1. (a) Use a graph to solve $x^2 + 4x < -3$.
   (b) For what values of k is the quadratic function $x^2 + 2x + 6k = 0$ positive definite?
   (c) A stone is thrown upwards so that at any time t seconds after throwing, the height of the stone is $h = 100 + 10t - 5t^2$ metres. Find the maximum height reached.
   (d) For what value(s) of b is the line $y = x + b$ a tangent to the curve $y = 2x^2 - 7x + 4$?

2. For what value(s) of k will the equation $x^2 - kx + (k - 1) = 0$ have: when .
   (a) one root equal to zero,
   (b) roots which are reciprocals of each other,
   (c) roots which are opposites of each other,
   (d) exactly one root.

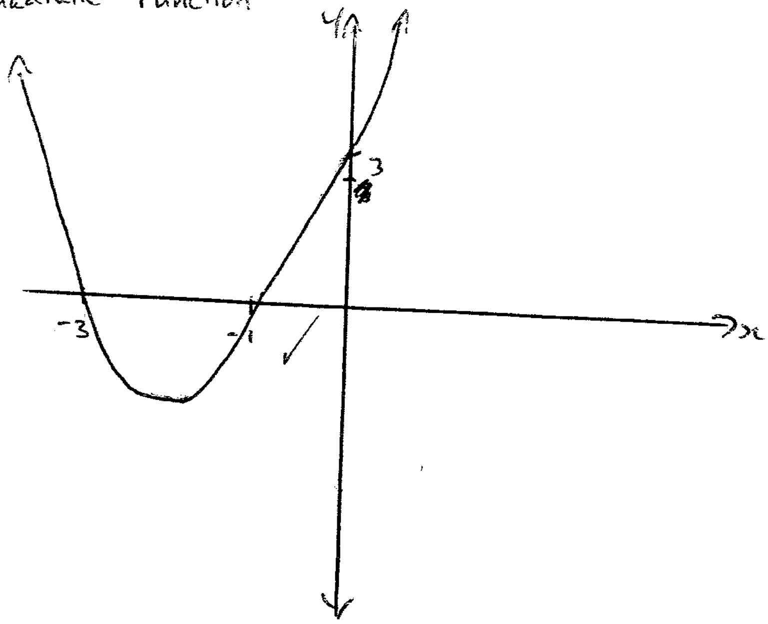
3. (a) Solve $3x = \sqrt{x} + 2$.
   (b) By substituting $u = x - \dfrac{6}{x}$, solve $\left(x - \dfrac{6}{x}\right)^2 - 6\left(x - \dfrac{6}{x}\right) + 5 = 0$.

4. (a) If α and β are the roots of $2x^2 - 5x + 1 = 0$, without solving the equation, find the values of:
      (i) $\alpha + \beta$
      (ii) $\alpha\beta$
      (iii) $(\alpha - 1)(\beta - 1)$
      (iv) $\alpha^2 + \beta^2$
      (v) $\dfrac{1}{\alpha} + \dfrac{1}{\beta}$
   (b) If α and β are the roots of $2x^2 - 5x + 1 = 0$, form the equation with integer coefficients having roots $\dfrac{1}{\alpha}$ and $\dfrac{1}{\beta}$.

5. (a) Find a, b and c so that:

$$ax(x - 1) + b(x - 2) + c = x^2 + 2x + 2, \text{ for all } x \in \mathbf{R}.$$

Quadratic Function



1a) $x^2+4x<-3$

$= x^2+4x+3<0$

$= (x+1)(x+3)$ ✓

x^2+4x+3

$x^2+3x+x+3$

$x(x+3)+1(x+3)$

$(x+1)(x+3)$

$-3<x<-1$

b) $x^2+2x+6k=0$

pos. def = no roots

$\cup$ concave up

so. ~~Δ~~

$\Delta = b^2-4ac$

$= 2^2 - 4\times 6k$

$4-24k$ ✓

so $\Delta<0$ for pos. def.

$4-24k<0$

~~$4<24k$~~

$-24k<-4$

$24k>4$

$k>\frac{1}{6}$.

c) Max value concave down $\cap$

axis of sym. $\frac{-b}{2a}$

so. $h=100+10t-5t^2$

$t=\frac{-b}{2a} = \frac{-10}{-5\times 2}$

$= \frac{-10}{-10} = 1$ ✓

$\therefore$ max height ~~= tm~~ $h_{max} = 100+10(1)-5(1)$

$= 105m.$

d) $y = x+b \qquad y = 2x^2 - 7x + 4.$

for tangent $\Delta = 0$, 1 point of intersection ✓

$ax^2 + bx + c = 0$

$\Delta = b^2 - 4ac$

so $x+b = 2x^2 - 7x + 4$

$0 = 2x^2 - 8x + (4+b)$

$\uparrow a \qquad \uparrow b \qquad c$

$\Delta = b^2 - 4ac$

$= (-8)^2 - 4 \times 2 \times (4-b)$

$= 64 - 8(4-b)$

$= 64 - 32 + 8b$

$= 32 + 8b$ ✓

$\Delta = 0$

$32 + 8b = 0$

$8b = -32$ ✓

$b = -4.$

2. a) $x^2 - kx + (k-1) = 0$

let $x = 0$

$0 - 0 + k - 1 = 0$

$k - 1 = 0$

$k = 1.$ ✓

b) let the roots be α and $\frac{1}{\alpha}$

$\alpha + \frac{1}{\alpha} = \frac{-b}{a}$

$\alpha + \frac{1}{\alpha} = \frac{k}{1}$

$\alpha \times \frac{1}{\alpha} = k - 1$

$1 = k - 1$ ✓

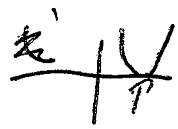
$k = 2,$

c). let the roots be α and $-\alpha$

$\alpha + -\alpha = \frac{-b}{a} = k$ ✓

$k = 0$ ✓

d.) for 1 root



so $\Delta = 0.$

$\Delta = b^2 - 4ac$

$(-k)^2 - 4(k-1)$

$k^2 - 4k + 4 = 0$

$(k-2)^2 = 0$

$k = 2$ ✓

3a) $3x = \sqrt{x} + 2$

$\cancel{9x^2 = x + 4}$ $\quad \cancel{(3x)^2} = \cancel{(\sqrt{x}+2)^2}$.

$\cancel{9x^2 - x - 4 = 0}$ $\quad \cancel{9x^2 = (\sqrt{x}+2)(\sqrt{x}+2)}$.

$\cancel{\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}}$ $\quad \cancel{9x^2 = x + 4\sqrt{x} + 4}$

$3x - 2 = \sqrt{x}$

$1 \pm \sqrt{1 - 4 \times 9 \times -4}$.

$(3x-2)^2 = x$

$(3x-2)(3x-2) = x$

$9x^2 - 12x + 4 = x$

$9x^2 - 13x + 4 = 0$ ✓

$9x^2 - 9x - 4x + 4 = 0$

$9x(x-1) - 4(x-1)$

$(9x-4)(x-1) = 0$ ✓

$x = 1$ or $\frac{4}{9}$.

b) let $u = x - \frac{6}{x}$

$u^2 - 6(u) + 5 = 0$

$u^2 - 6u + 5 = 0$

$(u-5)(u-1) = 0$ ✓

$x - \frac{6}{x} = 5$ $\qquad x - \frac{6}{x} = 1$

$= x^2 - 6 = 5x$ $\qquad x^2 - 6 = x$

$x^2 - 5x - 6 = 0$ $\qquad x^2 - x - 6 = 0$

$(x-6)(x+1)$ ✓ $\qquad (x+2)(x-3)$

$x = 6$ or -1 ✓ $\qquad x = -2$ or 3 ✓

so $x = 6, -2, -1$ or 3 ✓

$2x^2 + 5x + 1 = 0$

4 a) i) $\alpha + \beta = \frac{-b}{a}$.

$= \frac{-5}{2}$

$= \frac{5}{2}$ ✓

$= 2\frac{1}{2}$

ii) $\alpha\beta = \frac{c}{a}$

$= \frac{1}{2}$ ✓

iii) $(\alpha - 1)(\beta - 1)$

$= \alpha\beta - \beta - \alpha + 1$ ✓

$= \alpha\beta - (\beta + \alpha) + 1$

$\frac{1}{2} - 2\frac{1}{2} + 1 \quad = \underline{-1}$

$= 2\frac{1}{2} - \frac{1}{2} + 1$

$= 3.$ ✗

iv) $\alpha^2 + \beta^2$

$\cancel{= (\alpha+\beta)^2}$ $= (\alpha+\beta)^2$

$= (\alpha+\beta)(\alpha+\beta).$

$(\alpha+\beta)^2 = \alpha^2 + \alpha\beta + \alpha\beta + \beta^2 \Rightarrow$ $(\alpha+\beta)^2 - 2\alpha\beta = \alpha^2 + \beta^2$

$= \alpha^2 + \beta^2 + 2\alpha\beta$

$\alpha^2 + \beta^2 = (\alpha+\beta)^2 - 2\alpha\beta$

$= (2\frac{1}{2})^2 - 2 \times \frac{1}{2}$

$= 6\frac{1}{4} - 1$

$= 5\frac{1}{4}.$

v). $\frac{1}{\alpha} + \frac{1}{\beta} \quad = \frac{\beta + \alpha}{\alpha\beta}$

$= \alpha^{-1} + \beta^{-1} \quad = \frac{2\frac{1}{2}}{\frac{1}{2}}$

$= (\alpha + \beta)^{-1}$ ✗

$= (2\frac{1}{2})^{-1} \quad = 5.$ ✓

$= \frac{2}{5}.$

4. b)

$$\frac{1}{\alpha}+\frac{1}{\beta}=\frac{a}{-b}$$

$$2x^2-5x+1=0$$

$$5=\frac{1}{\alpha}+\frac{1}{\beta}$$

$$1=(\alpha\beta)$$

$$\frac{1}{\alpha\beta}=2$$

$$= \quad x^2-(5)x+2=0.$$

5a). $ax(x-1)+b(x-2)+c=x^2+2x+2$ for all real values of x

ie $x>0$.

$$ax^2-ax+bx-2b+c=x^2+2x+2$$

$$\cancel{a(x^2-x)+b(x-2)+c}=x^2+2x+2$$

$$ax^2+(b-a)x+(c-2b)\equiv 1x^2+2x+2$$

so $a=1$

$b-a=2$

$b-1=2$

$b=3$ ✓

$c-2b=2$

$c-6=2$

$c=8$ ✓