



FORM VI

MATHEMATICS

Examination date

Monday 16th May 2005

Time allowed

2 hours

Instructions

- All eight questions may be attempted.
- All eight questions are of equal value.
- All necessary working must be shown.
- Marks may not be awarded for careless or badly arranged work.
- Approved calculators and templates may be used.

Collection

- Write your name, class and master clearly on the front.
- Hand in all the writing paper in a single well-stapled bundle.
- Keep the printed examination paper and bring it to your next Mathematics lesson.

4A: TCW	4B: KWM	4C: JCM
4D: DS	4E: JMR	4F: REP
4G: FMW	4H: JNC	4I: GJ

Checklist

- Writing paper required.
- Candidature: 192 boys.

Examiner

JNC

QUESTION ONE Start a new page.

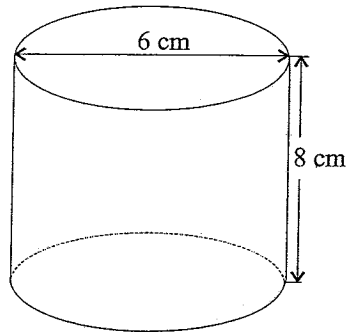
- Expand and simplify $(2x + 1)(x - 3)$.
- Write down the volume of a cube of edge 8 cm.
- Write down the gradient of the line $y = 1 - 2x$.
- Factorise $x^2 + 3x - 10$.
- Find the simple interest earned if \$5000 is invested at 4% p.a. for 5 years.
- Find the midpoint of the interval joining $A(-3, 8)$ and $B(-5, -1)$.
- What is the probability of choosing the ace of hearts from a normal deck of 52 playing cards?
- Expand and simplify $(x - 4)^2$.
- Write down the coordinates of the centre and the length of the radius of the circle with equation $(x - 2)^2 + (y + 4)^2 = 1$.
- Write down the solutions of the quadratic equation $2x(x + 5) = 0$.

QUESTION TWO Start a new page.

- Write down the equation of the line with gradient 3 and y -intercept $(0, -6)$.
- Max buys a stereo system with a marked price of \$2135. He pays for it by making 24 equal monthly payments of \$117.45.
 - Calculate the total amount that Max pays for the stereo.
 - How much interest did Max pay?
 - Find the annual rate of interest as a percentage of the original price. Give your answer correct to the nearest whole percent.
- Solve the following quadratic equations by the method specified. In each case leave your answers as exact values.
 - Solve $x^2 - 3x - 28 = 0$ by factorising.
 - Solve $2x^2 + x - 13 = 0$ by the quadratic formula.
 - Solve $x^2 + 6x + 8 = 0$ by completing the square.

QUESTION THREE Start a new page.

(a)



In the diagram above, the cylindrical can has a diameter of 6 cm and height of 8 cm.

- (i) Find the area of the curved surface of the can. Give your answer correct to the nearest square centimetre.
- (ii) Find the volume of the can. Give your answer correct to the nearest cubic centimetre.

- (b) (i) Copy and complete the table below of values for the graph of $y = \frac{2}{x}$.

x	-4	-2	-1	$-\frac{1}{2}$	$\frac{1}{2}$	1	2	4
y								

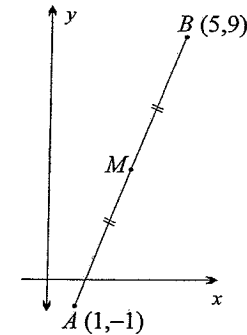
- (ii) Write down the equation of the horizontal asymptote.
 - (iii) Sketch the graph of $y = \frac{2}{x}$ using a scale of 1 unit = 1 cm on both axes.
- (c) In a raffle, 100 people bought 3 tickets each, 30 people bought 2 tickets each and 15 people bought 1 ticket each.
- (i) If Stan was one of those who bought 1 ticket, what is the probability that he wins the raffle?
 - (ii) What is the probability that someone who bought 2 tickets wins the raffle?
- (d) Write down the equation of the axis of symmetry of the parabola $y = (x - a)^2$.

QUESTION FOUR Start a new page.

- (a) Find the gradient of any line that is parallel to the line $3x + 2y - 12 = 0$.
- (b) Find in simplest form the gradient of the interval joining $A(2a, -2b)$ and $B(4b, -a)$.
- (c) Alan bought a new car valued at \$50 000. It depreciates in value at the rate of 18% per annum.
 - (i) What is the car worth after 2 years?
 - (ii) How much value will the car have lost after 4 years? Give your answer correct to the nearest dollar.
- (d) Consider the parabola $y = x^2 - 2x - 8$.
 - (i) Find the x -intercepts of the parabola.
 - (ii) Find the coordinates of the vertex of the parabola.
 - (iii) Sketch the graph of $y = x^2 - 2x - 8$.
 - (iv) What is the maximum value of the expression $8 + 2x - x^2$?

QUESTION FIVE Start a new page.

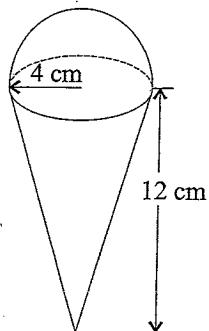
(a)



In the diagram above, M is the midpoint of the interval joining $A(1, -1)$ and $B(5, 9)$.

- (i) Show that the length of the interval AB is $2\sqrt{29}$.
- (ii) Write down the coordinates of M .
- (iii) Write down the equation of the circle with diameter AB .
- (iv) Find the gradient of AB .
- (v) Find the equation of the line that is parallel to AB and passes through the origin. Give your answer in general form.

(b)



In the diagram above, a hemi-spherical blob of ice cream of radius 4 centimetres is placed on top of a waffle cone of height 12 centimetres.

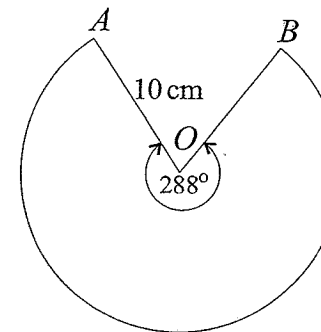
- (i) Find the volume of ice cream. Give your answer correct to the nearest cubic centimetre.
- (ii) The ice cream melts into the cone and the liquid has the same volume as the solid blob of ice cream. It fills the cone to a depth of H centimetres, and has a surface of radius R centimetres.

Given that $\frac{H}{R} = 3$, find the depth of the melted ice cream, giving your answer correct to the nearest millimetre.

QUESTION SIX Start a new page.

- (a) A different letter of the alphabet was written on each of 26 cards. One of these cards is then drawn at random. What is the probability that the card drawn is:
 - (i) a letter of the word *mathematics*?
 - (ii) either a letter of the word *trick* or a letter of the word *quick*?
- (b)
 - (i) If x is an odd integer, write down an expression for the next odd integer.
 - (ii) Form a quadratic equation and solve it to find two consecutive odd positive integers such that the sum of their squares is 394.

(c)



The diagram above shows a sector of a circle with centre O , radius 10 centimetres and an angle of 288° at the centre.

- (i) Show that the length of major arc AB is 16π centimetres.
- (ii) The sector is cut out and the edges OA and OB are joined so that a cone is formed. Show that the radius of the base of the cone is 8 centimetres.
- (iii) Find the exact volume of the cone in part (ii).

QUESTION SEVEN Start a new page.

- (a) A shell is fired from ground level. After t seconds, its height h in metres is given by $h = 50t - 5t^2$.
 - (i) How long is the shell in the air?
 - (ii) What is the maximum height reached?
 - (iii) Draw a graph of the parabola $h = 50t - 5t^2$, for $0 \leq t \leq 10$.
 - (iv) A second, similar shell is fired from the edge of a cliff, which is 375 metres above sea-level. How long will it take for the shell to hit the water after it is fired?
- (b) A cylinder has a radius of r centimetres and a height of h centimetres.
 - (i) Find the volume, in terms of r and h , of the cylinder if its radius is increased by 25% and the height is increased by 10%.
 - (ii) By what percentage has the volume of the cylinder increased?

QUESTION EIGHT Start a new page.

(a) (i) Simplify $\frac{1}{n} - \frac{1}{n+1}$.

(ii) Find the sum of the series $\frac{1}{1 \times 2} + \frac{1}{2 \times 3} + \frac{1}{3 \times 4} + \dots + \frac{1}{100 \times 101}$.

(b) An office has two envelope-filling machines. Working together, they can fill a batch of envelopes in 2 hours. Working separately, it will take the second machine 1 hour longer than the first machine to fill a batch of envelopes. How long will it take each machine to fill a batch of envelopes by themselves? Give your answer correct to the nearest minute.

(c) Michael invested a sum of money \$ P on January 1st, 1975. The account received interest, which was compounded annually. On January 1st, 1985 there was \$44 771.20 in the account and on January 1st 2005 the amount of money in the account had grown to \$143 587.28.

(i) Find the rate of interest that Michael received on his investment. Give your answer correct to the nearest whole percent per annum.

(ii) Find the amount \$ P that Michael invested in 1975.

(iii) In what year will the account have more than \$250 000 in it?

END OF EXAMINATION

Syd Grammar - Form ⁶ 2005 $\frac{1}{2}$ yrly

Question 1

a) $(2x+1)(x-3)$
 $= 2x^2 - 6x + x - 3$
 $= 2x^2 - 5x - 3$ ✓

b) $V = s^3$
 $= 8^3$

$\therefore \text{Volume} = 512 \text{ cm}^3$ ✓

c) $m = -2$ ✓

d) $(x+5)(x-2)$ ✓

e) $I = PRT$
 $= 5000 \times 0.04 \times 5$
 $= \$1000$ ✓

$\therefore \text{Simple Interest earned} = \1000

f) $M \left(\frac{-3-5}{2}, \frac{8-1}{2} \right)$ ✓

$= M \left(-4, \frac{7}{2} \right)$ ✓

g) $x = \frac{1}{52}$ ✓

h) $(x-4)^2 = 0$
 $= x^2 - 8x + 16$ ✓

i) centre $(2, -4)$ ✓
 radius = 1 units ✓

j) $2x(x+5) = 0$
 $\therefore x = 0, -5$ ✓

Question 2

a) $y = 3x - 6$ ✓

b) x

c) i) $x^2 - 3x - 28 = 0$
 $0 = (x-7)(x+4)$
 $x = 7, -4$ ✓

ii) $2x^2 + x - 13 = 0$

$a = 2, b = 1, c = -13$

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$= \frac{-1 \pm \sqrt{1 - (4 \cdot 2 \cdot -13)}}{4}$

$= \frac{-1 \pm \sqrt{105}}{4}$ ✓

iii) $x^2 + 6x + 8 = 0$

$x^2 + 6x + 8 + 1 - 1 = 0$

$(x+3)^2 - 1 = 0$ ✓

~~$x+3 = 1$~~

$(x+3)^2 = 1$

$x+3 = 1$

$x = -2$ ✓

$x+3 = -1$

$x = -4$ ✓

$\therefore x = -2, -4$

Question 3

a) i) $A = lb$

$L = 2\pi r$

$= 6\pi$

$b = 8$

$\therefore A = 48\pi$ ✓

$= 150.7$ ✓

$\therefore \text{Area} = 151 \text{ (cust cm}^2\text{) cm}^2$

ii) $V = Ah$

$= \pi r^2 h$

$= \pi 3^2 \cdot 8$

$= 226.1$ ✓

$\therefore \text{Volume} = 226 \text{ cm}^3$ ✓ (cust cm³)

b) i)

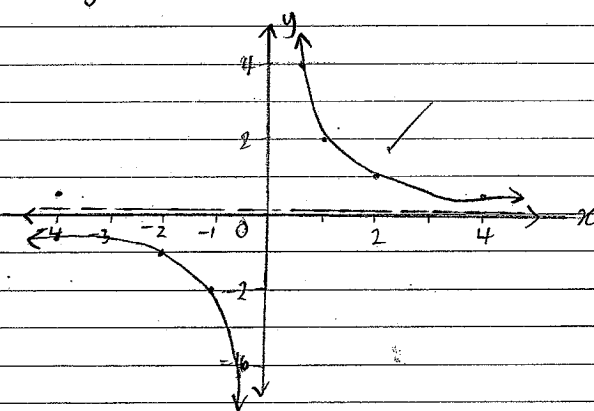
x	-4	-2	-1	$-\frac{1}{2}$	$\frac{1}{2}$	1	2	4
y	$-\frac{1}{2}$	-1	-2	-4	4	2	1	$\frac{1}{2}$

ii) $x \rightarrow \infty, y \rightarrow 0^+$

$x \rightarrow -\infty, y \rightarrow 0^-$ ✓

$\therefore y = 0$ is the HA.

iii)



c) x

d) $y = x^2 - 2ax + a^2$

$a = 1$

axis of symmetry:

$b = -2a$

$c = a^2$

$x = \frac{-b}{2a}$

$= \frac{2a}{2}$

$\therefore x = a$

Question 4.

a) $2y = -3x + 12$

$y = \frac{-3x}{2} + 6$

$\therefore m = -\frac{3}{2}$

\therefore the gradient of any such line is $-\frac{3}{2}$.

b) $m_{AB} = \frac{(-2b) - (-a)}{(2a) - (4b)}$

$= \frac{a - 2b}{2a - 4b}$

$= \frac{a - 2b}{2(a - 2b)}$

$= \frac{1}{2}$

c) x

d) $y = x^2 - 2x - 8$

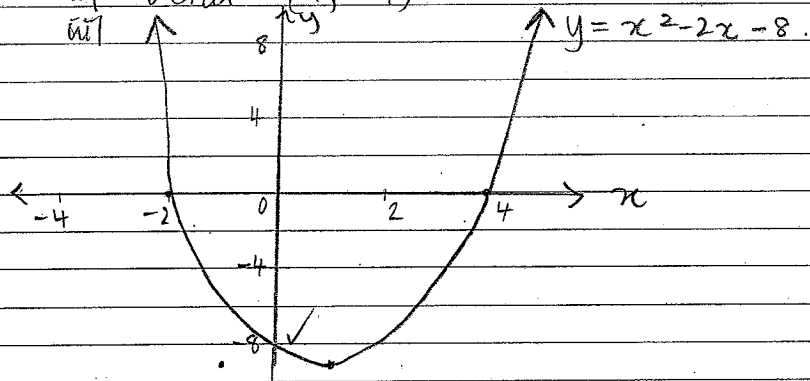
i) x intercepts:

$y = (x - 4)(x + 2)$

$\therefore x = 4, -2$

ii) vertex $(1, -9)$

iii)



iv) $y_{\text{max}} = 9$

Question 5

a) i) $d_{AB} = \sqrt{(5-1)^2 + (9-1)^2}$

$= \sqrt{16 + 100}$

$= \sqrt{116} = \sqrt{4 \times 29}$

$= 2\sqrt{29}$ units.

ii) $M \left(\frac{5+1}{2}, \frac{9+1}{2} \right)$

$= M(3, 4)$

iii) $(x-3)^2 + (y-4)^2 = 29$

iv) $m_{AB} = \frac{9-1}{5-1}$

$= \frac{10}{4}$

$= \frac{5}{2}$

$M y = \frac{5x}{2}$

$2y = 5x$

$0 = 5x - 2y$

b) i) $V = \frac{\pi r^2 h}{3} + \frac{2\pi r^3}{3}$

Voice cream only $= \frac{2\pi r^3}{3}$

$= \frac{\pi \cdot 4^2 \cdot 12}{3} + \frac{4\pi \cdot 4^3}{3}$

$= \frac{2\pi(4)^3}{3}$

$= 134 \text{ cm}^3$

$= 469.1 \dots$

\therefore Volume $\approx 469 \text{ cm}^3$ (not cm^2)

ii) $V = \frac{\pi r^2 h}{3}$

$\frac{134}{3} = \frac{\pi r^2 h}{3} \Rightarrow H = 3R$

$R = \frac{H}{3}$

$\therefore 134 = \pi \left(\frac{H}{3}\right)^2 H$

$= \frac{\pi H^3}{9}$

$\frac{1407}{\pi} = \frac{\pi r^2 h}{3}$

$= \left(\frac{H^2}{9}\right)(3R)$

$H = 3R$

$\therefore H^3 = \frac{9 \times 134}{\pi}$

$\frac{4221}{\pi} = H^2 R$

$R = \frac{H}{3}$

$H = \sqrt[3]{\frac{9 \times 134}{\pi}}$

$\frac{4221}{\pi} = 2R^3 H^2 \frac{H}{3}$

$-4-$

$= 7.267 \text{ cm}$

$= 73 \text{ mm}$

$$\frac{489}{\pi} = r^3$$

$$\frac{422}{\pi} = \frac{H^3}{3}$$

$$H^3 = \frac{12663}{\pi}$$

$$= 15.91$$

$\therefore H = 15.9 \text{ mm (c not mm)}$

Question 6

a) x

b) i) $x+2$ ✓

ii) ~~$(x)^2 + (x+2)^2 = 394$~~

let such numbers be $x, x+2$.

$$(x)^2 + (x+2)^2 = 394$$

$$x^2 + x^2 + 4x + 4 - 394 = 0$$

$$2x^2 + 4x - 390 = 0$$

$$x^2 + 2x - 195 = 0$$

$$(x+15)(x-13) = 0$$

$$x = -15, 13$$

\therefore two such ^{positive odd} sets of numbers are 13, 15

check

$$x = -15$$

$$x = 13$$

$$\therefore \text{next number} = -17$$

$$\therefore \text{next number } (x+2) = 15$$

$$-15 \times -17 = 255$$

$$13 \times 15 = 394$$

$$\therefore x \neq -15$$

$$\therefore x = 13$$

$$\therefore x = 13 \text{ ONLY}$$

e) d) ~~$a^2 = b^2 + c^2 - 2bc \cos A$~~

~~$$= 10^2 + 10^2 - (2 \times 10 \times 10 \times \cos 72)$$~~

~~$$= 138.196$$~~

~~$$\therefore a = 11.75$$~~

$$C = \frac{2\pi r}{360}$$

$$= \frac{2\pi \times 10}{360}$$

$$= \frac{4\pi}{36}$$

$$= 4\pi$$

$$= 4\pi$$

c) i) $C = \frac{2\pi r \cdot 288}{360}$

$$= \frac{2\pi \cdot 10 \cdot 288}{360}$$

$$\therefore \text{major arc} = 16\pi \text{ cm}$$

ii) $C = 2\pi r$

$$16\pi = 2\pi r$$

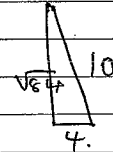
$$r = 8 \text{ cm}$$

iii) $V = \frac{\pi r^2 h}{3}$

$$= \frac{\pi \cdot 4^2 \cdot \sqrt{84}}{3}$$

$$= \frac{\pi \cdot 4^2 \cdot 2 \cdot \sqrt{21}}{3}$$

$$= \frac{32\sqrt{21} \cdot \pi}{3}$$



Question 7

a) i) $h = 5t(10-t)$

$$\therefore t = 0 \text{ if } h = 0$$

$$0 = 5t(10-t)$$

$$\therefore t = 0, 10$$

\therefore shell in air for 10 seconds.

ii) $h = 50t - 5t^2$ or sub. $t = 5$

$$0 = 5t^2 - 50t \quad h = 50 \times 5 - 5(5)^2$$

$$0 = t^2 - 10t \quad = 250 - 125$$

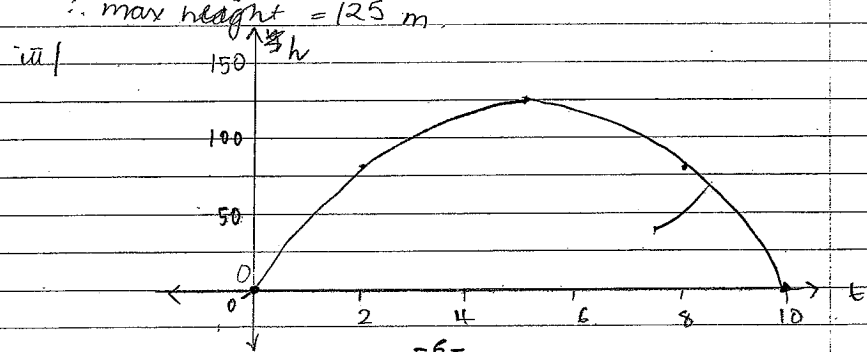
$$h = -5t^2 + 50t \quad = 125 \text{ m}$$

$$h = -5(t^2 - 10t + 25 - 25) + 125$$

$$h = -5(t^2 - 10t + 25) + 125$$

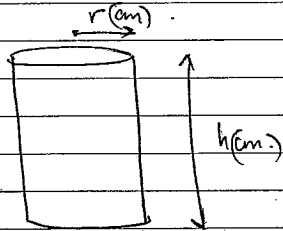
$$\therefore h = -5(t-5)^2 + 125$$

$$\therefore \text{max height} = 125 \text{ m}$$



iii) ?

b)



$$V = \pi r^2 h$$

$$\begin{aligned} \text{i) } V &= \pi (1.25r)^2 (h) \\ &= \frac{55\pi r^2 h}{32} \end{aligned}$$

$$\begin{aligned} \text{ii) } \text{percentage} &= \frac{\frac{55}{32} - \frac{32}{32}}{\frac{32}{32}} \times 100 \\ &= \frac{23}{32} \times 100 \\ &= 71.88\% \text{ (2dp) increase} \end{aligned}$$

Question 8.

$$\text{a) i) } \frac{n+1 - n}{n(n+1)} = \frac{1}{n(n+1)}$$

$$\text{ii) series } \frac{1}{2}, \frac{1}{6}, \frac{1}{12}, \frac{1}{20}, \frac{1}{30}, \frac{1}{42}, \frac{1}{56}, \frac{1}{72}$$

$$\begin{aligned} \frac{1}{6} &= \frac{1}{2 \times 3} \\ &= \frac{1}{2} - \frac{1}{3} \\ \frac{1}{2} &= \frac{1}{2} - \frac{1}{3} \end{aligned}$$

$$\text{Note: } \frac{1}{1 \times 2} + \frac{1}{2 \times 3} + \frac{1}{3 \times 4} + \dots + \frac{1}{100 \times 101}$$

$$= \frac{1}{1} - \frac{1}{2} + \frac{1}{2} - \frac{1}{3} + \frac{1}{3} - \frac{1}{4} + \dots + \frac{1}{100} - \frac{1}{101}$$

$$= \frac{1}{1} - \frac{1}{101}$$

$$= \frac{100}{101}$$

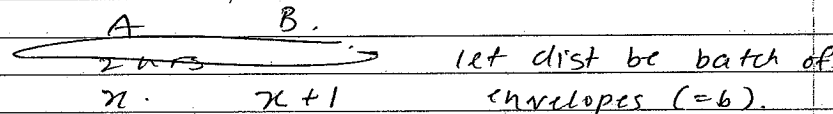
Question 8.

$$\text{a) i) } \frac{1}{n(n+1)} = \frac{1}{n} - \frac{1}{n+1}$$

$$\begin{aligned} \text{ii) } &\frac{1}{1} - \frac{1}{2} + \frac{1}{2} - \frac{1}{3} + \frac{1}{3} - \frac{1}{4} + \dots + \frac{1}{100} - \frac{1}{101} \\ &= \frac{1}{1} - \frac{1}{101} \\ &= \frac{100}{101} \end{aligned}$$

b). 2 machines
1 batch = 2 hrs.

Machine



let speed = $\frac{\text{dist}}{\text{time}}$

$$S_A = \frac{b}{x} \quad S_B = \frac{b}{x+1}$$

working together.

time = $\frac{\text{dist}}{\text{speed}}$

$$2 = \frac{b}{\left(\frac{b}{x}\right) + \left(\frac{b}{x+1}\right)}$$

$$= \frac{b}{b(x+1) + bx}$$

$$= \frac{b(x)(x+1)}{b[(x+1) + (x)]}$$

$$2 = \frac{x^2 + x}{2x + 1} \Rightarrow 4x + 2 = x^2 + x$$

$$4x + 2 = x^2 + x \Rightarrow x^2 - 3x - 2 = 0$$

$$x = \frac{3 \pm \sqrt{9+8}}{2} = \frac{3 \pm \sqrt{17}}{2}$$

$$a=1 \quad b=4 \quad c=-1$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{4 \pm \sqrt{(4)^2 - (4 \cdot 1 \cdot -1)}}{2}$$

$$= \frac{4 \pm \sqrt{16 + 4}}{2}$$

$$= \frac{4 \pm 2\sqrt{5}}{2}$$

$$= 2 \pm \sqrt{5}$$

Check

$$x = 2 - \sqrt{5}$$

$$x = 2 + \sqrt{5}$$

$$95 < -\sqrt{5} < 0$$

$$t_A = \frac{\text{dist}}{\text{speed}}$$

$$= \frac{b}{x}$$

$$= \frac{2 + \sqrt{5}}{1}$$

$$= 4.24 \text{ (2dp)}$$

$$= 4 \text{ hrs } 14 \text{ min}$$

$$= 3.6 \text{ hrs}$$

$$= 3 \text{ hrs } 34 \text{ mins}$$

$$t_B = \frac{d}{s}$$

$$= \frac{b}{x+1}$$

$$= \frac{2 + \sqrt{5}}{2}$$

$$= 5.24 \text{ (2dp)}$$

$$= 5 \text{ hrs } 14 \text{ min}$$

$$\text{or } 4 \text{ hrs } 34 \text{ mins}$$