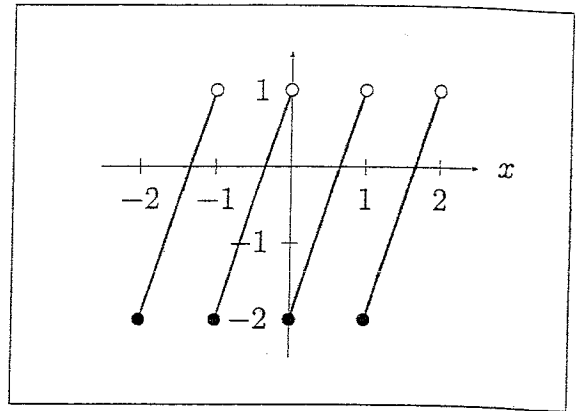


39. a)



2,  $f(-1\frac{1}{2}) = 1$ ,  $f(10) = 1$ .

b)



$f(7) = -2$ ,  $f(-2\frac{1}{4}) = \frac{1}{4}$ .