

CURVE SKETCHING No 6 ASYMPTOTES

An asymptote is a line that a curve approaches but does not cross at its extremities
(That means that a non vertical asymptote can be crossed in the middle regions of the graph)

a) Vertical: These occur when the denominator is zero

i) ODD
single (or odd number of factors)

$$y = \frac{1}{x+1}$$

ii) EVEN
double (or even number of factors)

$$y = \frac{1}{(x+1)^2}$$

b) Horizontal

i) $y = 0$
degree of denominator > deg numerator

$$y = \frac{1}{x+1}$$

ii) $y = k$
degree of denominator = deg numerator

$$y = \frac{x}{(x+1)}$$

c) Inclined

i) deg num - deg den = 1

$$y = \frac{x^2}{(x+1)}$$

ii) square root

$$y = \sqrt{x^2 - 4}$$

Exercises: Sketch the following curves without the use of calculus. Note that where the numerator is equal to zero, the equation will have a root.

1. $y = \frac{2}{x-2}$ ✓

2. $y = \frac{2}{2-x}$

3. $y = \left(\frac{2}{x-2}\right)^2$ ✓

4. $y = \frac{x-1}{x+1}$ ✓

5. $y = \frac{2x}{x-2}$ ✓

6. $y = \frac{2}{2x-1}$

7. $y = \frac{x^2+1}{x}$ ✓

8. $y = \frac{x^2+1}{x+1}$ ✓

9. $y = \frac{x^2+1}{x^2}$ ✓

10. $y = \frac{1}{x^2-1}$

11. $y = \frac{x-1}{(x+1)(x+3)}$

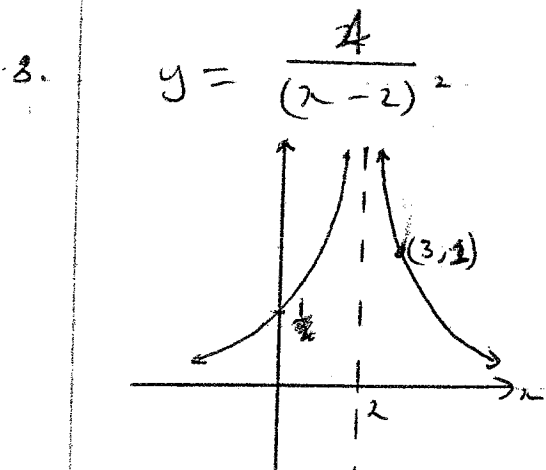
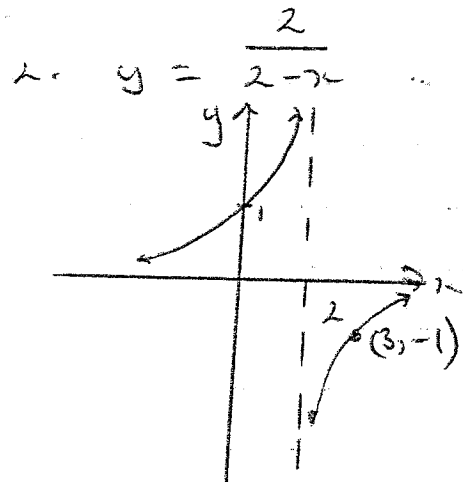
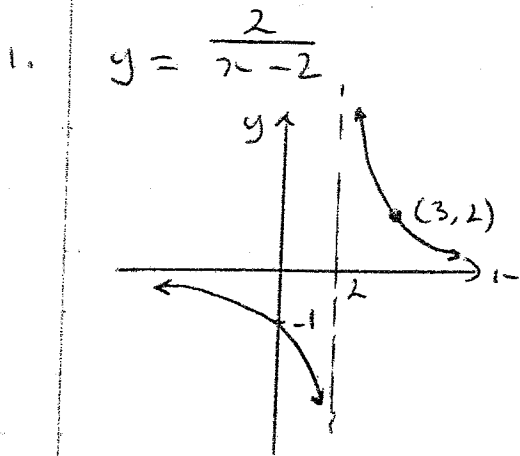
12. $y = \frac{x-1}{x(x+3)}$ ✓

13. $y = \frac{2x}{(x-1)^2}$

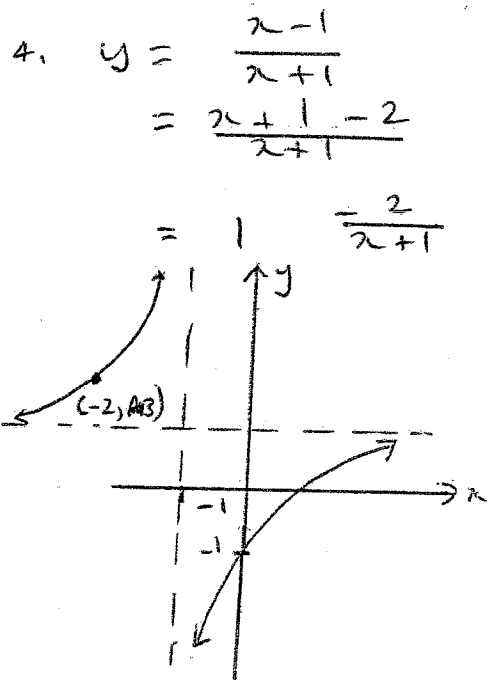
14. $y = \frac{2}{(x-1)(x+1)^2}$

15. $y = \sqrt{x^2 - 1}$

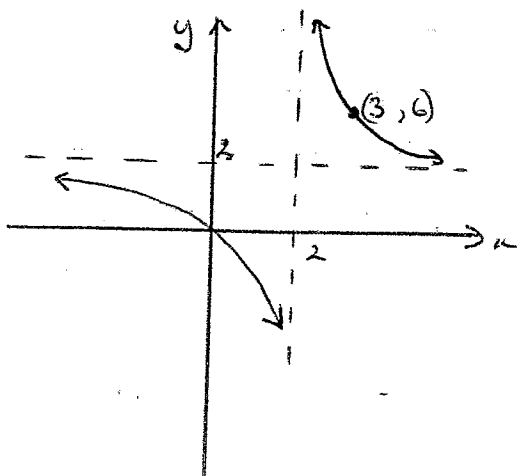
Solutions Curve sketching 6 ^{1/3}



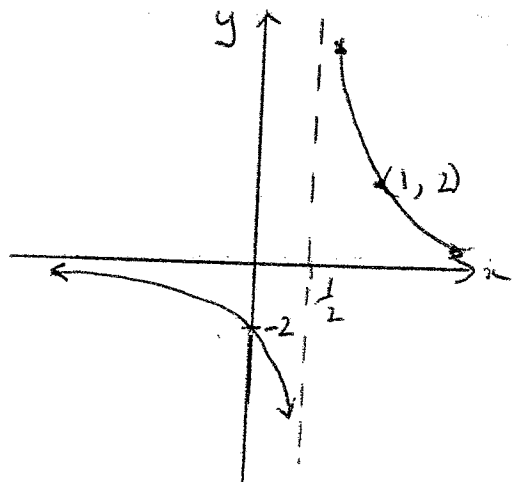
$x=2$ vert asympt (even)



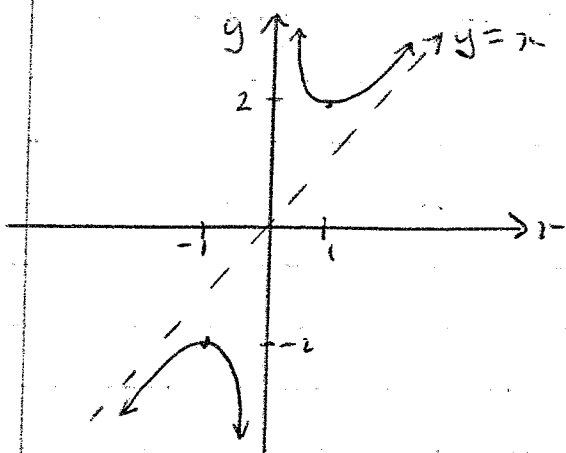
5. $y = \frac{2x}{x-2}$
 Horiz asympt $y=2$
 vert $x=2$



6. $y = \frac{2}{2x-1}$



7. $y = \frac{x^2+1}{x}$
 $= x + \frac{1}{x}$ Odd Fn

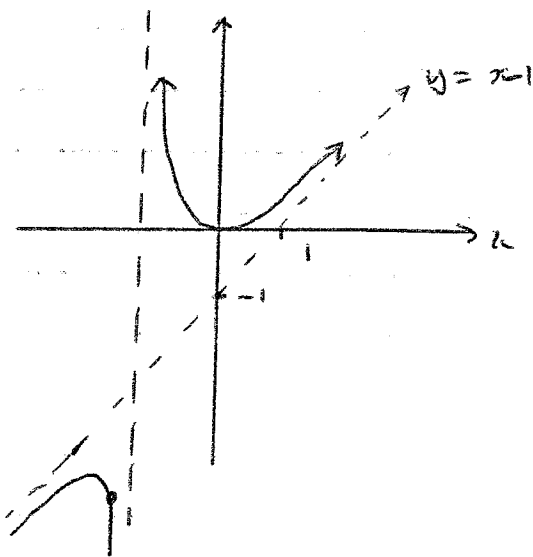


8. $y = \frac{x^2+1}{x+1}$

$$x+1 \overline{) \begin{array}{r} x^2 - 1 \\ \underline{x^2 + x} \\ -x + 1 \end{array}}$$

$$\begin{array}{r} -x + 1 \\ \underline{-x - 1} \\ 2 \end{array}$$

$y = x - 1 + \frac{2}{x+1}$

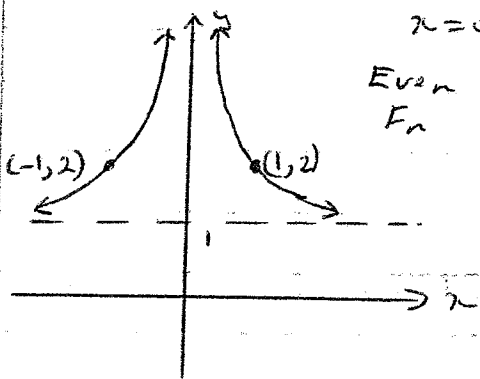


9. $y = \frac{x^2+1}{x^2}$

$y = 1 + \frac{1}{x^2}$

$x=0$ vert asympt (even)

Even Fn

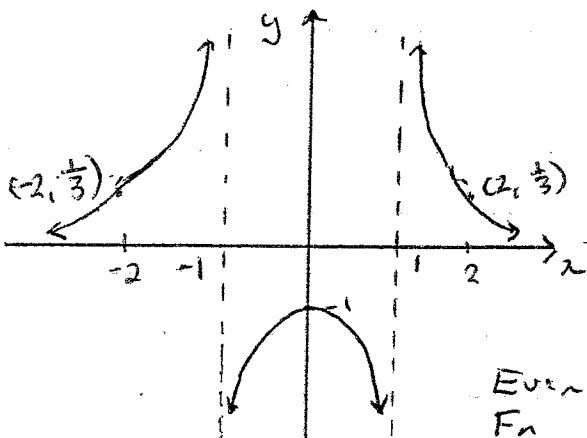


11.

$y = \frac{x-1}{(x+1)(x+3)}$

10. $y = \frac{1}{x^2-1}$

$= \frac{1}{(x+1)(x-1)}$

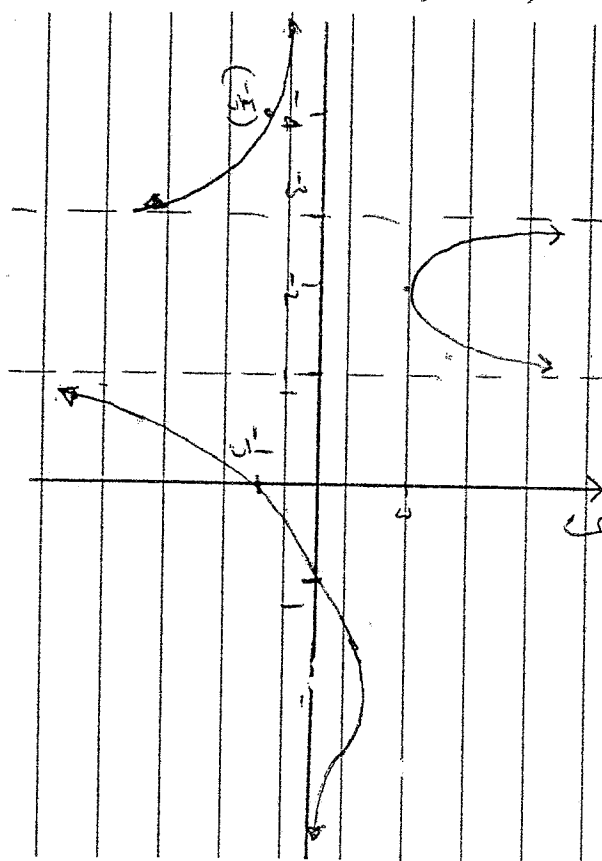


Even Fn

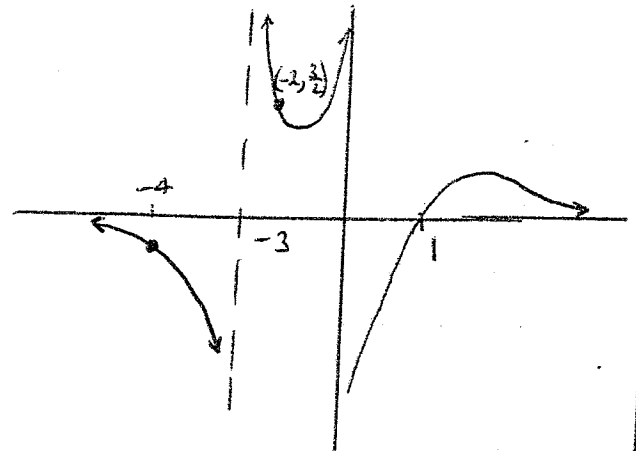
when $x=0, y = -1$

$x=2, y = 3$

$x=-2, y = 3$



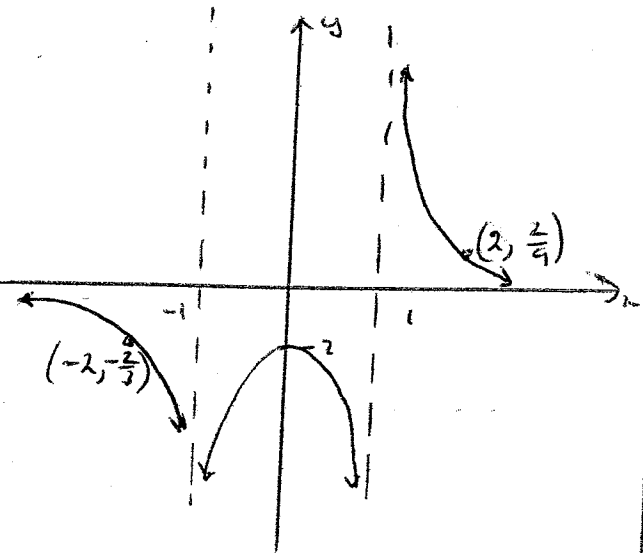
12. $y = \frac{x-1}{x(x+3)}$



Vert Asympt $x=0, x=-3$
 Horiz x axis
 x intercept 1 No y intercept
 $x=-2, y = \frac{-3}{-2(1)} = \frac{3}{2}$
 $x=4, y = \frac{-5}{(4)(-1)} = \frac{5}{4}$

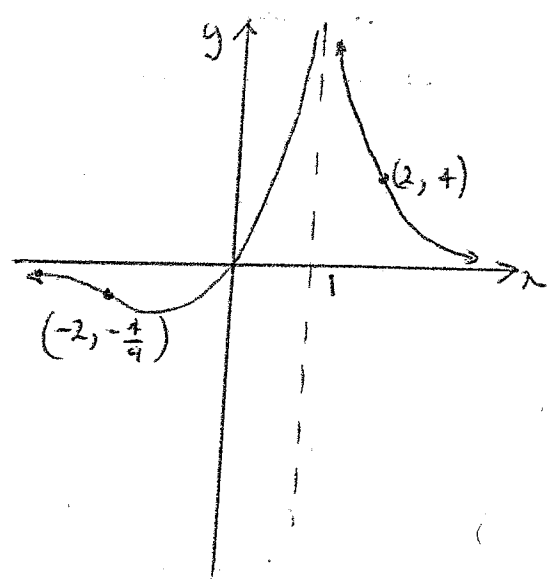
14. $y = \frac{2}{(x-1)(x+1)^2}$

Vert Asympt $x=1$ (odd)
 " " $x=-1$ (even)
 x axis horizontal



When $x=0, y = -2$
 $x=2, y = \frac{2}{9}$
 $x=-2, y = \frac{-2}{3}$

13. $y = \frac{2x}{(x-1)^2}$



Vert Asympt $x=1$ (even)
 horiz x axis
 when $x=0, y=0$
 $x=-2, y = \frac{-4}{9}$

15. $y = \sqrt{x^2 - 1}$
 defined for $x^2 - 1 \geq 0$
 $x \leq -1, x \geq 1$
 as $x \rightarrow \pm\infty, y \rightarrow \pm x$

