



2010
Higher School Certificate
Trial Examination

Mathematics

General Instructions

- Reading time – 5 minutes
- Working time – 3 hours
- Board approved calculators may be used.
- Write using black or blue pen
- A table of standard integrals is provided at the back of the paper
- All necessary working should be shown in every question
- Write your student number and/or name at the top of every page

Total marks – 120

- Attempt Questions 1 – 10
- All questions are of equal value

This paper **MUST NOT** be removed from the examination room

STUDENT NUMBER/NAME:

Total marks – 120
Attempt Questions 1 - 10
All questions are of equal value.

Answer the questions on your own paper or writing booklet, if provided.
Start each question on a new page.

Question 1 (12 marks)

- | | Marks |
|--|--------------|
| (a) Calculate $\sqrt{\frac{3.9^2 + 2.5^3}{14 - \sqrt{5}}}$. Give your answer correct to three significant figures | 2 |
| (b) Solve $\frac{3x+4}{x-1} = 2$. | 2 |
| (c) Find the gradient of the tangent to the curve $y = \frac{2}{x}$ at the point $(-1, -2)$. | 2 |
| (d) Solve $ 3x-1 = 10$. | 2 |
| (e) Sketch the graph of $2x+3y=9$, showing the intercepts on both axes. | 2 |
| (f) Find the exact value of x such that $\sec x + 1 = 3$ where $0 \leq x \leq \frac{\pi}{2}$. | 2 |

Question 2 (12 marks) Start a new writing booklet.

Marks

(a) Shade the region in the plane defined by $y \leq 0$ and $y \geq x^2 + 3x$

2

(b) Differentiate with respect to x :

(i) $\ln \sqrt{3x^2 - 1}$

2

(ii) $x \cos 2x$

2

(c) (i) Find $\int dt$

1

(ii) Find $\int \frac{4}{(2x-1)^3} dx$

2

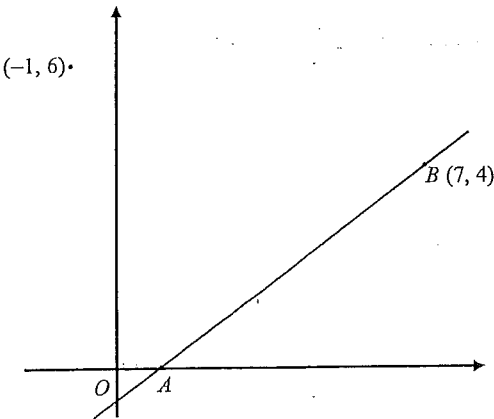
(iii) Evaluate $\int_1^e 2x + \frac{1}{x} dx$. Leave your answer in exact form.

3

Question 3 (12 marks) Start a new writing booklet.

Marks

(a) $P(-1, 6)$



The line AB has a gradient of $\frac{2}{3}$. The point B has coordinates $(7, 4)$.

(i) Find the equation of AB in the form $ax + by + c = 0$.

2

(ii) Find the shortest distance of the point $P(-1, 6)$ from the line AB .

2

(iii) Find the coordinates of A , the point where the line AB intersects with the x -axis.

1

(iv) Find the distance AB .

1

(v) Find the angle the line AB makes with the positive direction of the x -axis.

1

(b) Evaluate $\sum_{m=2}^6 10 - m^2$

2

(c) Use the Trapezoidal Rule with 4 subintervals to find an approximation for $\int_1^3 f(x) dx$ given that

3

x	1	$1\frac{1}{2}$	2	$2\frac{1}{2}$	3
$f(x)$	11.2	17.8	9.3	4.1	11.6

Marks

Question 4 (12 marks) Start a new writing booklet.

- (a) Find the values of k for which the expression $x^2 - (k-2)x + (k+13)$ is positive definite.

3

Question 4 (continued)

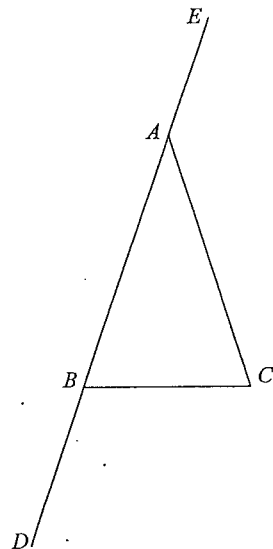
Marks

(c) (d) Juan started work at 20 and at the beginning of each month he invested \$150 into a superannuation fund. Interest was paid at 6% p.a. compounded monthly on the investment. Juan retired at 65 after having contributed to the fund for 45 years.

- (i) How much did Juan contribute to the fund over the 45 years? 1
- (ii) How much did Juan's investment amount to after 45 years? 3
- (iii) Juan plans to reinvest some of the money into an account which offers 8% p.a. compound interest compounded annually. The final amount will be \$200 000 at the end of the investment period. How much does Juan need to reinvest to achieve this amount. (Give your answer to the nearest \$10). 2

End of Question 4

(b)



NOT TO SCALE

ABC is an isosceles triangle in which $AB = AC$. E is a point in BA produced. D is a point in AB produced such that $BD = BC$.

Copy or trace the diagram into your answer booklet showing all given information.

Show, giving reasons, that $\angle CAE = 4\angle BDC$.

3

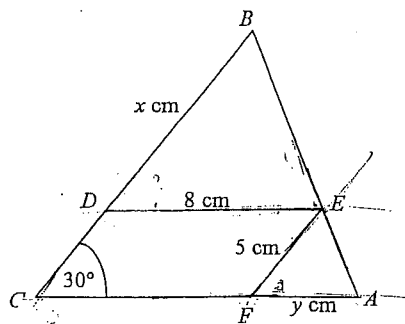
Question 4 continues.

Marks

Marks

Question 5 (12 marks) Start a new writing booklet.

- (a) In the diagram ABC is a triangle in which $\angle ACB = 30^\circ$ and D, E and F lie on the lines BC, BA and AC respectively. $CDEF$ is a parallelogram with $DE = 8$ cm and $EF = 5$ cm.



Let $BD = x$ cm and $AF = y$ cm,

- (i) Show triangles BDE and EFA are similar. 2
- (ii) Show that $xy = 40$. 1
- (iii) Show that the area, A , of triangle ABC is given by 3

$$A = 20 + 2x + \frac{50}{x}$$

- (iv) Find the values of x and y which will minimise the area of triangle ABC . Justify your answer. 3

Question 5 (continued)

- (b) During July the probability that it rains on any day is $\frac{1}{3}$. Find the probability that during a 7 day week
- (i) it rains only on the first 3 days. 1
- (ii) there is at least one rainy day. 2

End of Question 5

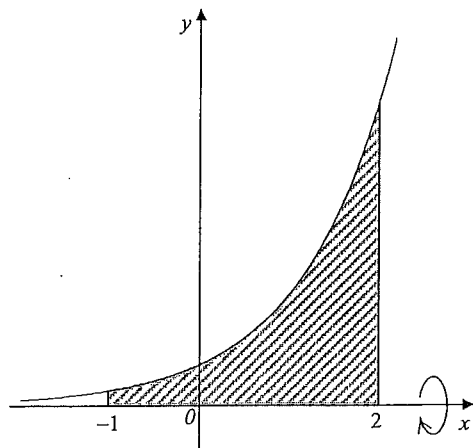
Question 5 continues

Question 6 (12 marks) Start a new writing booklet.

Marks

- (a) The diagram shows the region bounded by the curve $y = e^x$, the lines $x = -1$ and $x = 2$, and the x -axis.

3



The region is rotated about the x -axis. Find the volume of the solid of revolution formed. Leave your answer in exact form.

- (b) Draw a one third page sketch of $y = 1 + \cos(x - \frac{\pi}{4})$ for $0 \leq x \leq \pi$.

5

Indicate clearly any intercepts on the x and y axes.

- (c) For the curve $y = 2x^3 - 12x^2 - 5x - 3$ find:

- i) Any points of inflexion
ii) The equation of the normal to this curve at the point of inflexion.

2

2

Question 7 (12 marks) Start a new writing booklet.

Marks

- a) (i) Differentiate $\sqrt{(2x^2 + 1)^3}$

2

- (ii) Hence evaluate $2 \int_0^2 x \sqrt{2x^2 + 1} dx$

2

- b) For the curve $y = \sin \pi x$, state the period and amplitude

2

- c) i) Differentiate $y = 3^{4x-2}$ with respect to x

3

- ii) Hence find:

$$\int 3^{4x-2} dx$$

1

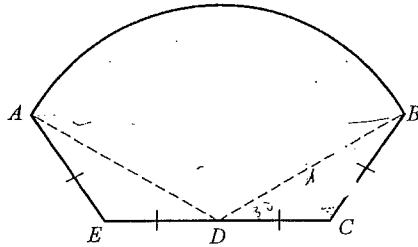
- (d) The area bounded by $y^2 = 3 - 2x - x^2$, $y \geq 0$ and between $x = -3$ and $x = 1$ is revolved about the x axis. Calculate the volume of the solid formed if this area is rotated about the x axis.

2

Question 8 (12 marks) Start a new writing booklet.

Marks

- a) Find the equation of the parabola with vertex (1, 2) and focus (1, 4). 2
- b) The Trumpets are building an unusually shaped pool on their country property.



In the diagram, $ABCDE$, represents the shape of the surface of the pool.

The sector ABD has centre D and $\angle ADB = \frac{2\pi}{3}$.

The points C, D, E lie on a straight line.

The arc AB has a length of 6π metres. $AE = ED = DC = CB$.

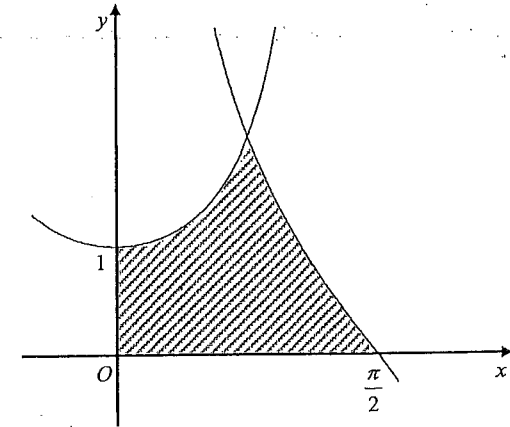
- (i) Show that $AD = 9$ metres. 1
- (ii) Find the length of BC . 2
- (iii) Find the area of the pool's surface. 2

Question 8 continues

Question 8 ((continued)

Marks

c)



The diagram shows the region bounded by the curves $y = \sec^2 x$, $y = 2 \cot x$ and the coordinate axes.

- (i) Verify, by substitution, that the point $(\frac{\pi}{4}, 2)$ lies on both $y = \sec^2 x$ and $y = 2 \cot x$. 1
- (ii) Differentiate $\ln(\sin x)$. 1
- (iii) Hence, or otherwise, find the exact area of the shaded region. 3

End of Question 8

Question 9 (12 marks) Start a new writing booklet.

Marks

(a) Evaluate $\int_0^{\frac{\pi}{6}} (x^2 + \sin 2x) dx$

2

- (b) Pedro is playing in a tennis competition. He is required to play 8 matches and he must win all 8 to win the competition. He has a 10% chance of winning the competition. In each match his probability of winning decreases by 5% of the preceding match's probability.

3

What is the probability that he wins the first match?

(c) Consider the function $y = (x^2 + 1)e^{-x}$.

The first derivative and second derivatives of this function are:

$$\frac{dy}{dx} = -e^{-x}(x-1)^2 \quad \text{and} \quad \frac{d^2y}{dx^2} = e^{-x}(x^2 - 4x + 3) \quad (\text{You don't need to show these})$$

- (i) Find any stationary points and determine their nature.

2

- (ii) Find the points of inflexion.

2

- (iii) Sketch the function clearly showing the stationary point, any points of inflexion and any intercepts.

3

Question 10 (12 marks) Start a new writing booklet.

Marks

a) For the function $y = 2\sin 3x + 4\cos 2x$ find p if $\frac{d^2y}{dx^2} + 4y = p\sin 3x$.

2

- (b) (i) Show that the point (e, e) lies on the curve with equation $f(x) = x \ln x$

1

- (ii) Determine the equation of the normal to the curve at this point.

2

- c) At the beginning of 2010 the Watersheds borrowed \$ 1 500 000 to purchase a new race horse. The annual interest rate on their loan is 6% pa compounded monthly. The loan is to be repaid by equal annual repayments of \$ 243 161, the first repayment being made at the end of 2010.

- (i) Show that the Watersheds still owe \$ 1 189 420 after they have made their 2nd repayment. (Answer correct to the nearest \$10.)

2

- (ii) Let A_n be the balance owing after the n th repayment.

3

$$\text{Show that } A_n = 1500000(1.005)^{12n} - 243161 \left(\frac{1.005^{12n} - 1}{1.005^{12} - 1} \right)$$

- (iii) In which year will the Watersheds pay off their debt?

2

End of Paper