Student Number: ______________

St. Catherine's School
Waverley

2007

HIGHER SCHOOL CERTIFICATE
ASSESSMENT TASK 4– 40%
TRIAL HSC EXAMINATION

Mathematics

General Instructions

- Working time – 3 hours + 5 minutes reading time
- Start each question on a new page in your answer booklet.
- If any additional booklet is used, please label it clearly and attach it to the appropriate booklet.
- Write using black or blue pen only.
- Board-approved calculators may be used.
- All necessary working must be shown.
- Marks may be deducted for careless or badly arranged work.

Total marks – 120

- Attempt Questions 1–10
- All questions are of equal value

STANDARD INTEGRALS

$$\int x^n\,dx = \frac{1}{n+1}x^{n+1},\ n\neq -1;\ x\neq 0,\ \text{if } n<0$$

$$\int \frac{1}{x}\,dx = \ln x,\ x>0$$

$$\int e^{ax}\,dx = \frac{1}{a}e^{ax},\ a\neq 0$$

$$\int \cos ax\,dx = \frac{1}{a}\sin ax,\ a\neq 0$$

$$\int \sin ax\,dx = -\frac{1}{a}\cos ax,\ a\neq 0$$

$$\int \sec^2 ax\,dx = \frac{1}{a}\tan ax,\ a\neq 0$$

$$\int \sec ax\tan ax\,dx = \frac{1}{a}\sec ax,\ a\neq 0$$

$$\int \frac{1}{a^2+x^2}\,dx = \frac{1}{a}\tan^{-1}\frac{x}{a},\ a\neq 0$$

$$\int \frac{1}{\sqrt{a^2-x^2}}\,dx = \sin^{-1}\frac{x}{a},\ a\neq 0,\ -a<x<a$$

$$\int \frac{1}{\sqrt{x^2-a^2}}\,dx = \ln(x+\sqrt{x^2-a^2}),\ x>a>0$$

$$\int \frac{1}{\sqrt{x^2+a^2}}\,dx = \ln(x+\sqrt{x^2+a^2})$$

NOTE: $\ln x = \log_e x,\ x>0$

QUESTION 1 (12 Marks) Start question on a NEW page — Marks

a) Find correct to two decimal places: $\left(\dfrac{4.71+3.02}{23.24}\right)^{-1}$. — 1

b) Simplify the expression: $8x^2-7x-2x(x-5)$. — 2

c) Factorise completely: $5x^2-20$. — 2

d) Evaluate: $\log_5 100-\log_5 4$. — 2

e) Express $\dfrac{2}{\sqrt{5}+\sqrt{7}}$ with a rational denominator. — 2

f) Solve the inequality $|2x-1|\le 3$.
Illustrate your solution on a number line. — 3

Question 2 starts on next page

QUESTION 2 (12 Marks) Start question on a NEW page — Marks

(i) On a number plane, plot the points $A(1, 5)$, $B(-2, -4)$ and $C(10, -4)$. — 1

(ii) Show that the gradient of the line AC is -1. — 1

(iii) Show that the length of the interval AC is $9\sqrt{2}$ units. — 2

(iv) Show that the equation of the line AC is $x+y-6=0$. — 2

(v) Find the perpendicular distance from B to AC.
Leave your answer in surd form. — 2

(vi) Find the area of the triangle ABC. — 1

(vii) Find the midpoints M and N of AB and AC respectively.
Hence show that $MN \parallel BC$. — 3

Question 3 starts on next page

QUESTION 3 (12 Marks) Start question on a NEW page — Marks

a) Differentiate:

(i) $f(x)=(3e^{x}-5)^{10}$, — 2

(ii) $y=x^{2}\tan x$, — 2

(iii) $y=\log_{e}\left(\dfrac{e^{x}}{\cos x}\right)$. — 3

b) Find the primitive function of $6x^{2}+\dfrac{2}{x}$. — 2

c) Find $\displaystyle\int_{1}^{2}\frac{x+1}{\sqrt{x}}\,dx$. — 3

Question 4 starts on next page

QUESTION 4 (12 Marks) Start question on a NEW page — Marks

a) Consider the curve $y=2x^{3}-3x^{2}-12x+2$.

(i) Show that $y=2$ is the y intercept for $y=2x^{3}-3x^{2}-12x+2$. — 1

(ii) Determine the stationary points and their nature. — 4

(iii) Find any points of inflexion. — 1

(iv) Sketch and label the curve for $-3\le x\le 3$. — 2

(v) Hence determine the maximum value of $y=2x^{3}-3x^{2}-12x+2$ over the domain $-3\le x\le 3$. — 1

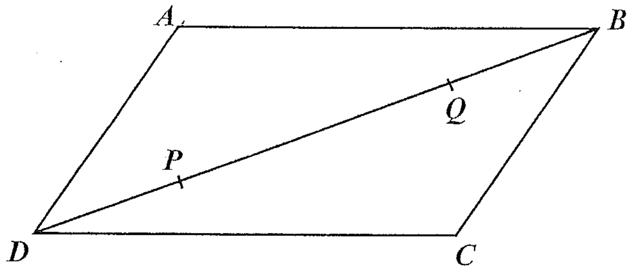
b) (i) In the lounge room of Amy's house there are three different light switches. If each switch has only two positions (OFF/ON), draw a tree diagram to show all the possible combinations for which these switches can be set to. — 1

(ii) What is the probability that at least two switches are ON? *Assume that the chance of the switch being ON or OFF is the same.* — 2

Question 5 starts on next page

QUESTION 5 (12 Marks) Start question on a NEW page **Marks**

a) $ABCD$ is a parallelogram. Points P and Q lie on the diagonal DB such that $DP = BQ$.



Copy the diagram into your answer booklet.

(i) Prove that ΔABQ is congruent to ΔCDP. **2**

(ii) Hence or otherwise, prove that $AQCP$ is a parallelogram. **3**

b) Find the values A, B and C such that: **3**

$$4x^2 - x + 1 = Ax(x+1) + B(x+1) + C.$$

c) State the domain and range for each of the following functions:

(i) $y = \dfrac{5}{x-2}$. **2**

(ii) $y = 3\cos \pi x - 2$. **2**

Question 6 starts on next page

QUESTION 6 (12 Marks) Start question on a NEW page **Marks**

a) A car moves in a straight line covering x metres in t seconds, in such a way that:

$$x = t^3 - 4t^2 + 5$$

(i) Find x', the velocity of the car at t seconds. **1**

(ii) Find the velocity of the car at $t = 5$ seconds. **1**

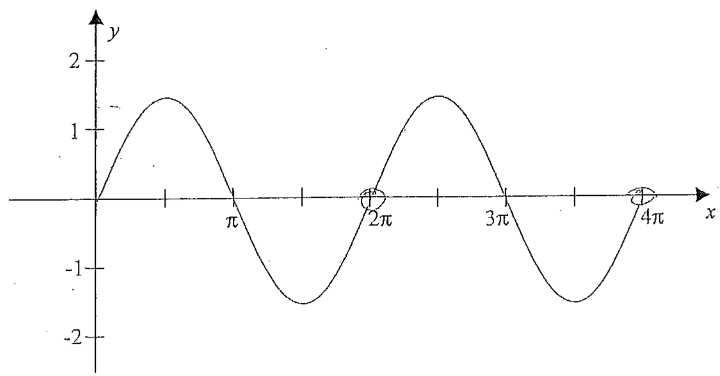
(iii) Find the acceleration of the car at $t = 5$ seconds. **2**

b) A boat travels 5 km on a bearing of 270°T from P to Q. It then turns at Q and travels a further 8 km on a bearing of 200°T to point R.

(i) Draw and label a neat diagram showing all given information. **1**

(ii) Calculate how far the boat is from the starting point accurate to 3 significant figures. **2**

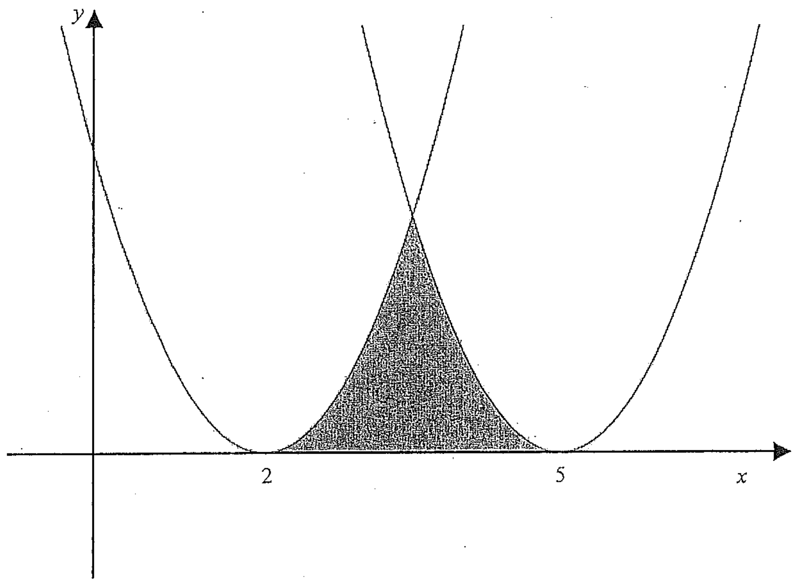
c) The diagram below shows the graph of $y = \dfrac{3\sin x}{2}$ for $0 \le x \le 4\pi$. **1**



Write down **one** equation for a Cosine graph that would look **exactly** the same as the graph shown above for $0 \le x \le 4\pi$.

Question 6 continues on next page

d) Find the area enclosed between the two curves $y=(x-2)^2$, $y=(x-5)^2$ and the x-axis. **4**

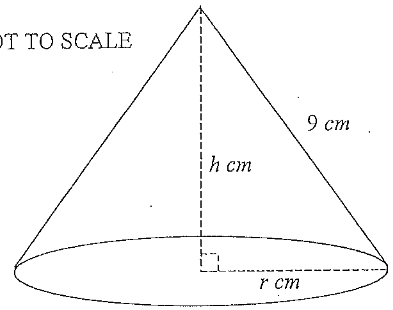


Question 7 starts on next page

QUESTION 7 (12 Marks) Start question on a NEW page Marks

a) The slant length of a right circular cone of height h cm and base radius r cm, is 9 cm.

DIAGRAM NOT TO SCALE



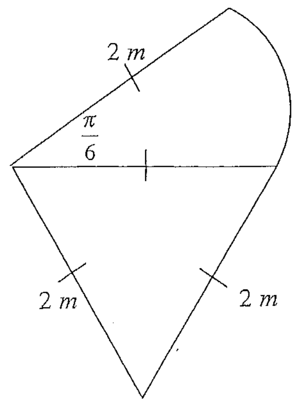
(i) Write down an equation linking r and h **1**

(ii) Show that the volume of the cone can be given by the equation: **2**

$$V = 27\pi h - \frac{\pi h^3}{3}$$

(iii) Find the height of the cone which gives a maximum volume. **2**

b) The figure below is composed of an equilateral triangle surmounted by a sector with the dimensions as shown. **3**
Find the area of the shape in **exact form.**



Question 7 continues on next page

c) A particle travels in a straight line such that its displacement from the origin at time t is x. If its acceleration $(\ddot{x})$ is a constant 3 ms^{-2}, and initially it has a displacement of x_0 and has a velocity of u, show that:

(i) Its velocity at any time is given by $\dot{x} = u + 3t$. 2

(ii) The distance travelled in time t is given by $x - x_0 = ut + \frac{3}{2}t^2$. 2

Question 8 starts on next page

QUESTION 8 (12 Marks) Start question on a NEW page — Marks

a) Given that $f(x) = x^3 + x^2 + x + 1$, show that $\frac{f'(x) + 2x - 5}{f''(x) + 10} = \frac{3x - 2}{6}$. 2

b) (i) Show that $\int_{\ln 3}^{\ln a} 3e^{3x}\,dx = a^3 - 27$. 2

(ii) Hence evaluate a if $\int_{\ln 3}^{\ln a} 3e^{3x}\,dx = 37$. 1

c) Using the integration method for calculating volumes of solids, find the volume of the solid formed when the line $y = 2x - 1$ is rotated about the y-axis between $y = 1$ and $y = 5$. 4

d) Use Simpson's Rule with 3 function values to find an approximation (correct to 1 decimal place) to: 3

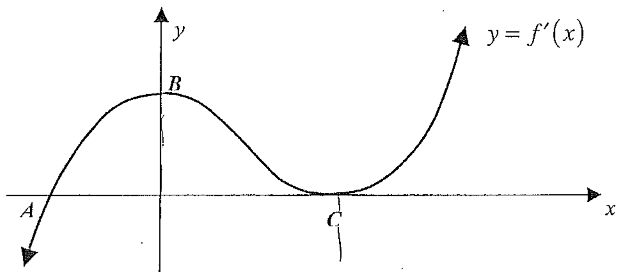
$$\int_0^4 \frac{3}{x^2 + 2}\,dx$$

x	0	1	2	3	4
$f(x)$	1.5	1	0.75	$0.\dot{2}\dot{7}$	$0.1\dot{6}$

Question 9 starts on next page

QUESTION 9 (12 Marks) Start question on a NEW page Marks

a) The curve below represents the graph $y = f'(x)$, the derivative function of the curve $y = f(x)$ — 2



Copy this sketch into your answer booklet and sketch a possible graph of $y = f(x)$ showing where the reference points A, B and C lie on the curve.

b) Solve for x: $3^{2x+1} = 27\sqrt{3}$. — 2

c) (i) Show that $\frac{2}{x-a} + \frac{k}{x} + \frac{8}{x+a} = 0$ may be expressed in quadratic form: $(10+k)x^2 - 6ax - ka^2 = 0$, where a is a constant. — 2

(ii) Hence find the value(s) of k if the roots are equal. — 2

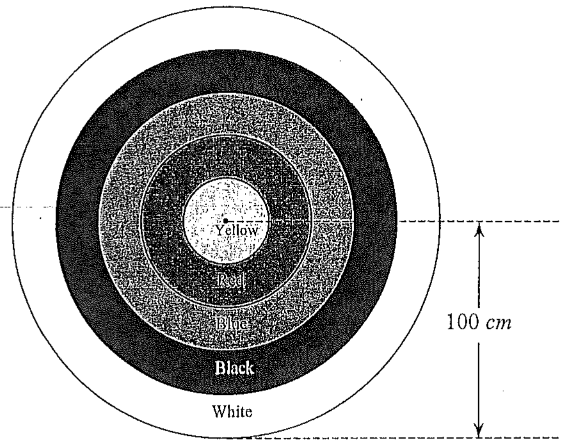
d) The curve $y = px^3 + \frac{q}{x^2}$ cuts the x-axis at $x = 1$, and the gradient of the tangent at this point is 2. — 4

Find the values of p and q.

Question 10 starts on next page

QUESTION 10 (12 Marks) Start question on a NEW page Marks

a) The following diagram shows an archery target that is made up of 4 concentric rings that surround a yellow "bullseye".



From the *centre* of the target to the outer edge, all circular bands are the same width.

The points score and probabilities, from the bullseye to the outer ring are:

POINTS	PROBABILITIES
Yellow = 10	$\frac{1}{25}$
Red = 8	$\frac{3}{25}$
Blue = 6	$\frac{1}{5}$
Black = 4	$\frac{7}{25}$
White = 2	$\frac{9}{25}$

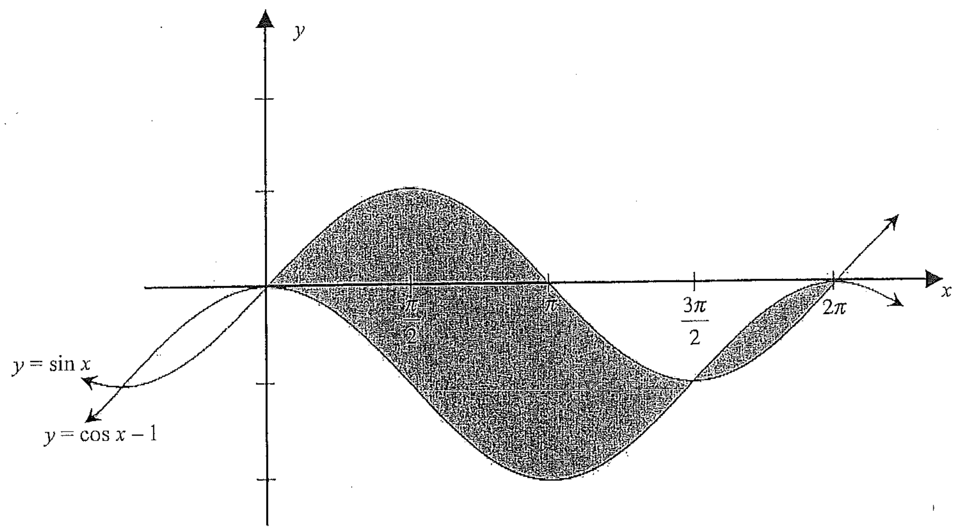
The probabilities of hitting each colour are directly proportional to the percentage area each colour represents on the target (*assuming every arrow hits the target*).

(i) What is the probability of the blue ring being hit after firing two arrows? — 1

(ii) Find the probability of scoring **more than** 11 points with two arrows. — 3

Question 10 continues on next page

b) Find the area in exact form, of the shaded region below enclosed between the two curves $y = \sin x$ and $y = \cos x - 1$. 3



c) Given that $x^2 + y^2 = 14xy$:

(i) Show that $\left(\frac{x+y}{4}\right)^2 = xy$. 2

(ii) Hence or otherwise, write: 3

$$2\log\sqrt{\left(\frac{x+y}{4}\right)} - \frac{1}{2}(\log x + \log y)$$

in simplest form.

End of Paper