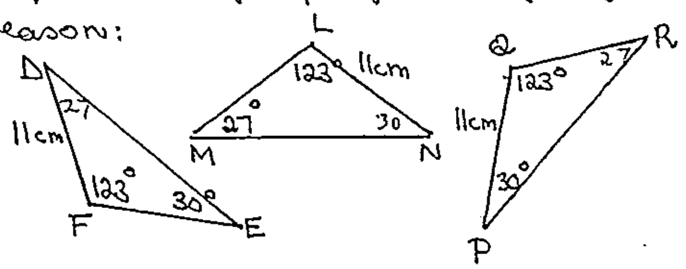
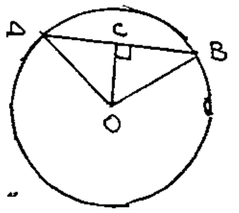
# YEAR 10 TEST: FURTHER REASONING in GEOMETRY + NUMBER

Name:

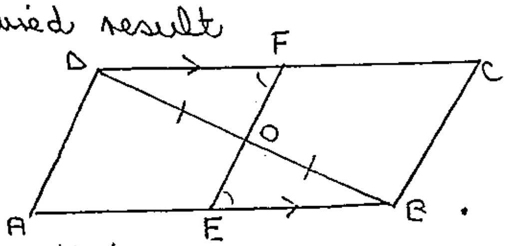
Q1: Select the two congruent triangles from this group of three, giving a reason:




Q2: Prove that the two triangles are congruent, setting out your proof correctly with a reason for each step.

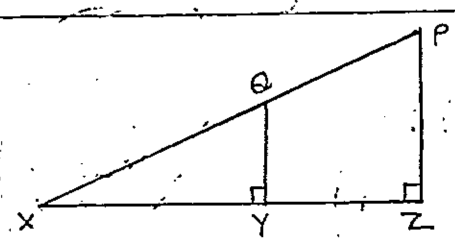



Q3: First prove that the two triangles are congruent, and then prove the required result




Prove that DF = EB

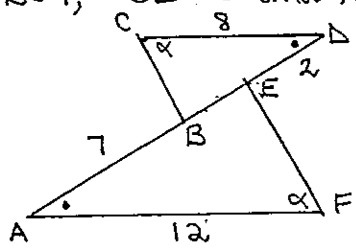
Q4:




(a) If ∠QYX and ∠PZY are both 90°, prove that ΔXYQ is similar to ΔXZP.

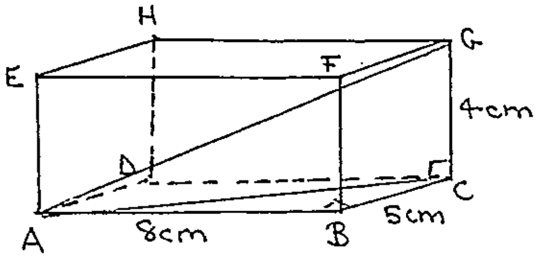
(b) Given that QY = 2.4 m, and that XY = 3.6 m and YZ = 4.8 m, find the length PZ

Q5: Calculate BE, given that DE = 2, AB = 7, CD = 8 and AF = 12




Q6: Find the last digit of $7^{51}$

Q7: Find the distance AG.




Q8: If $x$, $y$ and $z$ are numbers with $x > y$, which of the following are always true? Explain. $\frac{1}{3} < \frac{1}{-2}$

a) $\frac{1}{x} < \frac{1}{y}$

(b) $xz > yz$

c) $x+z > y+z$

Q9: Simplify the following

a) $x\sqrt{x} + \sqrt{x^3} - \sqrt{4x}$

b) $\sqrt{3} \times \sqrt{5} \times \sqrt{3}$

c) $2\sqrt{3}(\sqrt{3} - 2)$

d) $(3\sqrt{2} + \sqrt{3})^2$

e) $64^{\frac{1}{2}}$

f) $4^{-5/2}$

g) $3a^0$

h) $\left(\frac{2}{5}\right)^{-1}$

$3^{-1} = \frac{1}{3}$

Q10: Rationalise the denominator:

a) $\frac{4\sqrt{2}}{3\sqrt{5}}$

b) $\frac{5+\sqrt{2}}{4\sqrt{2}}$

Q11: Solve if possible

a) $2x + \sqrt{3} = \sqrt{15}$

b) $8\sqrt{x} = 40$

Q12: If $x$ and $y$ are positive integers and $x+y<11$, how many different values are there for the product $xy$? (Show all necessary working)

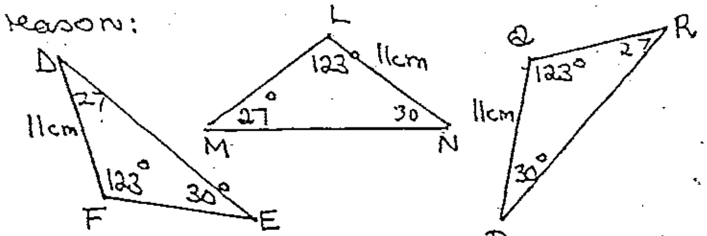
V. Good!

See corrections!

# YEAR 10 TEST: FURTHER REASONING in GEOMETRY + NUMBER

Name: ~~Amy Ow.~~ Olivia Lien

1: Select the two congruent triangles from this group of three, giving a reason:




In $\Delta$'s LMN & QRP

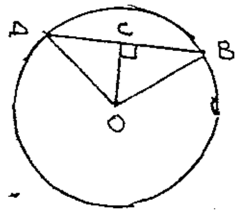
LN = QP (given) ✓

$\angle MLN = \angle PQR = 123°$ (given)

$\angle LMN = \angle QRP = 27°$ (given)

$\therefore \Delta LMN \equiv QRP$ (AAS) ✓

2: Prove that the two triangles are congruent, setting out your proof correctly with a reason for each step.




In $\Delta OCD$ & $\Delta OCB$

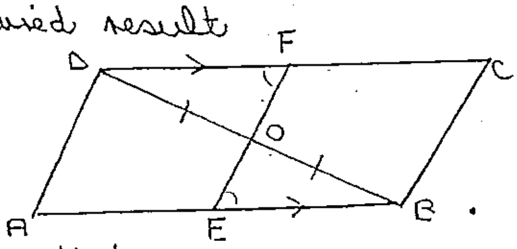
OC is common ✓

OD = OB (radii of circle) ✓

$\angle DCO = \angle BCO = 90°$ ✓ (given)

$\therefore \Delta OCD \equiv \Delta OCB$ (RHS)

3: First prove that the two triangles are congruent, and then prove the required result




Prove that DF = EB

In $\Delta$'s DFO & BEO

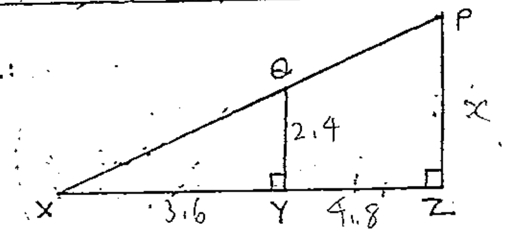
$\angle DFO = \angle BEO$ (given) ✓

DO = BO (given) ✓

$\angle DOF = \angle BOE$ (vert. opp $\angle$'s) ✓

$\therefore \Delta DFO \equiv \Delta BEO$ (AAS) ✓

Q4:




(a) If $\angle QYX$ and $\angle PZY$ are both 90°, prove that $\Delta XYQ$ is similar to $\Delta XZP$.

In $\Delta$'s XYQ & XZP

$\angle PXZ$ is common ✓

$\angle QYX = \angle PZY$ (given) ✓

$\therefore \Delta XYQ \ ||| \ \Delta XZP$ (equiangular) ✓

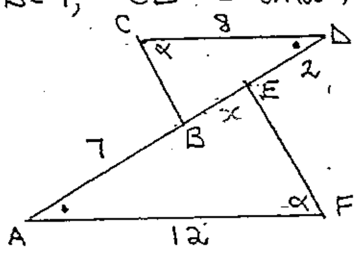
(b) Given that QY = 2.4 m, and that XY = 3.6 m and YZ = 4.8 m, find the length PZ

$$\frac{PZ}{QY} = \frac{x}{2.4} = \frac{8.4}{3.6} ✓$$

$$\frac{XZ}{XY} = \frac{8.4}{3.6} ✓$$

$\therefore x = 5.6$ m ✓

Q5: Calculate BE, given that DE = 2, AB = 7, CD = 8 and AF = 12




$\Delta BCD \ ||| \ \Delta EFA$ ✓ (equiangular)

$$\frac{AF}{CD} = \frac{12}{8} = \frac{3}{2} ✓$$

$$\frac{AE}{BD} = \frac{7+x}{x+2} = \frac{3}{2} ✓$$

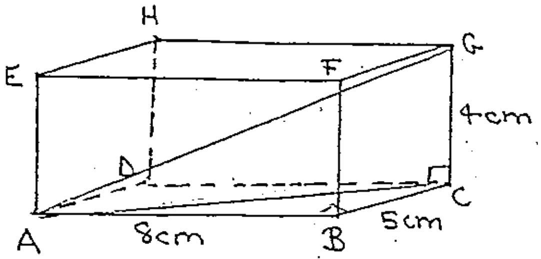
$14 + 2x = 3x + 6$

$-x = -8 \quad \therefore x = 8$ ✓

Q6: Find the last digit of $7^{51}$

3 ✓

: Find the distance AG.




$AC = \sqrt{89}$

$AG = \sqrt{(\sqrt{89})^2 + 16}$ ✓

$= \sqrt{105}$ ✓

:) If $x$, $y$ and $z$ are numbers with $x > y$, which of the following are always true? Explain. $\frac{1}{3} < \frac{1}{2}$

) $\frac{1}{x} < \frac{1}{y}$ $\quad x - y > 0$

$\frac{1}{x} - \frac{1}{y} < 0$

$= \frac{y-x}{xy} < 0$ /

$\frac{-x+y}{xy} < 0$

$xz > yz$

$\frac{xz}{z} > \frac{yz}{z}$

If $z > 0$, $x > y$ is true

If $z < 0$, $x < y$ is false ✓

$x + z > y + z$

If $z > 0$, $x > y$ is true

If $z < 0$, $x > y$ is true. /

Q9: Simplify the following

a) $x\sqrt{x} + \sqrt{x^3} - \sqrt{4x}$ $\quad$ Factorise $\sqrt{x}$

$x\sqrt{x} + x\sqrt{x} - 2\sqrt{x}$ $\quad = \sqrt{x}(x + x - 2)$

$= 2x^2 - 2\sqrt{x}$ $\quad = \sqrt{x}(2x - 2)$.

b) $\sqrt{3} \times \sqrt{5} \times \sqrt{3}$ $\quad 3\sqrt{5}$ ✓

c) $2\sqrt{3}(\sqrt{3} - 2)$ $\quad 6 - 4\sqrt{3}$ ✓

d) $(\cancel{3\sqrt{2} + \sqrt{3}})^2$ $12 + 6\sqrt{6} + 3$

$= 15 + 6\sqrt{6}$ ✓

e) $64^{\frac{1}{2}}$ $\quad 8$ ✓

f) $4^{-5/2} = 2^{-5} = \frac{1}{32}$ ✓

g) $3a^0$ $\quad 3$ ✓

h) $\left(\frac{2}{5}\right)^{-1}$ $\quad \frac{5}{2}$ ✓

$3^{-1} = \frac{1}{3}$

Q10: Rationalise the denominator:

a) $\frac{4\sqrt{2}}{3\sqrt{5}} = \frac{4\sqrt{10}}{15}$ ✓

b) $\frac{5+\sqrt{2}}{4\sqrt{2}} = \frac{5\sqrt{2}+2}{8}$ ✓

Q11: Solve if possible

a) $2x + \sqrt{3} = \sqrt{75}$

$2x = 5\sqrt{3} - \sqrt{3}$ ✓

$2x = 4\sqrt{3}$

$x = 2\sqrt{3}$ ✓

b) $8\sqrt{x} = 40$

$\sqrt{x} = 5$ $\quad \Longrightarrow \quad x = 5^2$

$= 25$.

$x = \pm\sqrt{5}$ X

Q12: If $x$ and $y$ are positive integers and $x+y<11$, how many different values are there for the product $xy$? (Show all necessary working)

Possible Values of $x + y = 1+9, 2+8, 3+7, 4+6, 5+5, 1+8, 2+7, 3+6, 4+5, 1+7, 2+6, 3+5, 4+4, 1+6, 2+5, 3+4, 1+5, 2+4, 3+3, 1+4, 2+3, 1+3, 2+2, 1+2, 1+1,$ ~~10+0, 9+0, 8+0, 7+0, 6+0, 5+0, 4+0, 3+0, 2+0, 1+0,~~

0 is neither positive nor negative.

Possible products = 9, 16, 21, 24, 25, 8, 14, 18, 20, 7, 12, 15, 6, 10, 5, 4, 3, 2, 1, ~~0~~