GEOMETRICAL APPLICATIONS OF CALCULUS

1) Find all values of x for which the curve $y = 2x^2 - 8x + 3 = 0$ is increasing.
2) Find the domain over which the function $f(x) = 4 - x^2$ is decreasing.
3) Find the stationary point on the curve $y = 3x^2 + 12x - 11$.
4) Show that $f(x) = \dfrac{2}{x-1}$ has no stationary points.
5) The curve $y = ax^2 + bx - 1$ has a stationary point at (1,-5). Find values of a and b.
6) Find the stationary point on the curve $y = x^2 - 4x + 3$ and show that it is a minimum turning point.
7) Find the stationary point on the curve $y = x^3 - 1$ and determine its nature.
8) Find the stationary points on the curve $y = x^3 + 3x^2 - 9x + 4$ and determine their nature.
9) Find the first and second derivatives of $2x^5 - 7x^3 + 2x - 1$.
10) Find $f^{1}(x)$ and $f^{11}(x)$ if $f(x) = 4x^3 - 9$.
11) Find $f^{1}(2)$ and $f^{11}(2)$ if $f(x) = 3x^4 - x^2 + 2x - 3$.
12) Find all values of x for which the curve $y = x^3 - 3x^2 + x - 4$ is concave downwards.
13) For the curve $y = f(x)$, $f^{1}(a) = 0$ and $f^{11}(a) > 0$. Describe the shape of the curve at the point where $x = a$.
14) The number, N, of people with flu is increasing over time t. Also, the rate at which people are catching flu is also increasing.

    (a) Describe the sign of $\dfrac{dN}{dt}$ and $\dfrac{d^2N}{dt^2}$.

    (b) Sketch a graph that describes the information.
15) The population P, of birds in a certain area is increasing over time t, but the rate of population growth is slowing. Describe the sign of $\dfrac{dP}{dt}$ and $\dfrac{d^2P}{dt^2}$.
16) Find the stationary point on the curve $y = x^3$ and show that it is a point of inflexion.
17) Find the point of inflexion on the curve $y = 2x^3 - 12x^2 + 5$.
18) Consider the curve $y = x^3 - 3x^2 + 1$.

    (a) Find any stationary points on the curve.

    (b) Determine their nature.

    (c) Find any points of inflexion on the curve.

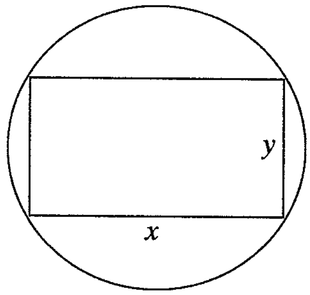
    (d) Hence sketch the curve in the domain $-2 \le x \le 3$

    (e) Find the minimum value of the curve in this domain.
19) A closed box with a square base is to be made so that its volume is 100 cm^3.

    (a) Show that the surface area of the box is given by $A = 2x^2 + \dfrac{400}{x}$.

    (b) Find the minimum possible surface area of the box, to one decimal place.

20)

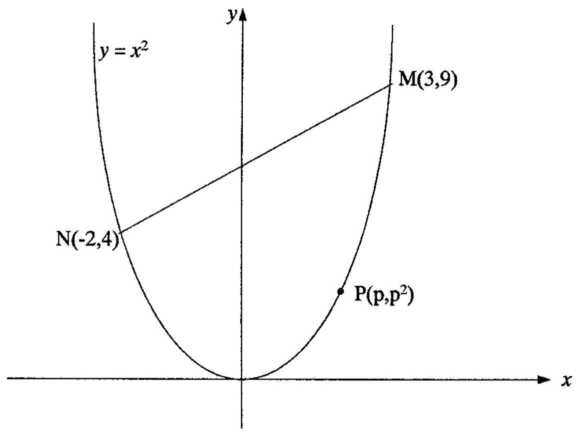


A rectangle with sides x and y is cut out of a circle with diameter 50 cm.

(a) Show that the area of the rectangle is given by $A = x\sqrt{2500 - x^2}$

(b) Find the maximum possible area of the rectangle.

21) Fixed points M(3,9) and N(-2,4) lie on the parabola $y = x^2$, and $P(p,p^2)$ is a variable point on the parabola.



(a) Find the exact perpendicular distance from P to the line MN.

(b) If P always lies under the line MN, show that the area of triangle MNP is given by $A = \dfrac{5(p - p^2 + 6)}{2}$.

(c) Hence find the maximum area of triangle MNP.

22) Find the primitive function of $2x - 3$.

23) Find the primitive function of $5x^4 + 2x^3 - x^2 + 1$.

24) If $\dfrac{dy}{dx} = 4x + 1$ and $y = 2$ when $x = 3$, find y when $x = 5$.

25) Given $f^1(x) = 6x^2 - 1$, and $f(0) = 4$, find $f(1)$.

26) If a curve has $\dfrac{d^2y}{dx^2} = 12x - 12$ and $\dfrac{dy}{dx} = 2$ and $y = 1$ when $x = 1$, find the equation of the curve.

27) The tangent to a curve at point N has equation $5x - y - 1 = 0$.

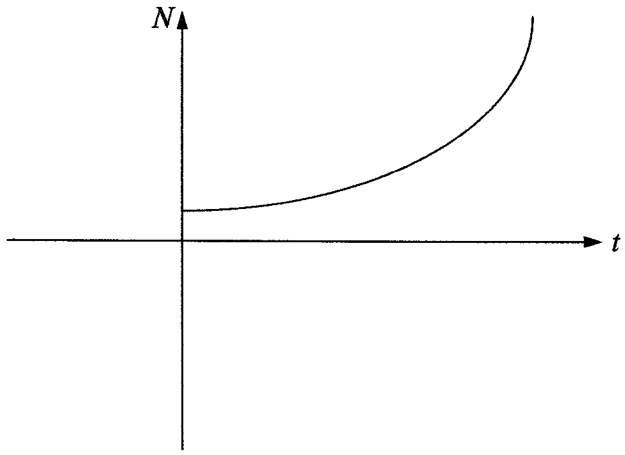
(a) If $\dfrac{dy}{dx} = 4x - 3$, find the coordinates of N.

(b) Find the equation of the curve.

ANSWERS

1) $x > 2$
2) $x > 0$
3) (-2,-23)
4) $\frac{-2}{(x-1)^2} \neq 0$
5) a = 4, b = -8
6) (2,-1); LHS $\frac{dy}{dx}<0$, RHS $\frac{dy}{dx}>0$.
7) (0,-1); point of inflexion
8) (-3,31) maximum, (1,-1) minimum
9) $y^1 = 10x^4 - 21x^2 + 2$; $y^{11} = 40x^3 - 42x$
10) $f^1(x) = 12x^2$; $f^{11}(x) = 24x$
11) $f^1(2) = 94$; $f^{11}(2) = 142$
12) $x < 1$
13) There is a minimum turning point where x = a.
14) (a) $\frac{dN}{dt} > 0, \frac{d^2N}{dt^2} > 0$

(b)

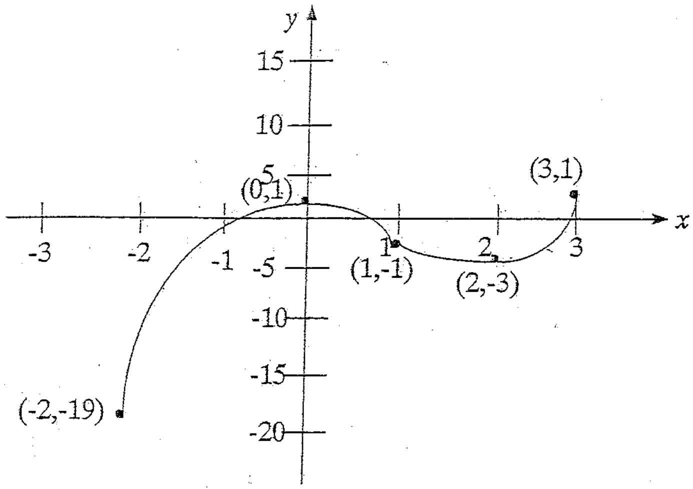


15) $\frac{dP}{dt} > 0, \frac{d^2P}{dt^2} < 0$
16) (0,0); At (0,0) $\frac{d^2y}{dx^2} = 0$. On LHS $\frac{d^2y}{dx^2} < 0$, on RHS $\frac{d^2y}{dx^2} > 0$.
17) (2,-27)
18) (a) (0,1), (2,-3)

(b) (0,1) maximum, (2,-3) minimum

(c) (1,-1)

(d)



(e) -19

19) (a) $x^2y = 100$

So $y = \dfrac{100}{x^2}$

$A = 2x^2 + 4xy$

$= 2x^2 + 4x\dfrac{100}{x^2}$

$= 2x^2 + \dfrac{400}{x}$

(b) 129.3 cm^2

20) (a) $x^2 + y^2 = 50^2$

So $y^2 = 2500 - x^2$

$y = \sqrt{2500 - x^2}$

Now $A = xy$

$= x\sqrt{2500 - x^2}$

(b) 1250 cm^2

21) (a) $\dfrac{p - p^2 + 6}{\sqrt{2}}$

(b) $MN = \sqrt{(3 - -2)^2 + (9 - 4)^2} = \sqrt{50}$

$A = \dfrac{1}{2}bh$

$= \dfrac{1}{2}\sqrt{50} \times \dfrac{p - p^2 + 6}{\sqrt{2}}$

$= \dfrac{5(p - p^2 + 6)}{2}$

(c) 15.625 $units^2$

22) $x^2 - 3x + C$

23) $x^5 + \dfrac{x^4}{2} - \dfrac{x^3}{3} + x + C$

24) $y = 36$

25) $f(1) = 5$

26) $y = 2x^3 - 6x^2 + 8x - 3$

27) (a) $N = (2,9)$ (b) $2x^2 - 3x + 7$