Year 11 Extension I assessment task 2 25/3/'04

Show all necessary working. Set out your steps clearly Hand in two bundles -Section A and Section B Attach the question paper to Section A	Examiners' Use Section A /20 Section B /21 Total /41

Name: Teacher:

Section A

Q.1) (a) Show that

(i) $\dfrac{1+\tan^2\alpha}{1+\cot^2\alpha} = \tan^2\alpha$ (3m)

(ii) $\dfrac{2\cos^2\theta-1}{\cos^2\theta-\sin^2\theta} = 1$ (2m)

(iii) Simplify $\dfrac{1+\cot\theta}{\operatorname{cosec}\theta}-\dfrac{\sec\theta}{\tan\theta+\cot\theta}$ (4m)

Q. 2) Solve for x :

(a) $\tan(x-45^o)=\dfrac{1}{\sqrt{3}}$ where $-180^o < x \le 180^o$ (3m)

(b) $2\cos^2 x+\sin x-2=0$ where $0^o \le x \le 360^o$ (4m)

Q.3) (i) On the same set of axes sketch the graphs of y-sin x and y= cosec x, $0^o \le x \le 360^o$ (3m)

(ii) Infer from the graph, the points of intersection in the stated domain. (1m)

(Please turn over for Section B)

Section B

Q.4) a) Find the perpendicular distance between the point (2,3) and the line
2x-3y+7=0 (2m)
(b) Given that the perpendicular distance between the point (1,-3) and the line
3x-+4y- p=0 is 4, find the possible values of p. (3m)
3x + 4y

Q.5) The line 2x+y-5=0 and x-y+2 = 0 intersect at a point P.
(i) Show that the gradient of any line passing through P is given by $\frac{2+k}{k-1}$, (2m)
(ii) Hence find the equation of the line that passes through P and is parallel to the y axis. (2m)

Q.6) (a) Find the acute angle between the lines 2x+y-5=0 and x-y+2 = 0 (3m)
(b) Find the acute angle between the lines x = 3 and $y=\sqrt{3}x+1$ (2m)

Q.7) Shade the region determined by the following inequalities
$y \le x^2,\ 2x-3y+6 \ge 0$ (3m)

Q.8) (a) A (1,3) and B(6,-2) are two points.
Find the coordinates of the point that divides AB internally in the ratio 1:4 (2m)
(b) Point B(3,5) divides the interval joining point A and point C (6,8) internally in the ratio 4:3. Find the coordinates of the point A. (2m)

2x-3y+6≥0
2x-3y+6=0
3y = 2x ~~+6~~
y = $\frac{2}{3}$x +2

Yr 11 Extension 1 Assessment Task 2

Section A

i) $LHS = \dfrac{1+\tan^2\alpha}{1+\cot^2\alpha}$

$= \dfrac{\sec^2\alpha}{\operatorname{cosec}^2\alpha}$

$= \dfrac{1}{\cos^2\alpha} \div \dfrac{1}{\sin^2\alpha}$

$= \dfrac{1}{\cos^2\alpha} \times \sin^2\alpha$ ✓ 3/3

$= \tan^2\alpha = RHS$

ii) $LHS = \dfrac{2\cos^2\theta - 1}{\cos^2\theta - \sin^2\theta}$

$= \dfrac{2\cos^2\theta - 1}{\cos^2\theta - (1-\cos^2\theta)}$

$= \dfrac{2\cos^2\theta - 1}{\cos^2\theta - 1 + \cos^2\theta}$ ✓ 2/2

$= \dfrac{2\cos^2\theta - 1}{2\cos^2\theta - 1}$

$= 1 = RHS$

iii) $\dfrac{1+\cot\theta}{\operatorname{cosec}\theta} - \dfrac{\sec\theta}{\tan\theta + \cot\theta}$

$\left[\left(1+\dfrac{\cos\theta}{\sin\theta}\right) \div \dfrac{1}{\sin\theta}\right] - \left[\dfrac{1}{\cos\theta} \div \left(\dfrac{\sin\theta}{\cos\theta} + \dfrac{\cos\theta}{\sin\theta}\right)\right]$

$= \left(\dfrac{\sin\theta + \cos\theta}{\sin\theta} \times \sin\theta\right) - \left(\dfrac{1}{\cos\theta} \times \dfrac{\sin\theta\cos\theta}{\sin^2\theta + \cos^2\theta}\right)$

$= (\sin\theta + \cos\theta) - \left(\dfrac{1}{\cos\theta} \times \sin\theta\cos\theta\right)$

$= \sin\theta + \cos\theta - \sin\theta$

$= \cos\theta$ ✓ Great 4/4

PTO →

$\tan -\theta = -\tan\theta$

2a) $\tan(x-45) = \frac{1}{\sqrt{3}}$ $\qquad -180° \le x \le 180°$

$-225° \le x - 45° \le 135°$

$x - 45° = 30°, -150°$

$x = 75°, -105°$

3/3

b) $2\cos^2 x + \sin x - 2 = 0 \qquad 0 \le x \le 360$

$2(1-\sin^2 x) + \sin x - 2 = 0$

$2 - 2\sin^2 x + \sin x - 2 = 0$

$-2\sin^2 x + \sin x = 0$

$\sin x(-2\sin x + 1) = 0$

$\sin x = 0$ or $\frac{1}{2}$

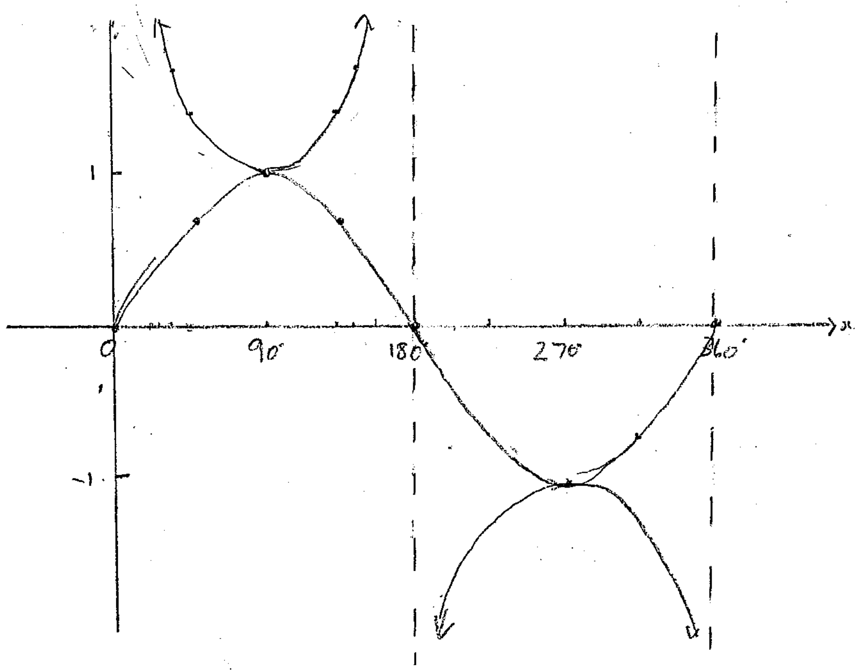
If $\sin x = 0$ $\qquad$ If $\sin x = \frac{1}{2}$

$x = 0°, 180°, 360° \qquad x = 30°, 150°$

$\therefore \quad x = 0°, 30°, 150°, 180°, 360°$

4/4

Q 3.i)



4/4

Perfect

ii) points of intersection - ~~x=90~~ $(90°, 1)$ and $(270°, -1)$

Section B

4a) $d = \dfrac{|2\times2 - 3\times3 + 7|}{\sqrt{2^2+(-3)^2}}$ $\qquad d = \dfrac{|ax_1+by_1+c|}{\sqrt{a^2+b^2}}$

$= \dfrac{2}{\sqrt{13}} = \dfrac{2\sqrt{13}}{13}$ units

b) $4 = \dfrac{|3\times1 + 4\times-3 - p|}{\sqrt{3^2+4^2}}$

$4 = \dfrac{|-9-p|}{5}$

$\dfrac{-9-p}{5} = 4$ or $\dfrac{-(-9-p)}{5} = 4$

$-9-p = 20$ $\qquad \dfrac{9+p}{5} = 4$

$-p = 29$

$p = -29$ $\qquad 9+p = 20$

$p = 11$

$p = -29$ or 11

5 i) Any line that passes through the point of intersection of the lines $2x+y-5=0$ and $x-y+2=0$ is of the form $2x+y-5+k(x-y+2)=0$

$2x+y-5+kx-ky+2k=0$

$x(2+k)+y(1-k)-5+2k=0$

$m = -\dfrac{a}{b} = -\dfrac{2+k}{1-k} = \dfrac{2+k}{k-1}$

$\therefore$ grad of $P = \dfrac{2+k}{k-1}$

ii) If line is parallel to y axis, the the gradient $=\infty$

ie, $m = \dfrac{1}{0}$

$\dfrac{2+k}{k-1} = \dfrac{1}{0}$

$k-1 = 0$

$k = 1$

PTO →

6a) $2x+y-5=0$ $\qquad$ $x-y+2=0$

$m_1 = -\frac{2}{1} = -2$ $\qquad$ $m_2 = -\frac{1}{-1} = 1$

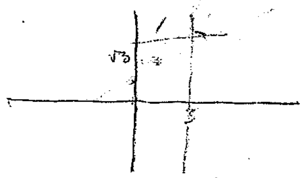
$$\tan\theta = \left|\frac{-2-1}{1+(-2\times 1)}\right|$$

$$= \left|\frac{-3}{-1}\right|$$

$$\tan\theta = 3$$

$$\theta = 71°34' \text{ (nearest min)}$$

b) $x=3$ $\qquad$ $y=\sqrt{3}x+1$



$m_1 = \infty = \frac{1}{0}$ $\qquad$ $m_2 = \sqrt{3}$

Because $x=3$ is parallel to the y-axis, it means that $\tan\theta = \frac{1}{\sqrt{3}}$

$$\theta = 30°$$

where is θ ?

7.

$2x-3y+6=0$

x-int $\quad 2x-0+6=0$

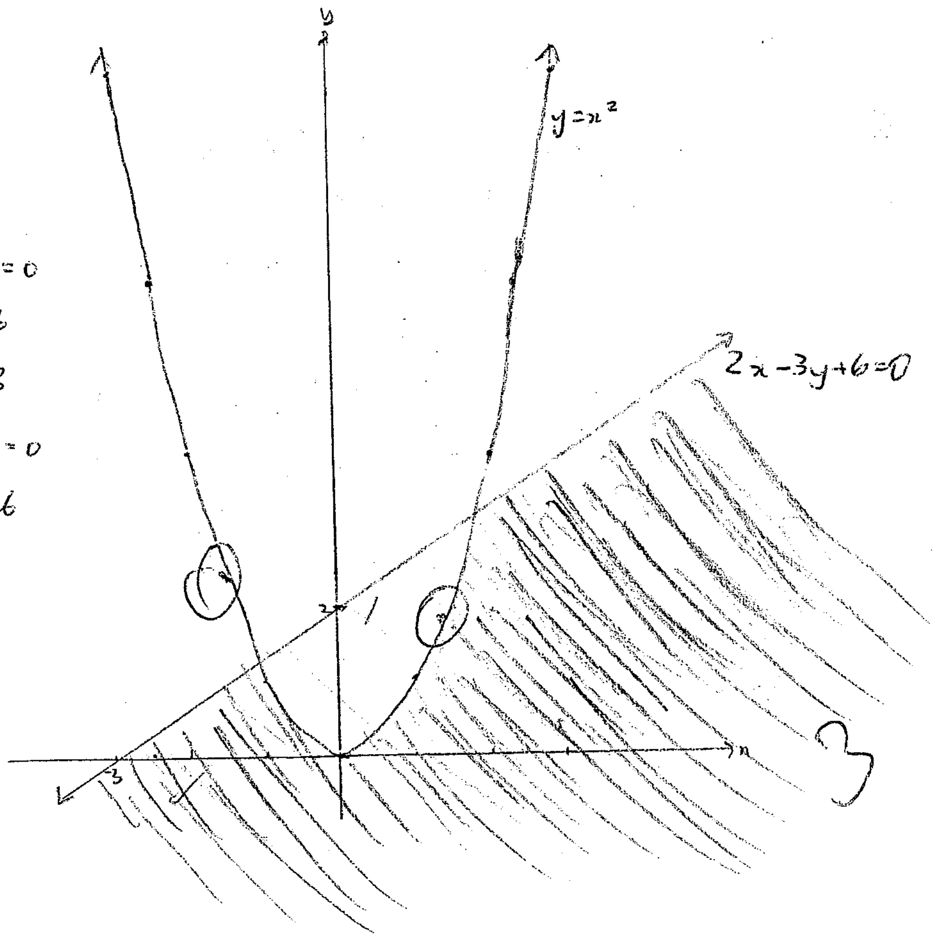
$2x=-6$

$x=-3$

y-int $\quad 0-3y+6=0$

$-3y=-6$

$y=2$



Section B cont'd ...

8a) $x = \dfrac{1 \times 6 + 4 \times 1}{1+4} = \dfrac{10}{5} = 2$

$y = \dfrac{1 \times -2 + 4 \times 3}{1+4} = \dfrac{10}{5} = 2$

$P = (2, 2)$

b) $A = (a, b) \quad B = (3, 5) \quad C = (6, 8)$

A —— B —— C

(a, b) (3, 5) (6, 8)

4:3

4:3

$\cancel{b = \dfrac{4 \times 6 + 3 \times a}{4+3} = \dfrac{24 + 3a}{7}}$

$3 = \dfrac{4 \times 6 + 3 \times a}{4+3} = \dfrac{24+3a}{7}$

$\dfrac{24+3a}{7} = 3$

$24 + 3a = 21$

$3a = -3$

$a = -1$

$5 = \dfrac{4 \times 8 + 3 \times b}{4+3} = \dfrac{32+3b}{7}$

$\dfrac{32+3b}{7} = 5$

$32 + 3b = 35$

$3b = 3$

$b = 1$

$A: (-1, 1)$