TEST 6 Angle Properties of Polygons

Marks: /80

Time: 1 hour 30 minutes

Name: .... Date: ....

INSTRUCTIONS TO CANDIDATES

Section A (40 marks) **Time: 45 minutes**

1. Answer all the questions in this section.
2. Calculators may **not** be used in this section.
3. All working must be clearly shown. Omission of essential working will result in loss of marks.
4. The marks for each question is shown in brackets [ ] at the end of each question.

---

1 Calculate the size of an interior angle of a regular polygon with 18 sides.

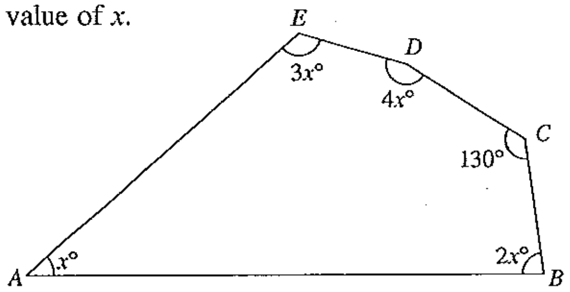
Answer .... ° [2]

---

2 Each of the exterior angles of a polygon is 70°, except for one which is 80°. How many sides does the polygon have?

Answer .... sides [3]

3 In the diagram, *ABCDE* is a pentagon. Find the value of x.



Answer $x =$ .... [3]

---

4 **(a)** Each interior angle of a regular polygon with n sides is 135°. Calculate the value of n.
(b) The exterior angles of a hexagon are $2x°$, $2x° + 10°$, $3x°$, $4x° + 20°$, $4x°$ and $5x°$. Find the value of x.

Answer (a) $n =$ .... [2]

(b) $x =$ .... [3]

---

5 Each interior angle of a regular polygon is 140° greater than each exterior angle. How many sides does the polygon have?

Answer .... sides [3]

6 The interior angles of a pentagon are in the ratio 2 : 3 : 4 : 5 : 6. Calculate the largest interior angle of the pentagon.

Answer .... ° [3]

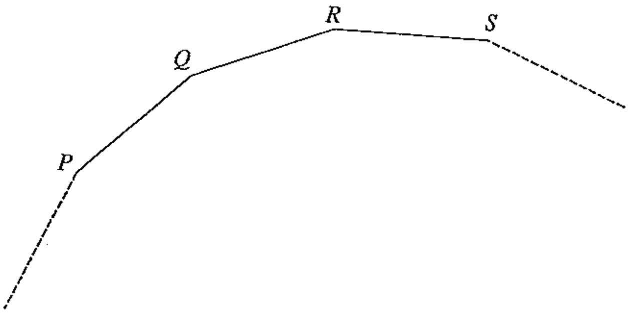
---

7 The diagram shows part of a regular polygon *PQRS* with 20 sides. Calculate

(a) $P\hat{Q}R$,

(b) $P\hat{R}S$,

(c) $P\hat{S}R$.



Answer (a) $P\hat{Q}R =$ .... ° [2]

(b) $P\hat{R}S =$ .... ° [2]

(c) $P\hat{S}R =$ .... ° [2]

8 The size of each interior angle of a regular polygon is 11 times the size of each exterior angle.

(a) Find the size of each exterior angle.

(b) How many sides does the polygon have?

Answer (a) .... ° [2]

(b) .... sides [2]

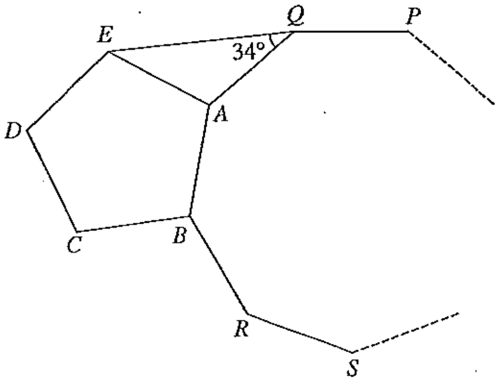
---

9 The diagram shows a regular pentagon $ABCDE$ and part of a regular polygon $PQABRS$ with n sides which are drawn on opposite sides of the common line AB. Given that $A\hat{Q}E = 34°$, find

(a) $E\hat{A}Q$,

(b) $B\hat{A}Q$,

(c) the value of n.



Answer (a) $E\hat{A}Q =$ .... ° [2]

(b) $B\hat{A}Q =$ .... ° [2]

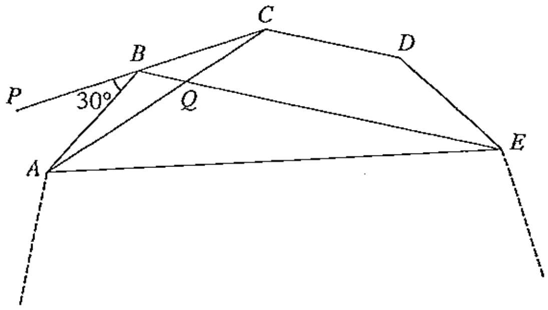
(c) $n =$ .... [2]

10 In the diagram, *ABCDE* is part of a regular polygon. *PBC* is a straight line with $P\hat{B}A = 30°$. Find

(a) $A\hat{C}B$,

(b) $B\hat{E}D$,

(c) $A\hat{Q}E$.



Answer (a) $A\hat{C}B$ = ... ° [2]

(b) $B\hat{E}D$ = ... ° [1]

(c) $A\hat{Q}E$ = ... ° [2]

INS

Sect

1.
2.
3.
4.

11

INSTRUCTIONS TO CANDIDATES

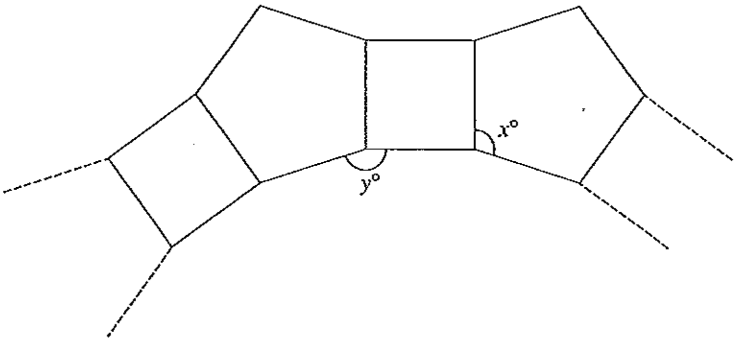
Section B (40 marks) **Time: 45 minutes**

1. Answer **all** the questions in this section.
2. Calculators may be used in this section.
3. All working must be clearly shown. Omission of essential working will result in loss of marks.
4. The marks for each question is shown in brackets [ ] at the end of each question.

---

11 **(a)** Each interior angle of a regular polygon of n sides is $\frac{3}{4}$ of each interior angle of a regular polygon of $2n$ sides. Find the value of n.

(b) The diagram shows part of a closed ring formed by alternating squares and regular pentagons.



Calculate

(i) the values of x and y,

(ii) the number of sides of the regular polygon formed by the closed ring.

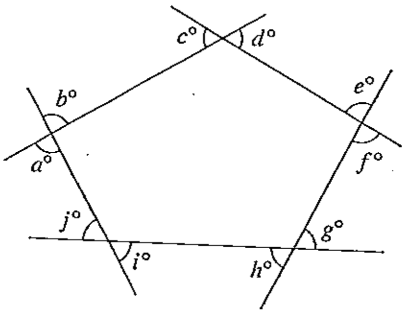
Answer (a) $n =$ .... [3]

(b) (i) $x =$ .... [2]

$y =$ .... [2]

(ii) .... sides [2]

12 **(a)** Find the sum of the angles $a°$, $b°$, $c°$, $d°$, $e°$, $f°$, $g°$, $h°$, $i°$ and $j°$.

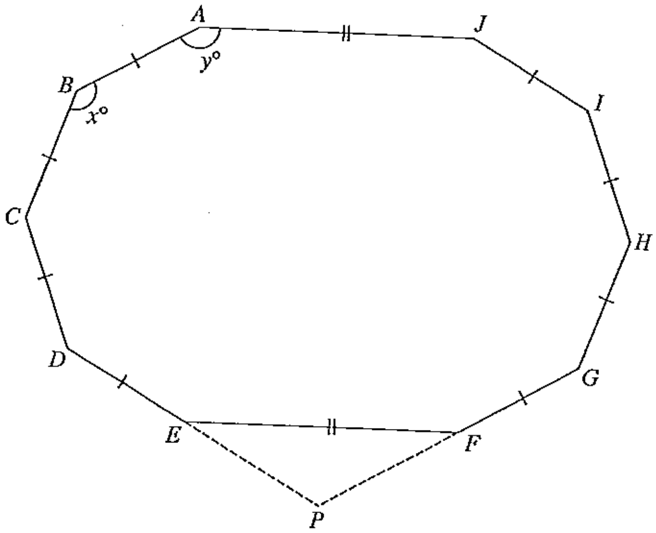


(b) The diagram below shows a decagon. $AJ = EF$ and $AB = BC = CD = DE = FG = GH = HI = IJ$. $A\hat{B}C = B\hat{C}D = C\hat{D}E = F\hat{G}H = G\hat{H}I = H\hat{I}J = x°$ and $B\hat{A}J = A\hat{J}I = D\hat{E}F = E\hat{F}G = y°$.

(i) Find an equation connecting x and y.

(ii) Calculate the value of y when $x = 140$.

(iii) DE produced meets GF produced at P. Find $E\hat{P}F$.



Answer (a) .... ° [2]

(b) (i) .... [2]

(ii) $y =$ .... [2]

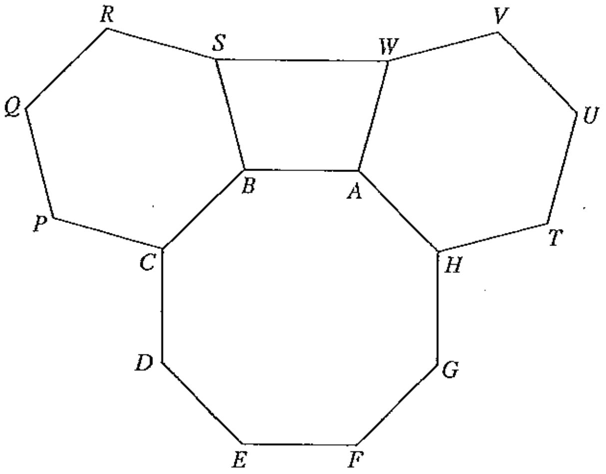
(iii) $E\hat{P}F =$ .... ° [2]

13 In the diagram, $ABCDEFGH$ is a regular octagon. $PQRSBC$ and $TUVWAH$ are regular hexagons.

(a) Find

- (i) reflex $A\hat{B}S$,
- (ii) obtuse $R\hat{B}H$,
- (iii) acute $B\hat{S}W$.

(b) Write down the special name given to quadrilateral $ABSW$.



Answer *(a)* (i) $A\hat{B}S =$ .... ° [2]

(ii) $R\hat{B}H =$ .... ° [3]

(iii) $B\hat{S}W =$ .... ° [2]

(b) .... [1]

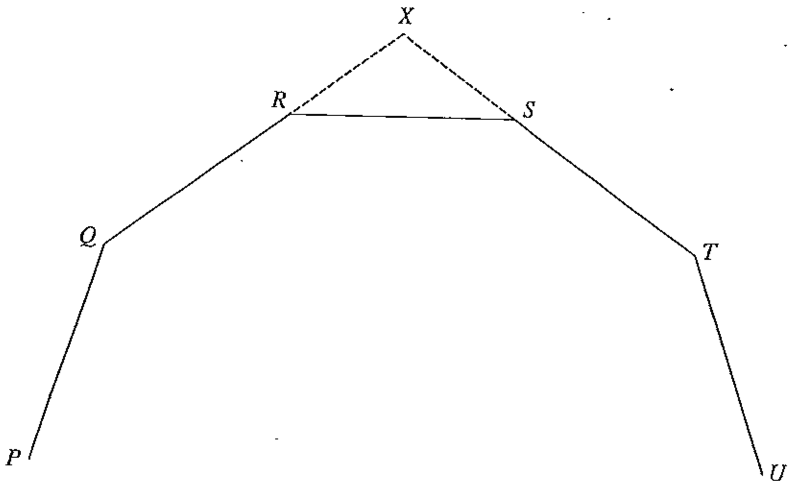
14 *PQRSTU* is part of a regular polygon with 10 sides. *QR* and *TS* produced meet at *X*. Calculate

(a) $X\hat{R}S$,

(b) $R\hat{S}Q$,

(c) $Q\hat{S}T$,

(d) $P\hat{Q}T$.



Answer (a) $X\hat{R}S$ = ... ° [1]

(b) $R\hat{S}Q$ = ... ° [2]

(c) $Q\hat{S}T$ = ... ° [2]

(d) $P\hat{Q}T$ = ... ° [2]

15 M

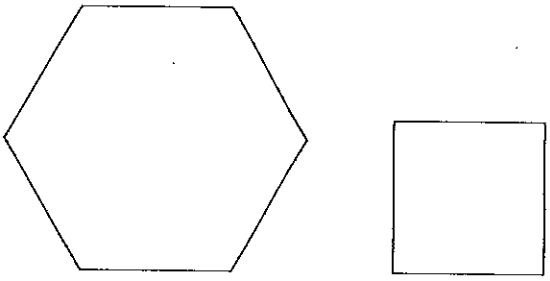
fit

(a

(b

(c

15 Mr Lee has some regular hexagonal tiles and some square tiles, each of side 20 cm. He wants to fit the tiles together to completely cover the floor of his living room.



(a) Find the size of each interior angle of the hexagonal tile.
(b) Explain why Mr Lee cannot fit a combination of square and hexagonal tiles together to cover the floor of his living room.
(c) Mr Lee needs another regular-shaped tile to fit one square and one hexagonal tile.
  (i) Find the size of each angle of this tile.
  (ii) How many sides would this tile have?

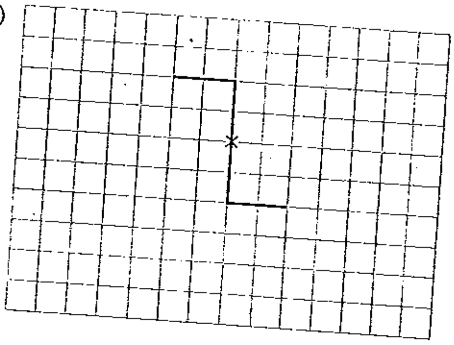
Answer (a) .... ° [2]

(b) .... [2]

(c) (i) .... ° [2]

(ii) .... sides [2]

(b)



14. (a) (i)

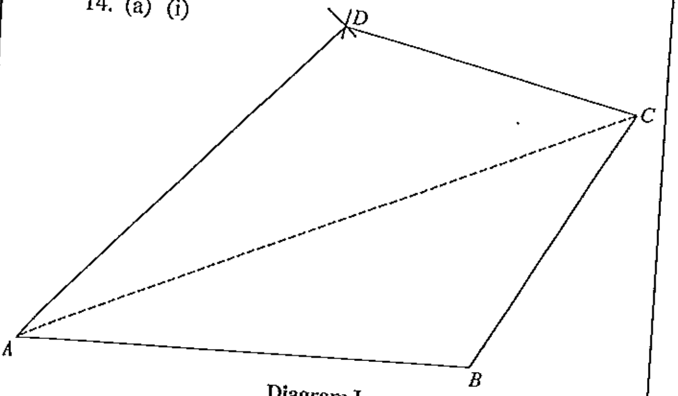


Diagram I

(ii) *ABCD* is a **kite.**

(b) (i)

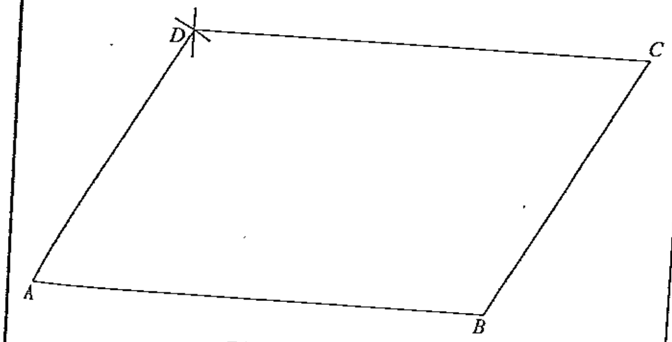


Diagram II

(ii) *ABCD* is a **parallelogram.**

15.

Solid	No. of plane symmetry	No. of axis (axes) of rotational symmetry
A	4	1
B	4	4
C	Infinite	1

Test 6: Angle Properties of Polygons

Section A

1.

Teacher's Tip

1. A regular polygon has all sides of the same length and all angles equal.
2. For a regular *n*-sided polygon,

each interior angle = $\frac{(n-2)\times 180^\circ}{n}$.

Each interior angle

$= \frac{(18-2)\times\cancel{180^\circ}^{10^\circ}}{\cancel{18}_1}$ $n = 18$

$= 16 \times 10^\circ$

$= \mathbf{160^\circ}$

Alternative method:

Each exterior angle

$= \frac{360^\circ}{18}$ Sum of the exterior angles of a polygon is 360°.

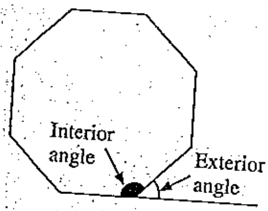
$= 20^\circ$

$\therefore$ each interior angle

$= 180^\circ - 20^\circ$ supplementary angles

$= \mathbf{160^\circ}$

Teacher's Tip



Notice that the two angles form a straight line.

$\therefore$ interior angle + exterior angle = 180° (supplementary angle).

2. Let *n* be the number of sides of the polygon.

$(n-1)\ 70^\circ + 80^\circ = 360^\circ$

$70n - 70 + 80 = 360$ Sum of the exterior angles of a polygon is 360°.

$70n = 350$

$n = \frac{350}{70}$

$= 5$

$\therefore$ the polygon has **5 sides.**

3. Sum of the interior angles of a pentagon

$= (5-2)\times 180^\circ$

$= 3\times 180^\circ$

$= 540^\circ$

For a n-sided polygon, sum of interior angles $= (n-2)\times 180^\circ$. Here, $n = 5$.

$$\therefore x^\circ + 2x^\circ + 130^\circ + 4x^\circ + 3x^\circ = 540^\circ$$

$$10x^\circ = 410^\circ$$

$$x^\circ = \frac{410^\circ}{10} = 41^\circ$$

$$\therefore x = \mathbf{41}$$

Teacher's Tip

The table below shows how the polygons are classified according to the number of sides they have.

No. of sides	Name of polygon
3	Triangle
4	Quadrilateral
5	Pentagon
6	Hexagon
7	Heptagon
8	Octagon
9	Nonagon
10	Decagon
n	n-gon

4. (a) $\frac{(n-2)\times 180^\circ}{n} = 135^\circ$

For a regular n-sided polygon, each interior angle $= \frac{(n-2)\times 180^\circ}{n}$.

$$(n-2)180 = 135n$$

$$180n - 360 = 135n$$

$$45n = 360$$

$$n = \frac{360}{45} = \mathbf{8}$$

Alternative method:

Each exterior angle $= 180^\circ - 135^\circ$

$= 45^\circ$

$$\therefore \frac{360^\circ}{n} = 45^\circ$$

$$n = \frac{360^\circ}{45^\circ}$$

$$= \mathbf{8}$$

For a regular n-sided polygon, each exterior angle $= \frac{360^\circ}{n}$.

(b) $2x^\circ + (2x^\circ + 10^\circ) + 3x^\circ + (4x^\circ + 20^\circ) + 4x^\circ + 5x^\circ = 360^\circ$

Sum of the exterior angles of a polygon is 360°.

$$20x^\circ + 30^\circ = 360^\circ$$

$$20x^\circ = 330^\circ$$

$$x^\circ = \frac{330^\circ}{20^\circ}$$

$$= 16.5^\circ$$

$$\therefore x = \mathbf{16.5}$$

5. Let each exterior angle be x°. Then each interior angle is $(x + 140)^\circ$.

$$x^\circ + (x + 140)^\circ = 180^\circ$$

$$2x^\circ = 40^\circ$$

$$x^\circ = \frac{40^\circ}{2}$$

$$= 20^\circ$$

The sum of the interior angle and exterior angle of a polygon is 180°.

Let n be the no. of sides of the regular polygon.

$$\therefore \frac{360^\circ}{n} = 20^\circ$$

$$n = \frac{360^\circ}{20^\circ}$$

$$= 18$$

Each exterior angle of a regular n-sided polygon $= \frac{360^\circ}{n}$.

$\therefore$ the polygon has **18 sides.**

6. Sum of angles of a pentagon

$= (5-2)\times 180^\circ$

$= 3\times 180^\circ$

$= 540^\circ$

Use $(n-2)\times 180^\circ$ where $n = 5$.

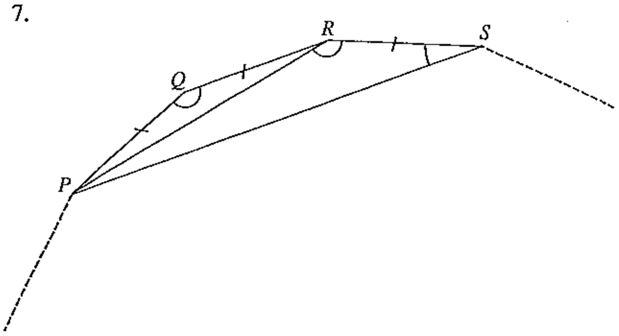
Largest interior angle

$$= \frac{6}{2+3+4+5+6}\times 540^\circ$$

$$= \frac{6}{20}\times 540^\circ$$

$$= \mathbf{162^\circ}$$

7.



(a) $P\hat{Q}R = \frac{(20-2)\times \overset{9^\circ}{\cancel{180^\circ}}}{\cancel{20}_1}$

$= 18\times 9^\circ$

$= \mathbf{162^\circ}$

Each interior angle $= \frac{(n-2)\times 180^\circ}{n}$.

(b) $P\hat{R}Q = \frac{180^\circ - 162^\circ}{2}$ base ∠s of isos. △

$= \frac{18^\circ}{2}$

$= 9^\circ$

$\therefore P\hat{R}S = 162^\circ - 9^\circ$

$= \mathbf{153^\circ}$

(c) $R\hat{P}S = P\hat{R}Q$ (alt. ∠s, $QR \parallel PS$)

$= 9^\circ$

$\therefore P\hat{S}R = 180^\circ - 9^\circ - 153^\circ$

$= \mathbf{18^\circ}$

8. (a) Let each exterior angle = x°,
then each interior angle = $11x^\circ$.

$\therefore x^\circ + 11x^\circ = 180^\circ$

$12x^\circ = 180^\circ$

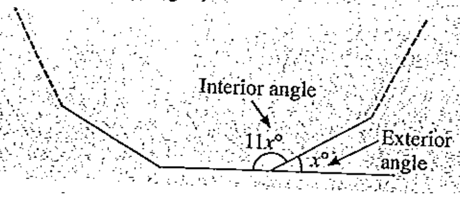
$x^\circ = \frac{180^\circ}{12}$

$= 15^\circ$

$\therefore$ the size of each exterior angle is **15°**.

Teacher's Tip

Use exterior angle + interior angle = 180° (supplementary angles).



(b) Let n be the number of sides of the polygon.

$\therefore \frac{360^\circ}{n} = 15^\circ$ Each exterior angle = $\frac{360^\circ}{n}$

$n = \frac{360^\circ}{15^\circ}$

$= 24$

$\therefore$ the polygon has **24 sides**.

(c) Each exterior angle of the polygon with n sides

$= 180^\circ - 140^\circ$

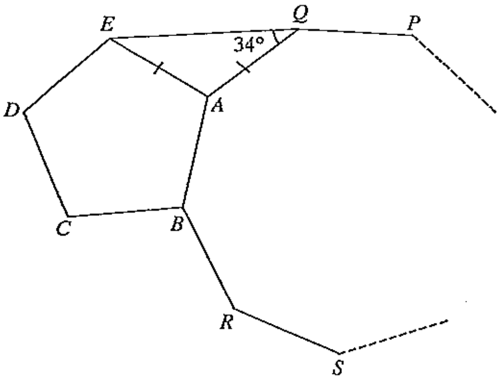
$= 40^\circ$

$\therefore \frac{360^\circ}{n} = 40^\circ$ Each exterior angle = $\frac{360^\circ}{n}$

$n = \frac{360^\circ}{40^\circ}$

$= 9$

9.



(a) **Teacher's Tip**

$\triangle AEQ$ is isosceles since $AE = AQ$.
$AE = AB$ (regular pentagon)
$AQ = AB$ (regular polygon)

$A\hat{E}Q = A\hat{Q}E = 34^\circ$ $\triangle AEQ$ is isosceles.

$E\hat{A}Q = 180^\circ - 34^\circ - 34^\circ$

$= \mathbf{112^\circ}$

(b) $E\hat{A}B = \frac{(5-2)\times \overset{36^\circ}{\cancel{180^\circ}}}{\cancel{5}_1}$ Use $\frac{(n-2)\times 180^\circ}{n}$, where $n = 5$. A pentagon has 5 sides.

$= 3 \times 36^\circ$

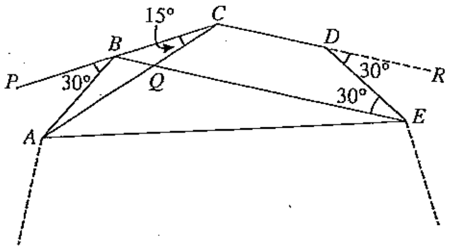
$= 108^\circ$

$B\hat{A}Q = 360^\circ - E\hat{A}B - E\hat{A}Q$ Sum of ∠s at a point is 360°.

$= 360^\circ - 108^\circ - 112^\circ$

$= \mathbf{140^\circ}$

10.



(a) $A\hat{C}B + B\hat{A}C = 30^\circ$ ext. ∠ = sum of int. opp. ∠s

$A\hat{C}B = \frac{30^\circ}{2}$ $A\hat{C}B = B\hat{A}C$, base ∠s of isos. △

$= \mathbf{15^\circ}$

Alternative method:

$A\hat{B}C = 180^\circ - 30^\circ$ supplementary angles

$= 150^\circ$

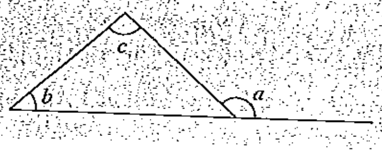
$A\hat{C}B = \frac{180^\circ - 150^\circ}{2}$ base ∠s of isos. △

$= \mathbf{15^\circ}$

Teacher's Tip

The exterior angle of a triangle is equal to the sum of the interior opposite angles.

$a = b + c$ (ext. ∠ = sum of int. opp. ∠s)



(b) $B\hat{E}D = E\hat{D}R$ alt. ∠s, $CD \parallel BE$

$= \mathbf{30^\circ}$ Ext. ∠s of a regular polygon are equal. $E\hat{D}R = P\hat{B}A = 30^\circ$

(c) $B\hat{Q}C = 180^\circ - Q\hat{B}C - B\hat{C}Q$

$= 180^\circ - B\hat{E}D - B\hat{C}Q$

$= 180^\circ - 30^\circ - 15^\circ$

$= 135^\circ$

$A\hat{Q}E = B\hat{Q}C$ vert. opp. ∠s

$= \mathbf{135^\circ}$

Section B

11. (a) $\frac{(n-2)180^\circ}{n} = \frac{3}{4}\left[\frac{(2n-2)180^\circ}{2n}\right]$

$8n(n-2) = 3n(2n-2)$ Cross multiply.

$8n^2 - 16n = 6n^2 - 6n$

$2n^2 - 10n = 0$

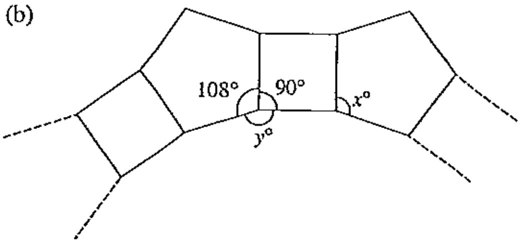
$2n(n-5) = 0$

$n = 0$ or $n = 5$

(rejected)

$\therefore$ **$n = 5$**

(b)



(i) $x^\circ = \frac{(5-2)\times 180^\circ}{5}$

$= 108^\circ$

$\therefore$ **$x = 108$**

$y^\circ = 360^\circ - 90^\circ - 108^\circ$ ∠s at a pt.

$= 162^\circ$

$\therefore$ **$y = 162$**

(ii) Each exterior angle of regular polygon formed

$= 180^\circ - 162^\circ$

$= 18^\circ$

No. of sides of the regular polygon formed

$= \frac{360^\circ}{18^\circ}$

$= 20$

12. (a) Sum of the angles $= 2 \times 360^\circ$

$= 720^\circ$

Teacher's Tip

Each of the 10 marked angles is an exterior angle of the pentagon. The sum of the exterior angles of a pentagon is 360° i.e. 5 exterior angles.

(b) (i) Sum of interior angles of a decagon

$= (10-2) \times 180^\circ = 1440^\circ$

$6x^\circ + 4y^\circ = 1440^\circ$ Divide both sides by 2.

$3x + 2y = 720$

(ii) When $x = 140$,

$3(140) + 2y = 720$

$2y = 300$

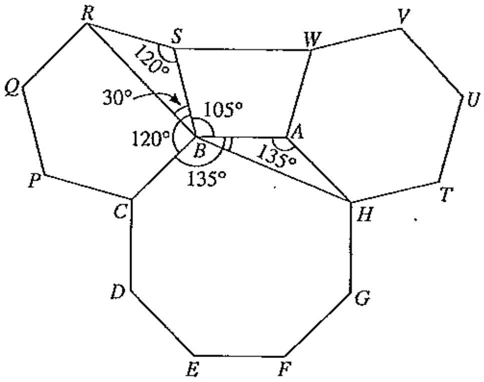
$y = \frac{300}{2}$ **$= 150$**

(iii) $P\hat{E}F = P\hat{F}E = 180^\circ - 150^\circ = 30^\circ$

$E\hat{P}F = 180^\circ - 30^\circ - 30^\circ$ ∠sum of Δ

$= 120^\circ$

13.



(a) (i) $A\hat{B}C = \frac{(8-2)\times 180^\circ}{8}$

$= 135^\circ$

$C\hat{B}S = \frac{(6-2)\times 180^\circ}{6}$

$= 120^\circ$

Reflex $A\hat{B}S = A\hat{B}C + C\hat{B}S$

$= 135^\circ + 120^\circ$

$= 255^\circ$

(ii) $S\hat{B}A = 360^\circ - 255^\circ$ ∠s at a pt.

$= 105^\circ$

$R\hat{B}S = \frac{180^\circ - 120^\circ}{2}$ base ∠s of isos. Δ

$= 30^\circ$

$A\hat{B}H = \frac{180^\circ - 135^\circ}{2}$ base ∠s of isos. Δ

$= 22.5^\circ$

Obtuse $R\hat{B}H = R\hat{B}S + S\hat{B}A + A\hat{B}H$

$= 30^\circ + 105^\circ + 22.5^\circ$

$= 157.5^\circ$

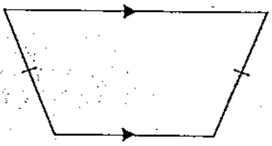
(iii) Acute $B\hat{S}W = \frac{360^\circ - 105^\circ - 105^\circ}{2}$

$= 75^\circ$

(b)

Teacher's Tip

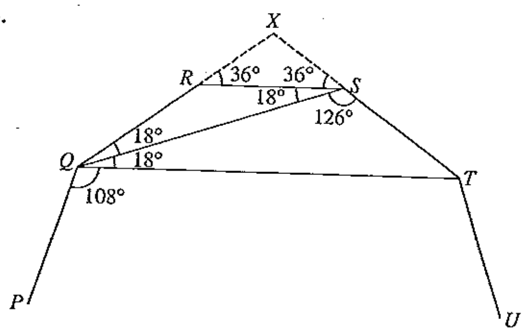
Isosceles Trapezium



- one pair of parallel sides
- non-parallel sides are equal in length

ABSW is an **isosceles trapezium.**

14.



(a) $X\hat{R}S = \frac{360°}{10}$
$= 36°$

Teacher's Tip

$X\hat{R}S$ is the exterior angle of a regular polygon with 10 sides.

(b) $\triangle RQS$ is isosceles.

$R\hat{Q}S + R\hat{S}Q = X\hat{R}S$ ext. ∠ = sum of int. opp. ∠s

$2R\hat{S}Q = 36°$ $R\hat{Q}S = R\hat{S}Q$

$R\hat{S}Q = \frac{36°}{2}$
$= \mathbf{18°}$

(c) $Q\hat{S}T = 180° - X\hat{S}R - R\hat{S}Q$ adj. ∠s on a str. line
$= 180° - 36° - 18°$
$= \mathbf{126°}$

(d) $R\hat{Q}S = R\hat{S}Q = 18°$ $\triangle QRS$ is isosceles.

$S\hat{Q}T = R\hat{S}Q$ alt. ∠s, $RS \parallel QT$
$= 18°$

Interior angle of polygon $= 180° - 36°$
$= 144°$

$\therefore P\hat{Q}T = 144° - 18° - 18°$
$= \mathbf{108°}$

15. (a) Each interior angle of the hexagonal tile
$= \frac{(6-2) \times 180°}{6}$
$= \mathbf{120°}$

(b) **Angles at any one point is 360°. No combination of squares and hexagons i.e. 90° and 120° gives 360°.**

(c) (i) Size of each angle
$= 360° - 90° - 120°$
$= \mathbf{150°}$

(ii) Exterior angle $= 180° - 150° = 30°$

No. of sides $= \frac{360°}{30°} = \mathbf{12}$

Test 7: Expansion and Factorisation of Algebraic Expressions

Section A

1. (a) $(5x + 3)^2$
$= (5x)^2 + 2(5x)(3) + 3^2$
$= \mathbf{25x^2 + 30x + 9}$

Use $(a + b)^2 = a^2 + 2ab + b^2$.

(b) $(2x + 5y)(2x - 5y)$
$= (2x)^2 - (5y)^2$ Use $(a + b)(a - b) = a^2 - b^2$.
$= \mathbf{4x^2 - 25y^2}$

2. (a) $8x^2 - 6xy$
$= \mathbf{2x(4x - 3y)}$

Teacher's Tip

Factorise by extracting common factor.
$2x$ is the common factor.

(b) $1 - 121x^2$
$= 1^2 - (11x)^2$ Use $a^2 - b^2 = (a + b)(a - b)$.
$= \mathbf{(1 + 11x)(1 - 11x)}$

(c) $x^2 + 12x + 36$
$= x^2 + 2(6)x + 6^2$ Use $a^2 + 2ab + b^2 = (a + b)^2$.
$= \mathbf{(x + 6)^2}$

3. (a) $35^2 - 280 + 16$
$= 35^2 - 2(35)(4) + 4^2$ Use $a^2 - 2ab + b^2 = (a - b)^2$.
$= (35 - 4)^2$
$= 31^2$
$= \mathbf{961}$

(b) $\frac{884^2 - 116^2}{219 \times 24 - 24 \times 211}$

$= \frac{(884 + 116)(884 - 116)}{24(219 - 211)}$

Use $a^2 - b^2 = (a + b)(a - b)$.

Use $a \times b - a \times c = a(b - c)$.

$= \frac{1000 \times \overset{\cancel{96}\ 4}{\cancel{768}}}{\underset{1}{\cancel{24}} \times \cancel{8}_1}$

$= \mathbf{4000}$

4.

$(2x + 5)(x - 2) + 3(x + 4)$
$= 2x^2 - 4x + 5x - 10 + 3x + 12$
$= \mathbf{2x^2 + 4x + 2}$

Expand the expression and collect like terms.

5. (a) $xt - yt$
$= \mathbf{t(x - y)}$ Extract common factor, t.

(b) $1896 \times 0.05489 - 896 \times 0.05489$
$= 0.05489(1896 - 896)$
$= 0.05489(1000)$
$= \mathbf{54.89}$

Extract common factor, 0.05489.