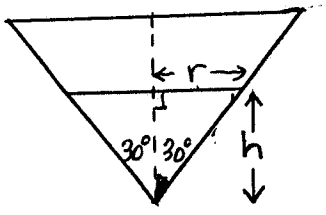
1. Differentiate the following with respect to x:
   (a) $2x^5 - 8x^4 + 9x - 11$,
   (b) $(4 - 5x)^{12}$,
   (c) $\dfrac{x^2 + 3}{7x - 4}$.

2. (a) State the product rule for differentiation.
   (b) Use the product rule to differentiate the function $y = 4x^2(3x - 5)^7$, writing your answer in factored form.
   (c) Write down the values of x for which the tangent is horizontal.

3. Find the equation of the tangent to the curve $y = x\sqrt{x}$ at the point $(4, 8)$.

4. For the curve $y = \dfrac{1}{2x^2}$,
   (a) find the gradient of the normal at the point (a,b),
   (b) if the gradient of the normal at (a,b) is 8, find the values of a and b.

5. A spherical balloon is being inflated so that its radius is increasing at a constant rate of 1·5 cm/s.
   (a) Find the rate at which the volume is increasing when the radius 8 cm.
   (b) Find the rate at which the surface area is of the balloon is increasing when the volume is 36π cm^3.

6. 

The diagram shows a conical vessel with sem-vertical angle 30°. The vessel is being filled with water at the rate of 5 cm^3/min, and at the same time water is leaking from a hole in the vessel at a rate of $\dfrac{\sqrt{h}}{2}$ cm^3/min, where h cm is the height of the water at time t.

(a) If r cm is the radius of the water surface at height h cm, show that the volume V cm^3 is given by $V = \dfrac{\pi h^3}{9}$.

(b) Find the rate at which the height of the water is increasing when $h = 4$.

(c) Find the rate at which the circumference of the water surface is increasing when $h = 4$.

Ningci Ho

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1a. $f(x) = 2x^5 - 8x^4 + 9x - 11$

$f'(x) = 10x^4 - 32x^3 + 9$

b. $(4-5x)^{11} = f(x)$

$f'(x) = -60(4-5x)^{10}$

c. ~~$f(x) = \frac{x^2+3}{7x-4}$~~

~~$\frac{dy}{dx} = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2} = \frac{(7x-4)(2x) - (x^2+3)(7)}{(7x-4)^2} = \frac{14x^2 - 8x - 7x^2 - 21}{(7x-4)^2}$~~

~~$= \frac{7x^2 - 8x - 21}{(7x-4)^2}$~~

c. $f(x) = (x^2+3)(7x-4)^{-1}$

$\frac{dy}{dx} = v\frac{du}{dx} + u\frac{dv}{dx} = (7x-4)^{-1} \times 2x + (x^2+3) \times -7(7x-4)^{-2}$

~~$= 2x(7x-4)^{-1} + (-7x^2-21)(7x-4)^{-1}(7x-4)^{-1}$~~

~~$= [2x + (-7x^2-21)(7x-4)^{-1}][7x-4]^{-1}$~~

$= \frac{2x}{7x-4} + \frac{-7(x^2+3)}{(7x-4)^2} = \frac{2x(7x-4)}{(7x-4)^2} + \frac{-7x^2-21}{(7x-4)^2}$

$= \frac{14x^2 - 8x - 7x^2 - 21}{(7x-4)^2}$

$= \frac{7x^2 - 8x - 21}{(7x-4)^2}$

4

2a. $\frac{dy}{dx} = v\frac{du}{dx} + u\frac{dv}{dx}$

b. $y = 4x^2(3x-5)^7$

Let $4x^2 = u$ and $(3x-5)^7 = v$

$\frac{dy}{dx} = v \times \frac{du}{dx} + u\frac{dv}{dx} = (3x-5)^7 \times 8x + (4x^2) \times 21(3x-5)^6$

6

$= (3x-5)^6 \times (3x-5) \times 8x + 84x^2(3x-5)^6$

$= (24x^2 - 40x + 84x^2)(3x-5)^6$

$= (108x^2 - 40x)(3x-5)^6$

$= 4x(27x - 10)(3x-5)^6$.

~~c. $f'(x) = 0$.~~

c. when $x = 0$, $\frac{10}{27}$ or $\frac{5}{3}$, $f'(x) = 0$.

3. $y = x\sqrt{x} = x(x)^{\frac{1}{2}} = x^{\frac{3}{2}}$

$f'(x) = \frac{3}{2}x^{\frac{1}{2}}$

$f'(4) = \frac{3}{2}(4)^{\frac{1}{2}} = \frac{3}{2} \times \pm 2$

$= 3$ or -3.

NB $\sqrt{4} = 2$

$-\sqrt{4} = -2$

X

2

Eqn 1: $y - 8 = 3(x-4)$

$y = 3x - 12 + 8$

$y = 3x - 4$

Eqn 2: $y - 8 = -3(x-4)$

$y = -3x + 12 + 8$

$3x + y = 20$.

4a. $y = \frac{1}{2x^2} = (2x^2)^{-1}$

$f'(x) = -(2x^2)^{-2} \times 4x$

$= -\frac{4x}{4x^4} = -\frac{1}{x^3}$

$f'(a) = -\frac{1}{a^3}$

m. of normal: $m \times -\frac{1}{a^3} = -1$

$m = a^3$ #

b. $m = a^3$

$8 = a^3$

$a = 2$ *

$f(x) = \frac{1}{2x^2}$

$f(a) = \frac{1}{2a^2}$

$f(2) = \frac{1}{2\times4} = \frac{1}{8}$

$b = \frac{1}{8}$ #.

4

5a. $V = \frac{4}{3}\pi r^3$

$\frac{dV}{dt} = \frac{dV}{dr} \times \frac{dr}{dt}$

$= 4\pi r^2 \times 1.5$

$= 4\pi \times 8^2 \times 1.5$

$= 384\pi \text{ cm}^3/\text{s}$.

b. $A = 4\pi r^2$

$V = \frac{4}{3}\pi r^3 = 36\pi$

$r^3 = 27$

$r = 3$ cm.

$\frac{dA}{dt} = \frac{dA}{dr} \times \frac{dr}{dt} = 8\pi r \times 1.5$

$= 8 \times \pi \times 3 \times 1.5$

$= 36\pi \text{ cm}^2/\text{s}$.

7

6a. $\tan 30° = \frac{r}{h}$

$\frac{1}{\sqrt{3}} = \frac{r}{h}$

$r = \frac{h}{\sqrt{3}}$ cm

$V = \frac{1}{3}\pi r^2 h = \frac{1}{3} \times \pi \times \left(\frac{h}{\sqrt{3}}\right)^2 \times h$

$= \frac{1}{3} \times \pi \times \frac{h^2}{3} \times h$

$= \frac{\pi h^3}{9}$

b. $\frac{dV}{dt} = \frac{dV}{dh} \times \frac{dh}{dt}$

$5 = \left[\frac{\pi h^2}{3} - \frac{\sqrt{16}}{2}\right] \times \frac{dh}{dt}$

$5 = \left[\frac{\pi \times 16}{3} - \frac{\sqrt{16}}{2}\right] \times \frac{dh}{dt}$

$5 = \left(\frac{16\pi}{3} - 2\right) \times \frac{dh}{dt}$

$5 \div \frac{16\pi - 6}{3} = \frac{dh}{dt}$

$\frac{dh}{dt} = \frac{15}{16\pi - 6}$ cm/s. $= 0.339$ cm/s (to 3 s.f.)

2

c. Circumference $= 2\pi r$

$\frac{dV}{dt} = \frac{dV}{dr} \times \frac{dr}{dt}$

$5 = \frac{3}{3}\pi h r \times$

b. $\frac{dV}{dt} = 5 - \frac{\sqrt{\pi}}{2}$

when $h = 4$, $\frac{dV}{dt} = 5 - 1 = 4 \text{ cm}^3/\text{min}$

$\frac{dV}{dt} = \frac{dV}{dh} \times \frac{dh}{dt}$

$\frac{dV}{dh} = \frac{\pi h^2}{3}$ → when $h = 4$, $\frac{dV}{dh} = \frac{\pi \times 4^2}{3}$

$\frac{dh}{dt} = \frac{1}{\frac{16\pi}{3}} = \frac{3}{4\pi}$ cm/min

c. $\frac{dc}{dt} = \frac{dc}{dh} \times \frac{dh}{dt}$

$c = 2\pi r = \frac{2\pi h}{\sqrt{3}}$

$\frac{dc}{dh} = \frac{2\pi}{\sqrt{3}}$

$\frac{dc}{dt} = \frac{3}{4\pi} \times \frac{2\pi}{\sqrt{3}} = \frac{\sqrt{3}}{2}$ cm/min.