EXAMINATIONS

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Examination 2

General instructions:

- Reading time 5 minutes
- Write using black or blue pen
- A formulae sheet is provided on pp 370-371
- Working time 2½ hours
- Calculators may be used

Time: 2½ hours Total marks: 100

Section I

Choose the correct answer (A, B, C or D) for each question.

Suggested time: 30 minutes

Attempt Questions 1–22 (each question is worth one mark)



In eight weekly tests, Maria scored the following marks: 8 9 6 8 5 7 8 7

What is the median?

A 6.5

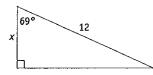
B 7

C 7.5

D 8



Which expression will give the value of x?



A 12 cos 69°

B 12 sin 69°

 $C \frac{12}{\sin 69^{\circ}}$



Simplify 8x - 3(5 - x).

A 5x - 15

B 7x - 15

C 9x - 15

D 11x - 15



How much compound interest, to the nearest dollar, is earned when \$30 000 is invested at 9% p.a. interest, compounded monthly, for seven years?

A \$18 900

B \$33 677

C \$24 841

D \$26 196



A box holds one blue, one yellow and two red ribbons. If two ribbons are chosen at random, what is the probability that both are red?

Given that $E = mc^2$, find E if m = 0.8 and $c = 3 \times 10^5$.

A 7.2×10^{10}

B 5.76×10^{10}

C 7.2×10^{5}

D 5.76×10^{5}



Cameron's pay-rate is \$21.60 per hour. What does Cameron earn in a week where he works $37\frac{1}{2}$ hours at the normal rate and $6\frac{1}{2}$ hours at time-and-a-half?

A \$835.92

B \$950.40

C \$1009.80

D \$1020.60



In a state-wide examination, the results are normally distributed with mean mark 62% and standard deviation 12%. What is the best approximation for the percentage of candidates who scored more than 50%?

A 34%

B 68%

C 84%

D 95%



Joanne borrows \$16 000 and repays it with 60 monthly repayments of \$332.

What annual flat rate of interest is charged?

A 4.9%

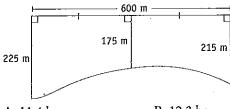
B 5.4%

C 24.5%

D 24.9%



Use Simpson's rule to find the approximate area of the land shown in the diameter. the land shown in the diagram.



A 11.4 ha

B 12.3 ha

C 22.8 ha

D 68.4 ha



A committee of three people is to be chosen from a group of 12 people. How many different committees are possible?

A 1320

B 479

C 220

D 36

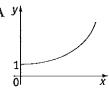


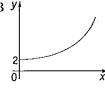
Simplify $\frac{a^2b}{2ab^2} \div ab$

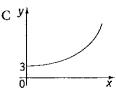
 $C \frac{b^2}{2}$

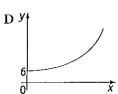


Which diagram could show the graph of $y = 3(2^x)$?











A sphere has diameter 18 cm. What is its volume to the nearest cubic centimetre?

A 3054 cm³

B 6107 cm³

C 12 215 cm³

D 24 429 cm³



A machine was bought for \$2400. It is being depreciated using the straight-line method, with the amount of depreciation apportioned per period being 12.5% of the original value. What is the salvage value of the machine after six years?

A \$600

B \$1100

C \$1300

D \$1800



The correlation between two sets of data has coefficient -0.6. Describe the correlation.

A weak negative

B moderatively-weak negative

C moderately-strong negative

D strong negative



Notebook entries have been made of a survey of a block of land. How long is the boundary from A to C?

A 30 m

B 40 m

C 50 m D 70 m

[100] D 50 70 30 40 C



Annabel has 3000 shares in a company and received a total of \$630 when dividends were paid. At the time the shares were valued at \$5.60 each. What was the dividend yield?

A 3.75%

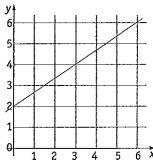
B 8.8%

C 11.25%

D 21%



What is the equation of the line?



$$A \quad y = \frac{3}{2}x + 2$$

$$B \quad y = 2x + \frac{3}{2}$$

C
$$y = \frac{2}{3}x + 2$$
 D $y = 2x + \frac{2}{3}$

D
$$y = 2x + \frac{2}{3}$$



A survey was taken of the number of cars in households. The results are shown in the table.

Number of cars	0	1	2	3	4
Frequency	15	23	30	5	1

What is the median?

A 1

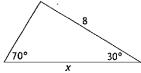
B 1.38

C 1.5

D 2



Which expression will give the value of x?



$$B = \frac{\sin 70^{\circ}}{8 \sin 30^{\circ}}$$

$$C = \frac{8 \sin 70^{\circ}}{\sin 30^{\circ}}$$

$$D = \frac{8 \sin 80}{\sin 70^{\circ}}$$



An amount of \$500 is paid into an account every month for a year. What single sum of money (to the nearest dollar) invested now would produce the same result, if both earn 9% p.a. interest, compounded monthly?

A \$5505

B \$5717

C \$6000

D \$6254

Section II

Suggested time: 2 hours

Total: 78 marks

Attempt Questions 23–28

Show all working.



- (13 marks)

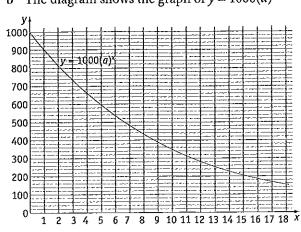
 a Ava borrows \$12 000 over three years. The flat rate of interest charged on the loan is 7% p.a.
 - i Find the total amount of interest Ava 1 mark will pay.
 - ii If Ava repays the loan with three equal annual instalments, find the amount of 2 marks each instalment.
 - iii Use the present value formula to find the amount of each annual instalment if the interest rate was reducible (7% p.a.) 2 marks instead of flat.
 - iv How much would Ava save altogether with the reducible rate of interest compared to the flat rate? 1 mark
 - b The formula $n = \frac{360}{180 a}$ is used to find the number (n) of sides of a regular polygon where all the angles measure a° . (A regular polygon has all its sides equal and all its angles equal.)
 - How many sides must a polygon have if each of its angles measure 135°? 1 mark
 - ii What type of polygon has all its angles 108°? 2 marks
 - iii Rearrange the formula so that a is the subject.
 - iv What is the size of each angle of a
- 2 marks 2 marks regular hexagon?



(13 marks)
a Solve
$$\frac{2x+3}{8} = 5$$

1 mark

The diagram shows the graph of $y = 1000(a)^x$



- What is the value of y when x = 2? 1 mark
- ii For what value of x is y approximately 1 mark

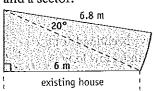
The expression $y = 1000(a)^x$ can be used to model the depreciation of an asset, originally valued at \$1000, under the declining-balance method.

- iii How much will an asset originally valued at \$1000 depreciate during the fourth 1 mark year?
- iv What is the rate of depreciation? 1 mark
- v If the asset was being depreciated by the straight-line method, and the value was the same as the declining-balance method after ten years, show the depreciation on 1 mark the graph.
- vi How much depreciation is allocated per period under the straight-line method? 1 mark
- vii For how many years could the straight-line method be used? Justify your answer. 2 marks
- c A helicopter leaves P and flies on a bearing of 070° to Q, a distance of 375 km. From Q it flies 515 km on a bearing of 130° to R.
 - Draw a diagram showing the path of the 1 mark helicopter.
 - ii Explain why the size of ∠PQR is 120°. 1 mark
 - iii Find the distance from R to P to the nearest kilometre. 2 marks



- (13 marks)

 a Dave plays a game in which he has one chance in five of winning \$200. If he doesn't win, Dave will lose \$50. What is Dave's financial expectation from playing the game? 2 marks
 - b The diagram shows the plan of the verandah extension that is being added to a house. The verandah is made up of a right-angled triangle and a sector.



Find the area of the verandah.

3 marks

ii Find the length of railing required if it is to be placed on all sides of the verandah except 2 marks where it adjoins the house.

c A teacher gave a test to her class where all the questions had to be answered as either true or false. She checked the answers and drew up the following table.

	Answered true	Answered false
Question true	104	78
Question false	<i>7</i> 5	193

i If 18 students did the test and each one answered every question, how many questions were on the test?

2 marks

ii What percentage of questions were answered correctly?

2 marks

iii What is the probability that a true question was answered true?

1 mark

iv What is the probability that a true answer was to a question that was actually true?

1 mark



(13 marks)

- A building has a rectangular roof 5 m long and 3 m wide. If 15 mm of rain falls, how much water should flow from the roof? 2 marks $(1 \text{ m}^3 = 1000 \text{ L})$
 - ii The water flows into a cylindrical tank. If the radius of the tank is 1.2 m, by how many centimetres will the water level rise 3 marks in the tank?
- b A plane flies from Sydney (34°S, 151°E) across the Pacific Ocean to Mexico City (19°N, 99°W). The flight takes 161/2 hours. If the plane left Sydney at 4:50 pm on Thursday Sydney time, what is the local time and day when the plane arrives in Mexico? (Ignore time zones.) 4 marks
- The masses of certain packets of chips are normally distributed, with mean mass 210 g and standard deviation 5 g.
 - i Explain what it means to have a z-score 1 mark
 - ii What mass would a packet have if it has a z-score of 3. I mark
 - iii The packets are labelled as having a weight of 200 g. Approximately what percentage of packets would have less than 200 g?



(13 marks)

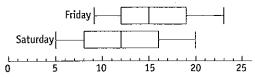
a Expand and simplify

 $2(x^2 + 5x + 1) + 3x(x - 1)$

2 marks

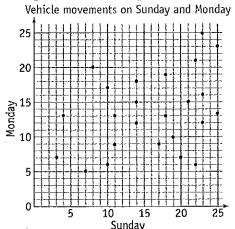
b A survey was taken on different days of the number of vehicles passing through a particular intersection in particular time periods during each day. The results for Friday and Saturday were analysed and two box-and-whisker plots were drawn on the same scale.

Number of vehicles on Friday and Saturday



- What was the highest number of vehicles and on what day did that occur?
- ii On Saturday, approximately what percentage of the time were there fewer than 8 vehicles? 1 mark
- iii Compare and contrast the patterns for 2 marks the two days.

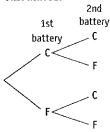
The results of the survey for Sunday and Monday were placed on a scatterplot.



- iv At the time when there were 20 vehicles on Monday, how many were there on 1 mark Sunday?
- v Briefly describe any correlation.

1 mark

c A box holds 10 batteries, seven are fully charged but the other three are flat. Natasha takes two batteries from the box and puts them in her calculator.



i Copy and complete the tree diagram by writing the probabilities on the branches.

2 marks

ii The calculator will only work if both batteries are charged. What is the probability that the calculator will work?

1 mark

iii Natasha comments that the chance that one battery is flat is exactly the same as the chance that both batteries are charged. Is she correct?

Justify your answer.

2 marks

(13 marks)

a The 75 students in Year 4 at a primary school were asked the number of pets in their household. The results are found in the table.

Number of pets	0	1	2	3	4	5	6
Frequency	18	22	11	9	8	4	3

i What is the mean?

1 mark

ii What is the standard deviation (σ_n) ? (Give the answer correct to one decimal place.)

1 mark

iii Is the data normally distributed? Justify your answer.

1 mark

- iv What percentage of scores are within one standard deviation of the mean? 2 marks
- b i Complete the table of values for $T = \frac{100}{S}$.

S	1	2	4	5	10	20	25	50	100
T									

1 mark

- ii Sketch the graph of $T = \frac{100}{S}$.
- 2 marks
- iii The rule $T = \frac{100}{S}$ can be used to give the

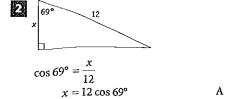
time (*T*) in seconds to travel 100 metres at a given speed (*S*) in metres per second. Use the graph to find the approximate time to travel 100 m at 15 m/s.

- c i If \$2000 is placed in an account every quarter at an interest rate of 6% p.a., compounded quarterly, how much is in the account at the end of ten years? 2 marks
 - ii Use the estimation and refinement technique to find the number of years an amount of \$2000 must be invested quarterly (at 6% p.a. interest compounded quarterly), to have \$550 000 in the account.

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C

8 9 6 8 5 7 8 7 In order: 5 6 7 7 8 8 8 9 median = $\frac{7+8}{2}$



- 8x 3(5 x) = 8x 15 + 3x= 11x 15 D
- $A = P(1+r)^{n}$ = \$30 000(1.0075)⁸⁴
 = \$56 196 (nearest dollar) I = \$56 196 \$30 000= \$26 196 D

- P(first is red) = $\frac{2}{4}$ = $\frac{1}{2}$ P(second is red) = $\frac{1}{3}$ P(both red) = $\frac{1}{2} \times \frac{1}{3}$ = $\frac{1}{6}$
- E = mc^2 If m = 0.8 and $c = 3 \times 10^5$, $E = 0.8 \times (3 \times 10^5)^2$ $= 7.2 \times 10^{10}$
- Pay = $37.5 \times \$21.60 + 6.5 \times 1.5 \times \21.60 = \$1020.60
- 50% is one standard deviation below the mean.

Percentage = 34% + 50% = 84%

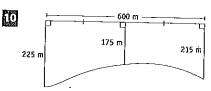
Total repaid =
$$60 \times $332$$

= \$19 920
Interest = \$19 920 - \$16 000

= \$3920 Annual interest = \$3920 ÷ 5 = \$784

Annual interest rate $= \frac{$784}{$2234} \times 100\%$

 $=\frac{16000}{$16000}$



$$A \approx \frac{h}{3}(d_f + 4d_m + d_1)$$

$$= \frac{300}{3}(225 + 4 \times 175 + 215)$$

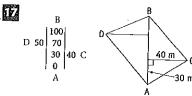
$$= 114\,000$$

Area $\approx 114\,000\,\mathrm{m}^2$ = 11.4 ha

Number of committees $= \frac{12 \times 11 \times 10}{3 \times 2 \times 1}$ = 220C

- $\frac{a^2b}{2ab^2} \div ab = \frac{a}{2b} \times \frac{1}{ab}$ $= \frac{1}{2b^2}$ A
- $y = 3(2^{x})$ When x = 0, $y = 3(2^{0})$ $= 3 \times 1$ = 3C is the correct graph.

- $V = \frac{4}{3}\pi r^3$ $= \frac{4}{3} \times \pi \times 9^3$ $= 3053.628 \ 059 \dots$ The volume is 3054 cm³, to the nearest cubic centimetre.
- D = 12.5% of \$2400 $= 0.125 \times \$2400$ = \$300 $S = V_0 Dn$ $= \$2400 \300×6 = \$600
- The correlation is moderately-strong negative.



By Pythagoras' theorem, $AC^2 = 30^2 + 40^2$ = 2500 AC = 50The length of AC is 50 m.

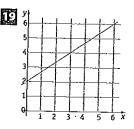
Dividend per share = \$630 + 3000= \$0.21Dividend yield = $$0.21 \times 100\%$ = 3.75% Ċ

A

C

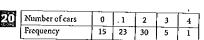
A

367



Gradient = $\frac{2}{3}$ Vertical intercept = 2

Equation is $y = \frac{2}{3}x + 2$



Total frequency = 15 + 23 + 30 + 5 + 1= 74

The middle scores are the 37th and 38th scores.

Median = 1

[15 + 23 = 38.

The 16th to 38th scores are all 1.]

A

A

b i
$$V_0 = $56\,000, r = 0.08, n = 5$$

 $S = V_0(1-r)^n$
 $= $56\,000(1-0.08)^5$
 $= $36\,908.5653...$

= \$36 900 (nearest \$100)

ii
$$$20\ 000 = $56\ 000(0.92)^n$$

 $n = 13$ [by calculator]
The tractor will fall below \$20\ 000 after 13 years.

c i Possible arrangements
=
$$26 \times 26 \times 10 \times 10 \times 26 \times 26$$

= $45 697 600$

ii There are 26 possibilities for the repeated letter. For each of these, there are 100 possibilities for the two numbers
$$(10 \times 10)$$
. \checkmark Number of arrangements = 26×100 = 2600 \checkmark

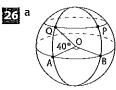
d i
$$B = kl^2$$

When $l = 30$, $B = 1000$
 $1000 = k \times 30^2$
 $= 900k$
 $k = \frac{1000}{900}$
 $= \frac{10}{9}$
 $B = \frac{10}{9}l^2$
When $l = 60$,
 $B = \frac{10}{9} \times 60^2$
 $= 4000$
There will be 4000 beads.

ii
$$B = \frac{10}{9}l^2$$

When $B = 6250$
 $6250 = \frac{10}{9}l^2$
 $l^2 = 6250 \div \frac{10}{9}$
 $= 5625$
 $l = \sqrt{5625}$ $(l > 0)$
 $= 75$

The skirt is 75 cm long.



iii
$$l = \frac{\theta}{360} 2\pi r$$

$$= \frac{90}{360} \times 2 \times \pi \times 6400$$

$$= 10\ 053.096\ 49\ ...$$
The distance from A to B is approximately 10 050 km.

iv Angular difference from P to
$$B=40^{\circ}$$
 Approximate distance

Plane flies at 500 knots
Time =
$$(2400 \div 500)$$
 hours

b i
$$M = \$500, r = 0.02, n = 24$$

$$A = M \left\{ \frac{(1+r)^n - 1}{r} \right\}$$

$$= \$500 \left\{ \frac{(1.02)^{24} - 1}{0.02} \right\}$$

$$= \$15 211 \text{ (nearest dollar)}$$

ii
$$N = \frac{A}{(1+r)^n}$$

$$= \frac{\$15 \ 211}{(1.02)^{24}}$$

$$= \$9457 \quad \text{(nearest dollar)}$$

c i
$$F = \frac{kQq}{r^2}$$

$$Fr^2 = kQq$$

$$r^2 = \frac{kQq}{F}$$

$$r = \sqrt{\frac{kQq}{F}} \quad (r > 0)$$

ii If
$$k = 9 \times 10^9$$
, $Q = 4 \times 10^{-12}$,
 $q = 1.2 \times 10^{-10}$ and $F = 2.7 \times 10^{-11}$,
 $r = \sqrt{\frac{9 \times 10^9 \times 4 \times 10^{-12} \times 1.2 \times 10^{-10}}{2.7 \times 10^{-11}}} \checkmark$
 $= 0.4$

a i
$$N = \$160\ 000, \ r = 0.006, \ n = 240$$

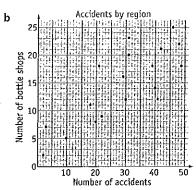
$$N = M \left\{ \frac{(1+r)^n - 1}{r(1+r)^n} \right\}$$

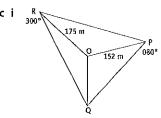
$$\$160\ 000 = M \left\{ \frac{(1.006)^{240} - 1}{0.006(1.006)^{240}} \right\}$$

$$\$160\ 000 = M \times 127.008\ 4321 \dots$$

$$$160\ 000 = M \times 127.008\ 4321...$$
 $M = $160\ 000$
 $\div 127.008\ 4321...$
 $= $1259.76\ (nearest cent)$

$$=$$
 \$1274.76
Total repaid $=$ \$1274.76 \times 240
 $=$ \$305 942.40





Q is due south of O.

The bearing of Q from O is 180°.✓

ii
$$\angle ROP = (360 - 300)^{\circ} + 80^{\circ}$$

= 140°

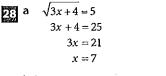
iii Let x m be the length of RP.
By the cosine rule:

$$x^2 = 175^2 + 152^2 - 2 \times 175 \times 152$$

 $\times \cos 140^\circ \checkmark$
 $= 94 482.56437 ...$
 $x = 307.380 1626 ...$ $(x > 0)$

$$x = 307.380 \ 1626 \dots (x > 0)$$

The length of the boundary RP is 307 m, to the nearest metre.



b i Types =
$$3 \times 2 \times 2$$

= 12

$$P(\text{ticket without black}) = \frac{3}{12}$$

$$= \frac{1}{4}$$

Expected number =
$$\frac{1}{4} \times 60$$

= 15

c i
$$(1.08)^n = 2$$

 $1.08^9 = 1.999\ 004\ 627...$
 $n = 9$

ii Arc length:
$$l = \frac{\theta}{360} 2\pi r$$

= $\frac{20}{360} \times 2 \times \pi \times 6.8$
= 2.373 647 783 ...
= 2.4 (1 d.p.)

Length of railing $\approx (3.2 + 6.8 + 2.4) \text{ m}$ = 12.4 m

2		Answered true	Answered false			
	Question true	104	78			
	Question false	75	193			

- i Total questions on all tests = 104 + 78 + 75 + 193 = 450 Questions per test = 450 + 18
- Questions per test = 450 ÷ 16 = 25 ii Questions answered correctly
- = 104 + 193 = 297% answered correctly = $\frac{297}{2} \times 100\%$

% answered correctly = $\frac{297}{450} \times 100\%$ = 66%

iji True questions = 104 + 78= 182 $P(\text{answered true}) = \frac{104}{100}$

 $P(\text{answered true}) = \frac{104}{182}$ $= \frac{4}{7}$

iv True answers = 104 + 75 = 179

 $P(\text{true question}) = \frac{104}{179}$

a i V = lbh= $5 \times 3 \times 0.015$ = 0.225Volume = 0.225 m^3 Amount of water = $0.225 \times 1000 \text{ L}$ = 225 litres

ii $V = \pi r^2 h$ $0.225 = \pi \times 1.2^2 \times h$ $h = \frac{0.225}{\pi \times 1.2^2}$ = 0.049 735 919 ...= 0.05 (2 d.p.)

The level of water in the tank will rise by approximately 5 cm.

b Angular difference = 151° + 99° = 250°

Time difference

$$= (250 \div 15) \text{ hours}$$

Sydney is 16 hours and 40 minutes ahead of Mexico City. The plane leaves at 12.10 am Thursday,

Mexico time.

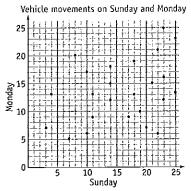
It arrives 16½ hours later. It arrives at 4:40 pm, Thursday.

- c i A z-score of −1 means that the raw score is one standard deviation below the mean.
 - ii Mass = $210 g + 3 \times 5 g$ = 225 g
 - iii 200 g is two standard deviations below the mean. 95% of packets will be within two standard deviations of the mean. 5% will be outside two standard deviations. 2.5% will be less than 200 g.
- a $2(x^2 + 5x + 1) + 3x(x 1)$ = $2x^2 + 10x + 2 + 3x^2 - 3x$ = $5x^2 + 7x + 2$
 - Number of vehicles on Friday and Saturday

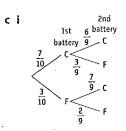
 Friday

 Saturday

 0 5 10 15 20 25
 - i The highest number was 23 on Friday.
 - ii 8 is the lower quartile. Approximately 25% of the time there were less than 8 vehicles.
 - iii Both displays are fairly symmetrical, showing similar patterns. The data for Friday was consistently higher than that for Saturday.



- iv There were 8 vehicles on Sunday.
- v There is no real pattern between the points. There is almost zero correlation, perhaps very weak positive.



ii P(both charged) = $\frac{7}{10} \times \frac{6}{9}$ = $\frac{7}{15}$ iii $P(\text{one is flat}) = \frac{7}{10} \times \frac{3}{9} + \frac{3}{10} \times \frac{7}{9}$ $= \frac{7}{15}$

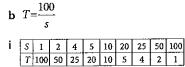
Yes, Natasha is correct.

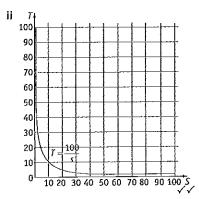
χį a	Number of pets	0	1	2	3	4	5	6
SSET.	Frequency	18	22	11	9	8	4	3

- i mean = 1.88 [by calculator]
- ii $\sigma_n = 1.70458206...$ = 1.7 (1 d.p.)
- iii No, the data is not normally distributed. It is skewed with more lower than higher scores.

iv
$$\bar{x} + s = 1.88 + 1.7$$

= 3.58
 $\bar{x} - s = 1.88 - 1.7$
= 0.18
Scores between 0.18 and 3.58
are 1, 2 and 3.
Number of scores = 22 + 11 + 9
= 42
Percentage within 1 standard
deviation = $\frac{42}{75} \times 100\%$





iii Approximately 7 seconds.

c i
$$M = $2000, r = 0.015, n = 40$$

$$A = M \left\{ \frac{(1+r)^n - 1}{r} \right\}$$

$$= $2000 \left\{ \frac{(1.015)^{40} - 1}{0.015} \right\}$$

$$= $108536 \text{ (nearest dollar)}$$

ii \$550 000 = \$2000 ×
$$\left\{ \frac{\left(1.015\right)^n - 1}{0.015} \right\}$$

n = 110 [by calculator] Number of years = $110 \div 4$ = 27.5

The money would need to be invested for 27½ years.