St Catherine's School

Year: 11
Subject: Mathematics
Time allowed: 55 minutes

Assessment Task No: 3
Date: June 2005

Student Name: _____

Teacher's Name: _____ _____

Directions to candidates:

- Answer all questions on the writing paper provided
- Start a new page for each question
- All questions are to be attempted.
- Marks may be deducted for careless or badly arranged work
- All necessary **working** must be shown in every question.

TEACHERS' USE ONLY

Q 1	Total	**/15**
Q 2	Total	**/12**
Q 3 - 4	Total	**/15**
	TOTAL	**/42**

Question 1 Start a new page **Marks**

(a) A(2,−2), B(−2,−3) and C(0,2) are the vertices of a triangle ABC.

(i) Draw a neat sketch of the triangle

(ii) Find the length of AC

(iii) Find the gradient of AC

(iv) Show that the equation of AC is $2x + y - 2 = 0$

(v) Find the shortest distance from the point B to the line AC.

(vi) Find the area of ΔABC

(vii) Find the equation of the median BF (the line joining B and F, the midpoint of AC)

(xiii) Find the coordinates of D if ABCD is a parallelogram /12

(b) Find the equation of the line through the intersection of the lines $x + y - 2 = 0$ and $2x - y - 1 = 0$ and the point $(-1,5)$ without finding the point of intersection of the lines. /3

Question 2 Start a new page

(a) If $f(x) = 2^{x-4}$

(i) evaluate $f(1)$

(ii) find the value of x if $f(x) = 1$ /3

(b) (i) Sketch the graph of the following function

$$f(x) = \begin{cases} x + 3 & \text{if } x \le 3 \\ 3 & \text{if } x > 3 \end{cases}$$

(ii) Evaluate $f(-2) + f(3) + f(6)$ /3

(c) (i) Sketch the graph of $y = |x| + 1$ showing all essential features.

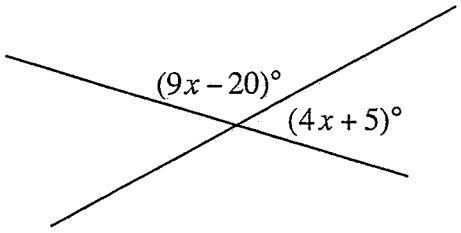
(ii) Shade the region defined by $y \le |x| + 1$ /3

(d) (i) What is the domain **and** range of $y = 2^x$?

(ii) What is the domain of $y = \dfrac{1}{x+1}$? /3

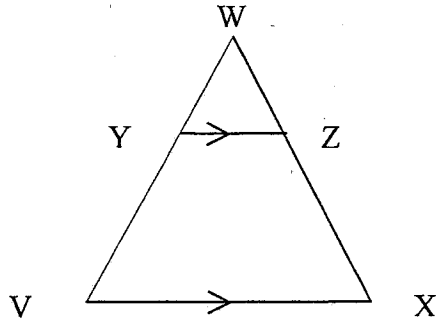
Question 3 Start a new page

(a) Find the value of x
Give reasons.



/2

(b)



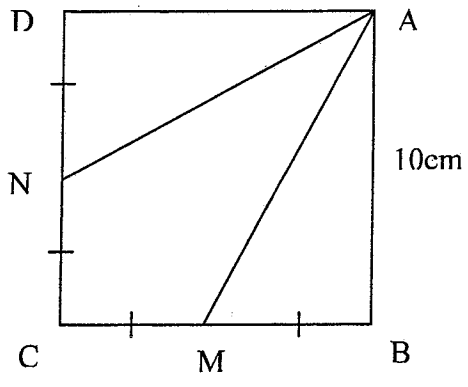
In the triangle WXV, YZ=9cm, VX=12cm, WX=8cm and YZ ||VX.

(i) Copy the diagram and add the given information onto your page

(ii) Prove that $\Delta WZY \,|||\, \Delta WXV$

(iii) Find the length of WZ /5

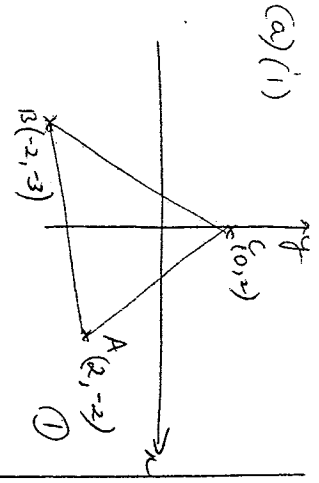
(c) ABCD is a square of side length 10cm. M is the midpoint of BC and N is the midpoint of CD.



(i) Prove that $\Delta ABM \equiv \Delta ADN$.

(ii) Find the area of the quadrilateral AMCN. Give reasons /5

1(a)(i)



(1)

(ii) $AC = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$

$= \sqrt{(2-0)^2+(-2-2)^2}$

$= \sqrt{4+16}$

$= \sqrt{20}$

$= 2\sqrt{5}$ (1)

(iii) $m = \frac{y_2-y_1}{x_2-x_1}$

$= \frac{2--2}{0-2}$

$= \frac{4}{-2}$

$= -2$ (1)

iv) $y-y_1 = m(x-x_1)$

$y-2 = -2(x-0)$

$y-2 = -2x$

$\therefore 2x+y-2=0$ (2)

(v) $d = \left|\frac{ax_1+by_1+c}{\sqrt{a^2+b^2}}\right|$

$a=2 \quad b=1 \quad c=-2$

$x_1=-2, \quad y_1=-3$

$d = \left|\frac{2x-2+1x-3-2}{\sqrt{2^2+1^2}}\right|$

$= \left|\frac{-4-3-2}{\sqrt{5}}\right|$

$= \left|\frac{-9}{\sqrt{5}}\right|$

$= \frac{9}{\sqrt{5}}$

$= \frac{9\sqrt{5}}{5}$ (2)

(vi) $A = \frac{1}{2} \times \text{base} \times \text{height}$

$= \frac{1}{2} \times 2\sqrt{5} \times \frac{9\sqrt{5}}{5}$

$= 9$ sq units (1)

(vii) midpoint AC

$= \left(\frac{0+2}{2}, \frac{2+-2}{2}\right)$

$= (1, 0)$

eqn BF $\Rightarrow \frac{y-y_1}{x_2-x_1} = \frac{y-y_1}{x-x_1}$

$\frac{0--3}{1--2} = \frac{y--3}{x--2}$

$\frac{3}{3} = \frac{y+3}{x+2}$

$x+2 = y+3$

$x-y-1=0$ (3)

1(a)(viii) $BF = DF$

$\therefore F$ is midpt of BD

$(1,0) = \left(\frac{x_1+-2}{2}, \frac{y_1+-3}{2}\right)$

$(1,0) = \left(\frac{x_1-2}{2}, \frac{y_1-3}{2}\right)$

$\frac{x_1-2}{2} = 1 \qquad \frac{y_1-3}{2} = 0$

$x_1 = 4 \qquad y = 3$

$\therefore D$ is $(4,3)$ (1)

1(b)

$x+y-2+k(2x-y-1)=0$

sub $(-1,5)$

$-1+5-2+k(2x-1-5-1)=0$

$2-8k=0$

$\therefore k = \frac{1}{4}$

$x+y-2+\frac{1}{4}(2x-y-1)=0$

$4x+4y-8+2x-y-1=0$

$6x+3y-9=0$

$\therefore 2x+y-3=0$ (3)

Q2

(a) If $f(x) = 2^{x-4}$

(i) $f(1) = 2^{1-4}$

$= 2^{-3}$ ✓

$= \frac{1}{2^3}$ ✓ or

$= \frac{1}{8}$ /1

(ii) $f(x) = 1$

i.e. $2^{x-4} = 1$

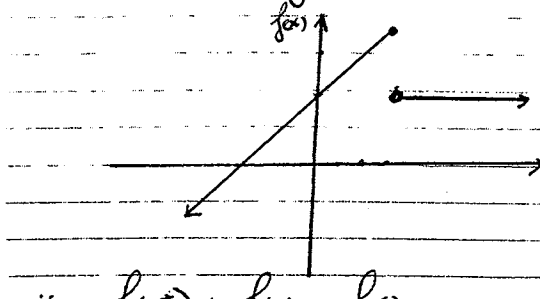
1 mark for showing that $2^0 = 1$ i.e. $x-4=0$

$x-4 = 0$

$x = 4$ /2

(b) (i)

$f(x) = \begin{cases} x+3 & \text{if } x \le 3 \\ 3 & \text{if } x > 3 \end{cases}$



½ mark for ● and ○

1 mark for both parts of the graph

(ii) $f(-2) + f(3) + f(6)$

$= (-2+3) + (3+3) + 3$

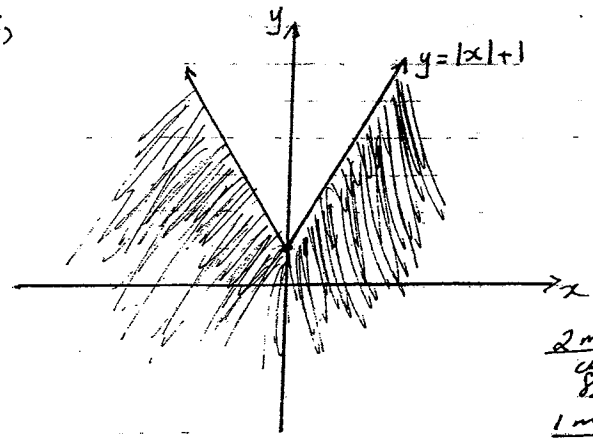
$= 1 + 6 + 3$

$= 10$

½ mark for each correct answer

/3

(c) (i)



2 marks for correct graph
1 mark for correct shading of the region.

(ii) $y \leq |x| + 1$

test (0,0)

$0 \leq |0| + 1$

$0 \leq 1$ TRUE ∴ include (0,0) in the shading.

/3

(d) (i) $y = 2^x$

D: For all real x /1

R: For all real y, $y > 0$ /1

(ii) $y = \frac{1}{x+1}$

D: For all real x, $x \neq -1$ /1

/3

3. (a) $9x - 20 + 4x + 5 = 180$ ✗

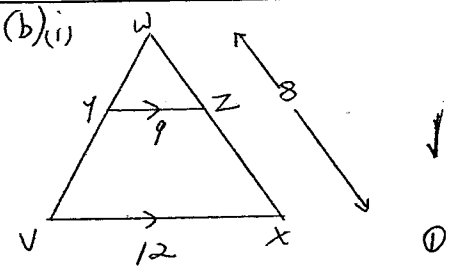
(Straight angle) ✗

$13x = 195$

$x = 15$ ✓

/2

(b) (i)



①

(ii) In $\triangle WZY$ and $\triangle WXV$

$\angle W$ is common ✓

$\angle WYZ = \angle WVX$ ✗ (corresponding angles on parallel lines ✗ YZ ∥ VX)

$\angle WZY = \angle WXV$ (as above)

$\triangle WZY \,|||\, \triangle WXV$ (equiangular) ✗ ②

(iii) $\frac{WZ}{8} = \frac{9}{12}$ ✗ equal ratio of similar triangles ✗ ②

$WZ = \frac{9}{12} \times 8$

$= 6$ cm ✓

/5

(c) (i) In $\triangle ABM$ and $\triangle ADN$

$AD = AB$ (equal sides of a square)

$\angle ADN = \angle ABM$ (angles of a square)

$DN = BM$ (given)

$\therefore \triangle ABM \equiv \triangle ADN$ (SAS) (3M)

(ii) area $\triangle ABM = \frac{1}{2} \times 5 \times 10$

$= 25$

area $\triangle ADN = 25$

area of square $ABCD = 100$

$\therefore$ AMCN $= 100 - 25 - 25$

$= 50$ cm^2 (2M)

(4) $\frac{EC}{FC} = \frac{3}{5}$

Let $EC = 3x$ ∴ $FC = 5x$

$\therefore BC = 3 \times EC$

$= 3 \times 3x$

$= 9x$

$\frac{DE}{EC} = \frac{AF}{FC}$ (intercepts cut by a family of ∥ lines are equal)

$\frac{3x}{3x} = \frac{AF}{5x}$

$\therefore AF = 5x$

now $AC = AF + FC$

$= 5 + 5$

$= 10x$

$\therefore \frac{AC}{BC} = \frac{10x}{9x}$

$\therefore AC = \frac{10}{9} BC$.

(3M)