2013
HIGHER SCHOOL CERTIFICATE
TRIAL EXAMINATION

Mathematics

General Instructions

- o Reading Time - 5 minutes
- o Working Time - 3 hours
- o Write using a blue or black pen. Black pen is preferred
- o Board approved calculators may be used
- o A table of standard integrals is provided at the back of this paper.
- o Show all necessary working in Questions 11-16

Total marks (100)

Section I

Total marks (10)
- o Attempt Questions 1-10
- o Answer on the Multiple Choice answer sheet provided.
- o Allow about 15 minutes for this section.

Section II

Total marks (90)
- o Attempt questions 11 – 16
- o Answer on the blank paper provided, unless otherwise instructed.
- o Start a new page for each question.
- o All necessary working should be shown for every question.
- o Allow about 2 hours 45 minutes for this section.

STANDARD INTEGRALS

$$\int x^n \, dx = \frac{1}{n+1} x^{n+1}, \quad n \neq -1; \quad x \neq 0, \text{ if } n < 0$$

$$\int \frac{1}{x} \, dx = \ln x, \quad x > 0$$

$$\int e^{ax} \, dx = \frac{1}{a} e^{ax}, \quad a \neq 0$$

$$\int \cos ax \, dx = \frac{1}{a} \sin ax, \quad a \neq 0$$

$$\int \sin ax \, dx = -\frac{1}{a} \cos ax, \quad a \neq 0$$

$$\int \sec^2 ax \, dx = \frac{1}{a} \tan ax, \quad a \neq 0$$

$$\int \sec ax \tan ax \, dx = \frac{1}{a} \sec ax, \quad a \neq 0$$

$$\int \frac{1}{a^2 + x^2} \, dx = \frac{1}{a} \tan^{-1} \frac{x}{a}, \quad a \neq 0$$

$$\int \frac{1}{\sqrt{a^2 - x^2}} \, dx = \sin^{-1} \frac{x}{a}, \quad a > 0, \ -a < x < a$$

$$\int \frac{1}{\sqrt{x^2 - a^2}} \, dx = \ln\left(x + \sqrt{x^2 - a^2}\right), \quad x > a > 0$$

$$\int \frac{1}{\sqrt{x^2 + a^2}} \, dx = \ln\left(x + \sqrt{x^2 + a^2}\right)$$

NOTE : $\ln x = \log_e x, \quad x > 0$

1.   What is 5.9893 correct to 2 significant figures?

A.   5.98

B.   5.99

C.   5.9

D.   6.0

2.   Which of the following is equivalent to $\dfrac{1}{\sqrt{7}+2\sqrt{3}}$?

A $\sqrt{7}-2\sqrt{3}$

B $\sqrt{7}+2\sqrt{3}$

C $\dfrac{\sqrt{7}-2\sqrt{3}}{-5}$

D $\dfrac{\sqrt{7}+2\sqrt{3}}{-5}$

3.   Simplify $\dfrac{x^2-5xy}{x^2-25y^2}$.
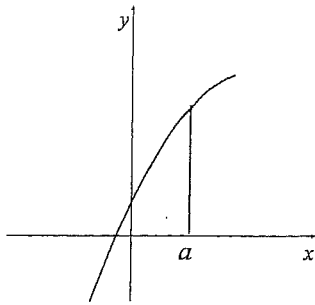
A $\dfrac{x}{x-5y}$

B $\dfrac{x}{x+5y}$
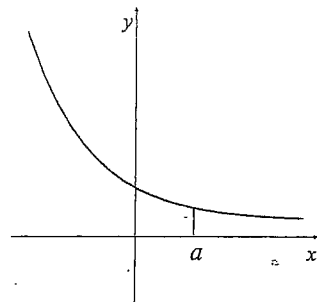
C $\dfrac{1-x}{1-5y}$

D $\dfrac{x-5y}{x+25y}$

4. Which diagram indicates $f'(a) > 0$ and $f''(a) < 0$?
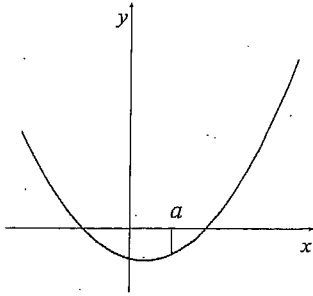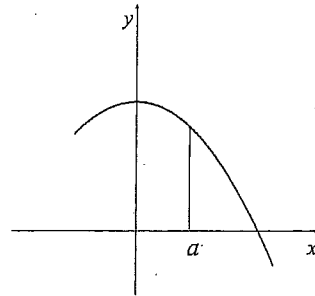
H6,7

A

B

C

D

5. What is the solution to the equation $2\cos 2x - 1 = 0$ in the domain $0 \le x \le 2\pi$?
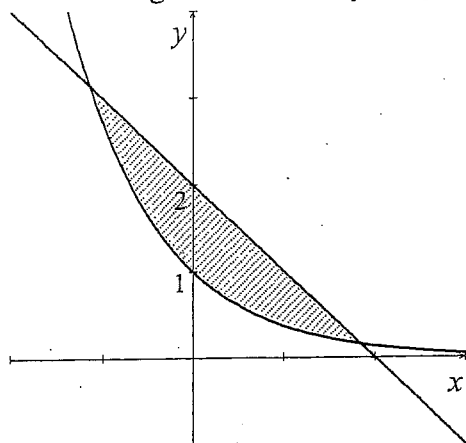
H5

A $\quad \dfrac{\pi}{3}, \dfrac{2\pi}{3}$

B $\quad \dfrac{2\pi}{3}, \dfrac{4\pi}{3}$

C $\quad \dfrac{\pi}{6}, \dfrac{5\pi}{6}, \dfrac{7\pi}{6}, \dfrac{11\pi}{6}$

D $\quad \dfrac{\pi}{3}, \dfrac{2\pi}{3}, \dfrac{4\pi}{3}, \dfrac{5\pi}{3}$

6. The diagram shows the region enclosed by $x+y=2$ and $y=e^{-x}$.

Which of the following pairs of inequalities describes the shaded region in the diagram?

A $x+y\leq 2$ and $y\leq e^{-x}$

B $x+y\leq 2$ and $y\geq e^{-x}$

C $x+y\geq 2$ and $y\leq e^{-x}$

D $x+y\geq 2$ and $y\geq e^{-x}$

7. What is the equation of the parabola with vertex $(4,2)$ and focus $(3,2)$?

A $(x-4)^2 = 4(y-2)$

B $(x-4)^2 = -4(y-2)$

C $(y-2)^2 = 4(x-4)$

D $(y-2)^2 = -4(x-4)$

8.   What is the angle of inclination of the line $3x + 2y = 7$ with the positive direction of the
     $x - axis$?

     A $33°41'$

     B $56°19'$

     C $123°41'$

     D $146°19'$

9.   What is the value of x if $\log x^2 - \log 3x = \log 9$?

     A   3

     B   9

     C   18

     D   27

10. The value of $\displaystyle\sum_{n=2}^{5} 2n^2$ is?

     A   50

     B   108

     C $205\dfrac{1}{32}$

     D $362\dfrac{21}{32}$

Section II

90 marks
Attempt Question 11 – 16
Allow about 2 hours 45 minutes for this section

Answer each question in a SEPARATE writing booklet. Extra writing booklets are available.

All necessary working should be shown in every question.

Question 11 (15 marks). Use a SEPARATE writing booklet.

a)   Solve $2^{3x-2} = 16$.   **2**   H3

b)   Solve $|5 - 2x| \geq 9$.   **2**   H3

c)   Factorise $2x^2 + 9x - 5$.   **2**   H3

d)   Find the equation of the tangent to the curve $y = 2x^2 - 1$ at the point where   **2**   H6
        $x = -1$.

e)   Differentiate with respect to x:

        (i) $e^{2x} \sin x$ **2**   H5

        (ii) $\dfrac{\cos x}{4 - x}$.   **2**   H5

f)   Find $\int \sec^2 3x\, dx$.   **1**   H8

g)   For the series 5, -1, -7,...

        i)   Which term will be equal to -61?   **1**   H5

        ii)   Find the sum of the first 20 terms.   **1**   H5

Question 12 (15 marks). Use a SEPARATE writing booklet.

a) Evaluate $\displaystyle\int_{2}^{6} \frac{dx}{\sqrt{x-1}}$ **3** H8

b) Five values of the function $f(x)$ are shown in the table.

x	2	6	10	14	18
$f(x)$	16	14	19	21	15

Use Simpson's Rule with the five values given in the table to estimate **3** H8

$$\int_{2}^{18} f(x)\,dx$$

c) There are 2 piles of playing cards. One pile contains 7 red and 4 black cards while the other pile contains 3 red and 4 black cards.

Felicity randomly chooses one card from each pile.

(i)   What is the probability that both cards are black? **1** HS

(ii)   What is the probability that at least one card is red? **1** HS

(iii) What is the probability that both cards are different colours? **2** HS

d)   In the diagram the coordinates of the points A and B are $(-2, -1)$ and $(1, -3)$ respectively. The line AD has equation $y = 2x + 3$ and the line CD has equation $2x + 3y = 17$. $BC \parallel AD$.
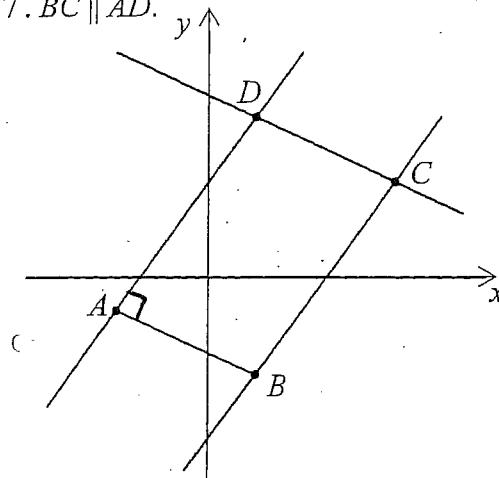
NOT TO SCALE.

Copy the diagram into your examination booklet clearly labelling all given information.

(i)   Find the perpendicular distance of $B.$ $(1, -3)$ from the line AD 1   HS
$y = 2x + 3$.

(ii)   Find the coordinates of D the point intersection of $y = 2x + 3$ and   2   HS
$2x + 3y = 17$.

(iii)   Hence, or otherwise, find the area of parallelogram $ABCD$.   2   H9

End of Question 12

Question 13 (15 marks). Use a SEPARATE writing booklet.

a) A function is given by $f(x) = 12x - 3x^2 - 2x^3$

    (i) Find the coordinates of the stationary points of $f(x)$ and determine their nature.   3   H6

    (ii) Hence sketch the graph $y = f(x)$ showing the stationary points and y-intercept.   2   H9

    (iii) For what values of x is the function decreasing?   1   H7

    (iv) For what values of k will $12x - 3x^2 - 2x^3 + k = 0$ have 2 real solutions?   2   H9

b) Find the exact area bounded by the curve $y = \dfrac{3}{x-2}$, the x-axis and the lines $x = 3$ and $x = 5$.   3   H8

c) Kevin has started an exercise program to lose weight. When he started the program he weighed 105 kg.
In the first month he lost 5 kg, in the second month he lost 4 kg and in the third month he lost 3.2 kg.
If this weight loss trend continues

    (i) how much will Kevin lose in the fourth month?   1   H5

    (ii) what will be his ultimate weight?   3   H5

Question 14 (15 marks). Use a SEPARATE writing booklet.

a)
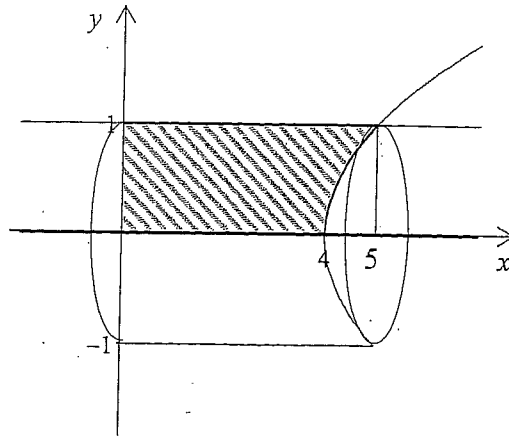
The shaded area bounded by $y = 1$, $y = 0$, $x = 0$ and $y = \sqrt{x-4}$ is rotated about the x – axis.
Find the volume generated.

3   H 8

b)   The acceleration of a particle is given by $\ddot{x} = 4\sin 2t$ where x is displacement in metres and t is time in seconds.
Initially the particle is 1 metre to the left of the origin with a velocity of 2 metres per second.

   (i)   Show that the velocity of the particle is given by $\dot{x} = 4 - 2\cos 2t$.   2   H5

   (ii)   Show the particle never comes to rest.   1   H5

   (iii)   Find the equation for the displacement of the particle.   2   H5

   (iv)   Find the distance travelled by the particle in the first 4 seconds.   2   H5

c)   A die is biased so that the probability of rolling a six is 0.1.
What is the minimum number of rolls of the die required to have a greater than 70% chance of rolling at least one 6?   3   H5

d)   A point $P(x, y)$ moves such that its distance from the point (2, 3) and the line $y = -1$ is equal.

   i)   Write the equation of the locus of P.   2   H4

Question 15 (15 Marks).   Use a SEPARATE writing booklet.

a)   Anthony, Bethany and Carl decided to meet for lunch at Bethany's house. Carl's house (C) is situated on a bearing of $046°T$ from Anthony's house (A).
From Bethany's house (B), the bearing to Carl's house is $341°T$.

C

NOT TO SCALE

510 m

A

620 m

B
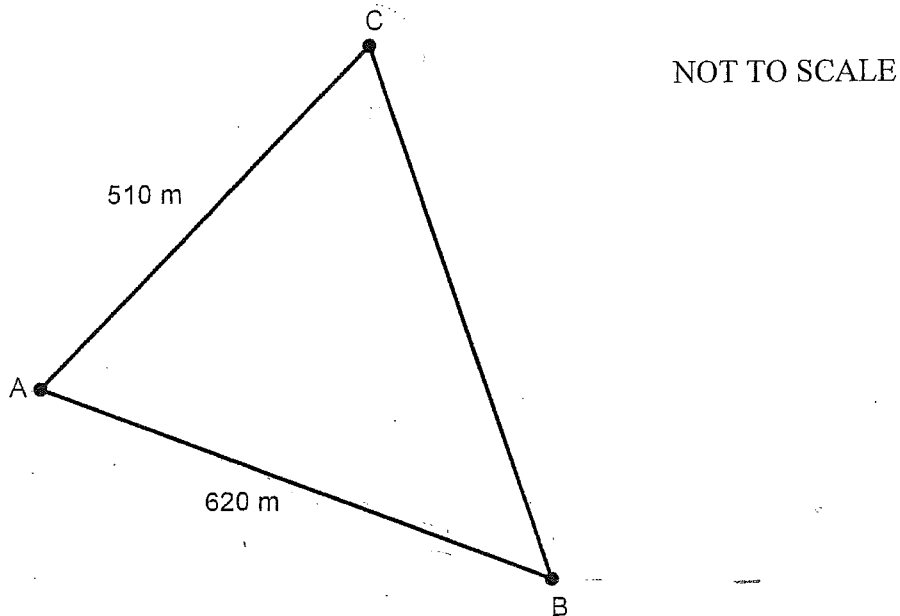
The distance from Anthony's house to Bethany's is 620 m.
The distance from Anthony's house to Carl's is 510 m.

3   HS

i)   What is the bearing to the nearest minute of Bethany's house from Anthony's?

1   HS

ii)   Use the Cosine Rule to show that the distance from Carl's house to Bethany's is given by:

$$BC = \sqrt{644500 - 632400\cos 66°48'} \text{ metres.}$$

b)   Solve for x where $0 \ll x \ll 2\pi$

2   HS

$$\cos x > \frac{1}{2}$$

Question 15 continues on the following page.

Question 15 (15 Marks). Use a SEPARATE writing booklet.

c)   A ball bearing is dropped into a vat of motor oil. The rate at which the ball bearing decelerates is proportional to its velocity. i.e. $\dfrac{dv}{dt} = -kv$ where v is the velocity in centimetres per second, t is the time in seconds and k is a constant.

   (i)   Show that $v = Se^{-kt}$ satisfies $\dfrac{dv}{dt} = -kv$.   1   H5

   (ii)   The ball bearing is travelling at 120 centimetres per second when it first enters the oil.   1   H5

   Show that the value of $S = 120$.

   (iii)   After travelling through the oil for 0.5 seconds the ball bearing has a velocity of 75 centimetres per second.   1   H5

   Show the value of k is 0.94, correct to 2 decimal places.

   (iv)   Find the distance travelled through the oil in the first 2 seconds.   1   H5

   (v)   If the vat is 1.25 metres deep, how long will it take for the ball bearing to reach the bottom of the vat?   2   H5

d)   Show that the quadratic equation in x, $\left(a^2 - b^2\right)x^2 + 2b\left(a - c\right)x + \left(b^2 - c^2\right) = 0$ has real and rational roots for all values of x, if a, b and c are rational.   3   H2

End of Question 15

a) 100 white and 100 blue marbles are mixed together. Some are placed in container A while the rest are placed in container B. The probability of selecting a white marble from container A is $\frac{2}{3}$. If a white marble is now taken from container A and placed in container B then the probability of selecting a blue marble from container B is also $\frac{2}{3}$.

(i) If w is the number of white marbles in container A and b is the number of blue marbles in container A, show that $w = 2b$. **1** H4

(ii) Show that $\dfrac{100-b}{201-w-b} = \dfrac{2}{3}$ **2** H4

(iii) Find the number of each colour originally in container A. **1** H4

b)
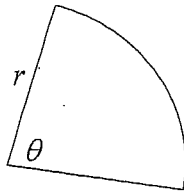
The diagram shows a sector of a circle with radius r cm. The angle at the centre is θ radians and the area is 18 cm^2.

(i) Find an expression for r in terms of θ. **1** H5

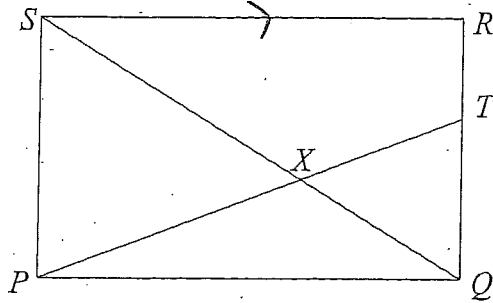(ii) Show that P, the perimeter of the sector in cm, is given by **1** H5

$$P = \frac{6(2+\theta)}{\sqrt{\theta}}.$$

(iii) Find the minimum perimeter and the value of θ for which this occurs. **3** H5

Question 16 continues on the next page.

Question 16 (15 Marks). Use a SEPARATE writing booklet.

c)

In the diagram $PQRS$ is a rectangle and $PQ = 3PS$. The point T lies on QR so that $2RT = TQ$. The line PT meets SQ at X.

(i)   Show that triangles XSP and XQT are similar.   2   H2

(ii)   Show that $2SQ = 5QX$.   2   H2

(iii)   Show that $8(PS)^2 = 5(QX)^2$.   2   H2

END OF EXAM