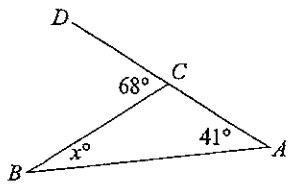
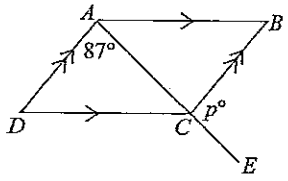
Worksheet 3-08 Deductive geometry

On a sheet of paper or in your book, carefully write out the solutions to each of the following deductive geometry problems, listing reasons for each step.

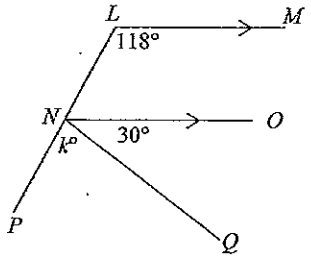
1 Find x.



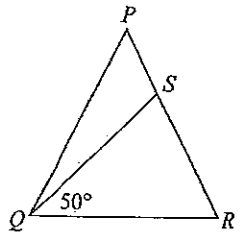
2 Find p.



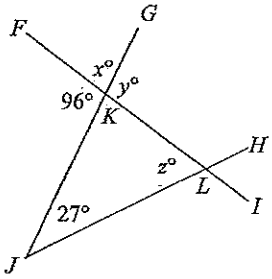
3 Find k.



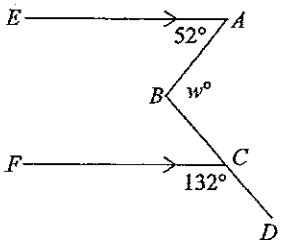
4 $PQ = PR$, $QR = QS$. Find $\angle QPR$.



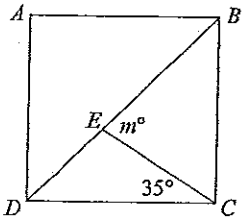
5 Find x, y, z.



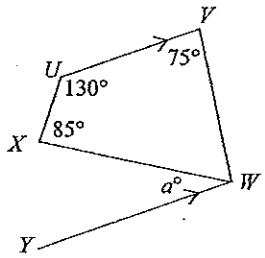
6 Find w.



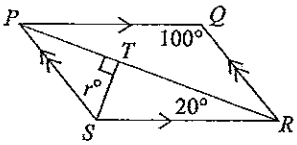
7 $ABCD$ is a square. Find m.



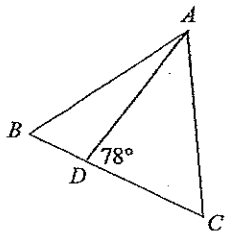
8 Find a.



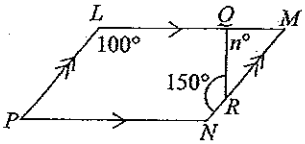
9 Find r.



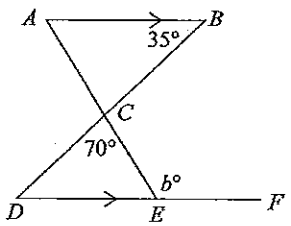
10 Triangle ABC is equilateral. Find all angles in ΔADC and in ΔABD.



11 Find n.



12 Find b.



SOLUTIONS

1) $x + 41° = 68°$ (ext ∠ of Δ)
$\therefore x° = 27$

2) $\angle BED = \angle DAC$ (alt ∠s DA ∥ CB)
$= 87°$
$\angle BCD + p° = 180°$ (adj, supp ∠s)
$\therefore p° = 93$

3) $\angle ONP = \angle MLN$ (corresp ∠s)
$= 118°$
$\angle ONQ + k° = \angle ONP$ (adj ∠s)
$= 118°$
$\therefore k° = 118° - 30°$
$= 88°$

4) $\angle BRQ = \frac{180 - 50}{2}$ (base ∠s of isosceles Δ, ∠ sum of Δ)
$= 65°$
$\angle PQR = \angle QRP$ (base ∠s of isosceles Δ)
$= 65°$
$\angle QPR = 180 - (\angle PQR + \angle QRP)$
$= 180 - 130$
$= 50°$

5) $\angle FKJ = \angle GKL$ (vert opp ∠s)
$= 96°$
$\therefore y° = 96$
$\angle FKG = 180 - \angle FKJ$ (adj, supp ∠s)
$\therefore x° = 84$
$\angle JKL = \angle FKG$ (vert opp ∠s)
$= 84°$

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$\angle KLJ = 180 - (\angle LKJ + \angle LJK)$ (∠ sum of Δ)
$= 180 - (84° + 27°)$
$\therefore z° = 69$

6) $\angle ABG = \angle FAB$ (alt ∠s EA ∥ HB)
$= 52°$
$\angle HBC = \angle FCD$ (corresp ∠s, BH ∥ EC)
$= 132°$
$\angle GBC = 180 - \angle HBC$ (adj, supp ∠s)
$= 48°$
$\therefore w° = \angle GBC + \angle ABG$ (adj ∠s)
$= 100$

7) $\angle BCE = 90° - 35°$ (∠s of a square $= 90°$)
$= 55°$
as BD bisects the square too
$\angle EBC = \frac{90}{2}$ (bisected ∠ of a square)
$= 45°$
$\therefore m° = 180° - (55° + 45°)$ (∠ sum of Δ)
$= 80°$

8) $\angle XWX = 360° - (75° + 130° + 85°)$ (∠ sum of quad)
$= 70°$
$\angle YWV = \angle WXW$ (co-int ∠s) WV ∥ YW
$= 105°$
$\therefore a° = \angle YWV - \angle YWX$ (adj ∠s)
$= 105° - 70°$
$= 35°$

9) $\angle RTS = \angle PTS$ (perp bisector)
$= 90°$ ✓

$\angle TSR = 180 - (\angle RTS + \angle TRS)$
$= 180° - 110°$
$= 70°$

$\angle PSR = \angle PQR$ (opp ∠s of a parallelogram)
$= 100°$

$\angle PST = \angle RSR - \angle TSR$ (adj ∠s)
$= 100° - 70°$
$= 30°$ ✓
$\therefore r° = 30$

10) $\angle ABC = \angle BAC = \angle BCA$ (∠s of equil △)
$= 60°$

In △ADC:
$\angle ADC = 78°$ (given)
$\angle ACD = 60°$ (∠ of equil △)
$\angle DAC = 180° - (78° + 60°)$ (∠ sum of △)
$= 42°$ ✓

In △ABD:
$\angle ABD = 60°$ (∠ of equil △) ✓
$\angle BAD = \angle BAC - \angle DAC$ (adj ∠s)
$= 60° - 42°$
$= 18°$ ✓
$\angle BDA = 180° - (60° + 18°)$ (∠ sum of △)
$= 102°$ ✓

11) $\angle MNP = \angle MLP$ (opp ∠s of parallelogram)
$= 100°$
$\angle MNS = 180° - \angle MNP$ (adj, supp ∠s)
$= 80°$ ✓
$\angle LMN = \angle MNS$ (alt ∠s $LM \parallel RN$)
$= 80°$
$\angle QRM = 360° - (180° + 180°)$ (revolution)
$= 30°$ ✓
$\angle MQR = 180° - (\angle QMR + \angle QRM)$
$= 180° - (80° + 30°)$
$= 70°$ ✓
$\therefore n° = 70$

12) $\angle ABF = \angle ABD$ (alt ∠s $AB \parallel DF$)
$= 35°$
$\angle CEF = \angle CDE + \angle DCE$ (ext ∠ of △)
$= 35° + 70°$
$= 105°$ ✓
$\therefore b° = 105$