International Grammar School

Year 11
Mathematics Extension 1
Yearly Examination
2011

General Instructions

- Reading time – 5 minutes
- Working time – 1.5 hours
- Write using black or blue pen
- Board-approved calculators may be used
- All necessary working should be shown in every question

Note: Any time you have remaining should be spent revising your answers.

Total marks – 60

- Attempt Questions 1 – 5
- All questions are of equal value
- Start each question in a new writing booklet
- Write your name on the front cover of each booklet to be handed in
- If you do not attempt a question, submit a blank booklet marked with your name and "N/A" on the front cover

DO NOT REMOVE THIS PAPER FROM THE EXAMINATION ROOM

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Total Marks – 60
Attempt Questions 1–5
All questions are of equal value

Answer each question in a SEPARATE writing booklet. Extra writing booklets are available.

Question 1 (12 marks) — Marks

(a) Simplify $\dfrac{1}{p^2 - pq} - \dfrac{1}{pq - q^2}$ — 2

(b) i) Expand $\sin(A - B)$ — 1

ii) Hence, or otherwise write the exact value of $\sin 15°$ — 2

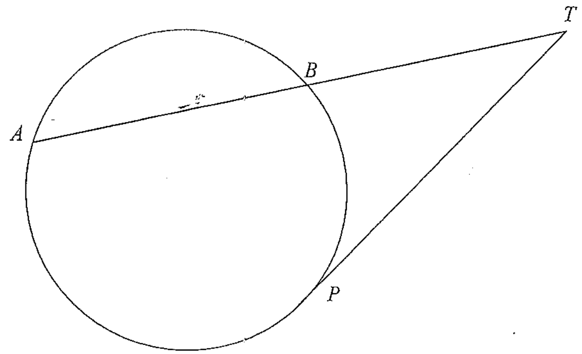
(c) Simplify $(n + 1)! - n!$ — 2

(d) Solve for x where $\dfrac{x+2}{x} \geq 0$ — 3

(e) Write the general solution to the equation $\tan x = 1$ — 2

Question 2 (12 marks) Use a SEPARATE writing booklet — Marks

(a)



In the diagram PT is a tangent to the circle at P.
$AB = 5cm$ and $PT = 8cm$. Find the length of the interval BT to the nearest tenth of a centimetre. — 3

(b) Find the coordinates of the point P which divides the interval AB, where $A = (-2, 1)$ and $B = (1, 7)$, internally in the ratio $2:3$ — 2

(c) Find $\displaystyle\lim_{x \to \infty} \frac{3x + 2x^3}{5x^2 - 1}$ — 2

(d) Find the numbers of ways in which the letters of the word **FUNCTIONS** can be arranged in a line. — 2

(e) i) Sketch $y = |x+5|$ and $y = 2x$ on the same number plane — 1

ii) Find the x coordinate(s) of the point(s) of intersection of the two functions $y = |x+5|$ and $y = 2x$. — 1

ii) Hence, or otherwise solve $2x \leq |x+5|$ — 1

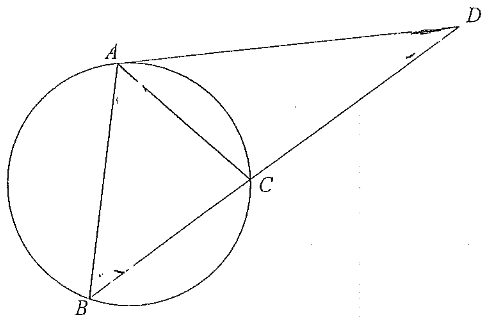
Question 3 (12 marks) Use a SEPARATE writing booklet **Marks**

(a) i) Show that the equation $\cos 2x + 3\cos x + 2 = 0$ is equivalent to the equation $2\cos^2 x + 3\cos x + 1 = 0$ 2

ii) Hence, or otherwise solve the equation $\cos 2x + 3\cos x + 2 = 0$ for $0^\circ \le x \le 360^\circ$ 2

(b) Find the domain of the function $f(x) = \sqrt{4 - x^2}$ 2

(c)



ABC is a triangle inscribed in a circle. The tangent to the circle at A meets BC produced at D. $\angle CAB = 60^\circ$, $\angle CDA = 30^\circ$ and $\angle ABC = x^\circ$. Copy the diagram, showing the given information, then find the value of x. 2

(d) In a game of Lotto four numerals are chosen from the numerals **1 2 3 4 5 6 7 8 9** without repetition. Order is not important. Find the number of ways in which this can be done so that

i) **No** odd numerals are chosen 2

ii) **At least** one odd numeral is chosen 2

Question 4 (12 marks) Use a SEPARATE writing booklet **Marks**

(a) i) Show that $\cos x - \sqrt{3}\sin x \equiv 2\cos(x + 60)^\circ$ 2

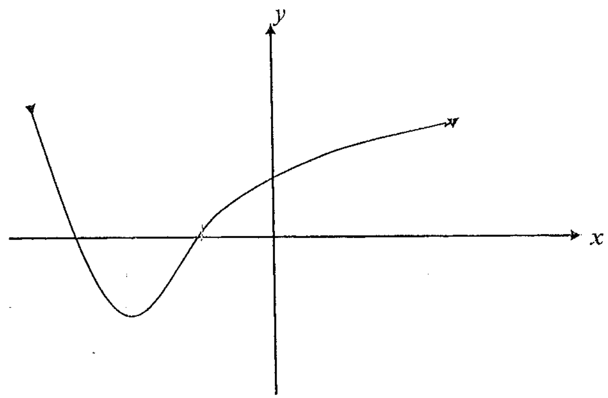
ii) Hence solve $\cos x - \sqrt{3}\sin x = 2$ over $0^\circ \le x \le 360^\circ$ 2

(b) i) Show that the acute angle θ between the lines $x - y = 0$ and $x - 2y = 0$ is such that $\tan\theta = \frac{1}{3}$. 2

ii) Find the equation of the other line $y = mx$ which is also inclined at the same acute angle θ to the line $x - 2y = 0$. 2

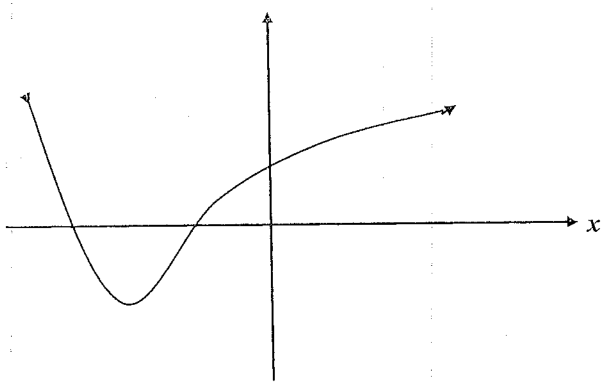
(c) Show that the line $y = x$ is a tangent to the circle $(x-2)^2 + y^2 = 2$ 2

(d) The graph shown is the function $y = f(x)$. On the answer sheet provided Sketch the gradient function $y = f'(x)$. 2



Answer sheet for Question 4 (d) **Name...................................**

The graph shown is the function $y = f(x)$. On the same number plane below sketch the gradient function $y = f'(x)$. 2

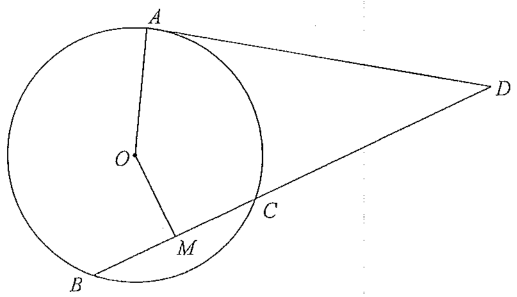


Question 5 (12 marks) Use a SEPARATE writing booklet **Marks**

(a) i) If $y = x\sqrt{x+3}$ find the gradient function $\dfrac{dy}{dx}$ 2

ii) Find the coordinates of the point, P on the curve $y = x\sqrt{x+3}$ where the tangent is parallel to the x axis. 2

(b)



A, B and C are points on a circle with centre O. The tangent to the circle at A meets BC produced at D. M is the midpoint of BC.

Copy the diagram into your answer book.

i) Show that $AOMD$ is a cyclic quadrilateral. 3

ii) Give a reason why OD is a diameter of the circle through A, O, M and D. 1

(c) If $t = \tan\dfrac{x}{2}$ show that $\dfrac{1-\cos x}{\sin x} = t$ 2

(d) 7 boys and 2 girls are to be seated around a circular table. In how many ways can this be done so that the girls are always seated together? 2

End of examination

Question 1

a) $\dfrac{1}{p^2 - pq} - \dfrac{1}{pq - q^2}$

$= \dfrac{1}{p(p-q)} - \dfrac{1}{q(p-q)}$

$= \dfrac{q-p}{pq(p-q)}$ 1 mark

$= \dfrac{-1}{pq}$ 2 marks

b) i). $\sin(A-B) = \sin A\cos B - \cos A \sin B$ 1 mark

ii). $\sin 15 = \sin(45-30)$

$= \sin 45 \cos 30 - \cos 45 \sin 30$ 1 mark

$= \dfrac{1}{\sqrt{2}} \cdot \dfrac{\sqrt{3}}{2} - \dfrac{1}{\sqrt{2}} \cdot \dfrac{1}{2}$

$= \dfrac{\sqrt{3}-1}{2\sqrt{2}}$ 2 marks

c) $(n+1)! - n!$

$= n!\,((n+1)-1)$ 1 mark - $n!$ as HCF

$= n!\,n$ 2 marks

d). $\dfrac{x+2}{x} \geq 0$

$\dfrac{x^2(x+2)}{x} \geq 0 \cdot x^2$

$x(x+2) \geq 0$ 1 mark

Consider $x = 0, -2$.

test: $x = -1$.

$-1(-1+2) \geq 0$ false.

$\therefore \{x : x \leq -2 \cup x > 0\}$

2 marks - $-2 \leq x < 0$.

or

2 marks - ~~correct~~ $x \leq -2 \cup x \geq 0$.

3 marks - correct sol.

e). $\tan x = 1$.

$(\tan x = \tan 45°)$. 1 mark

$x = 180n + 45$ 2 marks.

Question 2

a) $PT^2 = AT \times BT$ (Square of the tangent equal to product of intercepts of secants) — 1 mark - Correct reason or 1 mark - Correct formula

let $BT = x$

$8^2 = (5+x)x$

$64 = 5x + x^2$

$x^2 + 5x - 64 = 0$

$x = \frac{-5 \pm \sqrt{25+256}}{2}$ — 2 marks - Correct Quadratic

$= \frac{-5 \pm \sqrt{281}}{2}$ — ~~2 marks - Correct Sol~~

$= 5.9$ or -10.9

But $x > 0$ only

$\therefore BT = 5.9$ (both cm) — 3 marks - Correct Sol.

b) $P = \left(\frac{nx_1 + mx_2}{m+n}, \frac{ny_1 + my_2}{m+n}\right)$

$m:n = 2:3$

$P = \left(\frac{3 \times -2 + 2 \times 1}{2+3}, \frac{3 \times 1 + 2 \times 7}{2+3}\right)$ — 1 mark.

$= \left(-\frac{4}{5}, \frac{17}{5}\right)$ — 2 marks.

c) $\lim_{x \to \infty} \frac{\frac{3x}{x^2} + \frac{2x^3}{x^2}}{\frac{5x^2}{x^2} - \frac{1}{x^2}}$ — 1 mark

$= \lim_{x \to \infty} \frac{\frac{3}{x} + 2x}{5 - \frac{1}{x^2}}$

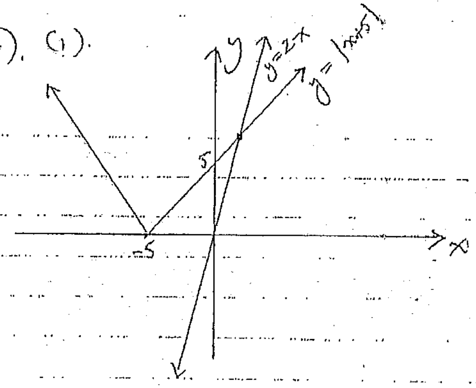
$= \frac{2}{5}x$ — 2 marks

d) FUNCTIONS.

tot $= \frac{9!}{2!}$ — 1 mark - 9!

$= 181\,440$ — 2 marks - $\frac{9!}{2!}$ or correct val.

e) (i) — 1 mark



(ii) Case 1: $x+5 \geq 0$

$2x = x+5$

$x = 5$ — 1 mark

(iii) Hence, if $2x \leq |x+5|$

$\{x : x \leq 5\}$ — 1 mark.

Year 11 Extension

Question 3

a i) $\cos 2x + 3\cos x + 2 = 0$

$2\cos^2 x - 1 + 3\cos x + 2 = 0$

$2\cos^2 x + 3\cos x + 1 = 0$

1, Correct use of $\cos 2x$ theorem

2, Simplification to answer.

ii) $(2\cos x + 1)(\cancel{\ }\cos x + 1) = 0$

$\cos x = -1, -\frac{1}{2}$

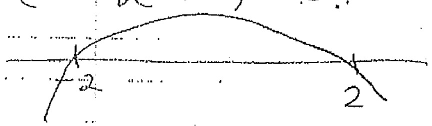
$x = 180°, 120°, 240°$

1, Correct factorisation or use of similar solving technique.

2, Answers.

b) $4 - x^2 \geq 0$

$(2-x)(2+x) \geq 0$

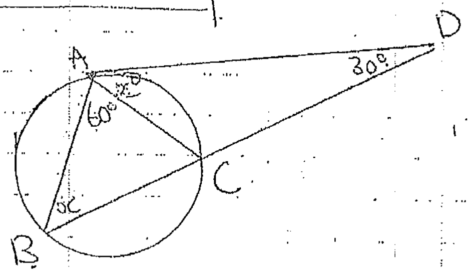


$\boxed{-2 \leq x \leq 2}$

1, Correct initial statement $4 - x^2 \geq 0$ or graph

2, Correct solve to $-2 \leq x \leq 2$.

c)



$\angle DAC = x$ (angle in the alternate segment)

$2x + 30 + 60 = 180$ (Angle sum of triangle)

$2x = 90$

$x = 45°$

1, Correct use of tangent theorem

2, Correct solution to $x = 45°$.

d i) $1 = {}^4C_4$

ii) ${}^9C_4 - 1 = 125$

1, Correct statement ${}^4C_4$

2, Answer.

1, Correct use of complementary events.

2, Answer.

Q4.

a) i) $\cos x - \sqrt{3}\sin x = A\cos(x+\alpha)$ $\quad 0° < \alpha < 90°$

$= A\cos x\cos\alpha + A\sin x\sin\alpha$.

equating coefficients:

$A\cos\alpha = 1$

$A\sin\alpha = \sqrt{3}$.

$A^2(\cos^2\alpha + \sin^2\alpha) = 1+3$
$= 4$.
$A = 2$.

$\frac{A\sin\alpha}{A\cos\alpha} = \tan\alpha = \sqrt{3}$

$\alpha = 60°$

$\therefore \cos x - \sqrt{3}\sin x = 2\cos(x+60°)$

1 mark each showing $A = 2$, $\alpha = 60°$

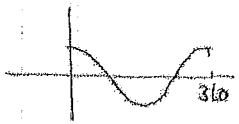
ii) $\cos x - \sqrt{3}\sin x = 2.$ $\quad 0° \le x \le 360°$

ie $2\cos(x+60°) = 2$ $\quad 60° \le x+60° \le 420°$

$\cos(x+60°) = 1$ 1 mark

$x + 60° = 360°$

$x = 300°$ 1 mark



b) i) $x - y = 0$ $\qquad x - 2y = 0$

$x = y \Rightarrow m_1 = 1$. $\qquad y = \frac{x}{2} \Rightarrow m_2 = \frac{1}{2}$.

$\tan\theta = \left|\frac{1-\frac{1}{2}}{1+\frac{1}{2}}\right|$ 1 mark correct use of formula

$= \left|\frac{\frac{1}{2}}{\frac{3}{2}}\right|$

$= \frac{1}{3}$. 1 mark correct.

ii) $y = mx$

$\frac{1}{3} = \left|\frac{\frac{1}{2} - m}{1 + 1.\frac{m}{2}}\right|$ 1 mark correctly setting equation

$\frac{1}{3} = \frac{1-2m}{2+m}$

$2 + m = 3 - 6m$

$7m = 1$

$m = \frac{1}{7}$

$\therefore y = \frac{x}{7}$ 1 mark.

c). $(x-2)^2 + y^2 = 2$ is a circle centre $(2,0)$ with radius $= \sqrt{2}$.

If $y = x$ a tangent, then distance b/w $y = x$ and centre of circle will be $\sqrt{2}$ u.

$x - y = 0$ $\quad (2,0)$

$d = \left|\frac{2+0+0}{\sqrt{2}}\right|$

$= \sqrt{2}$ u

$\therefore$ tangent.

2 marks correct any method.

Question 5

a i)

$u = x$
$u' = 1$
$v = (x+3)^{\frac{1}{2}}$
$v' = \frac{1}{2(x+3)^{\frac{1}{2}}}$

$y' = (x+3)^{\frac{1}{2}} + \frac{x}{2(x+3)^{\frac{1}{2}}}$

$y' = \frac{2(x+3) + x}{2(x+3)^{\frac{1}{2}}}$

$y' = \frac{3x+6}{2\sqrt{x+3}}$

1, Correct use of product rule
2, Answer from line 1

ii) Parallel to x-axis means $m = 0$

$\therefore y' = 0$

$0 = \frac{3x+6}{2\sqrt{x+3}}$

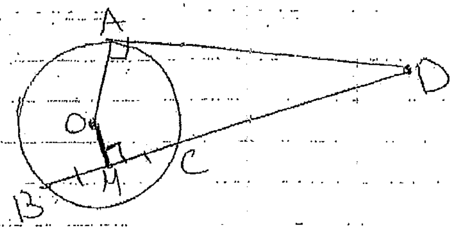
$3x + 6 = 0$
$3x = -6$
$x = -2$

@ $x = -2$, $y = -2\sqrt{-2+3}$
$y = -2$

$\therefore$ P is the point $(-2, -2)$

1, Correct sub of $y' = 0$
2, Answer $P(-2,-2)$

b i)



$\angle OAD = 90°$ (Angle between tangent and radius)

$\angle OMC = 90°$ (Line drawn from centre to chord bisects chord)

$\therefore \square AOMD$ is cyclic (opposite angles supplementary)

1, Use of tangent or chord theorem
2, Use of both tangent and chord theorem
3, statement of cyclic quad theorem

ii) Angles in a semi-circle are right angles

1, Semi-circle angle theorem

c) $\frac{1 - \frac{1-t^2}{1+t^2}}{\frac{2t}{1+t^2}} = t$

$\frac{1+t^2-(1-t^2)}{2t} = t$

$\frac{\cancel{2}t^{\cancel{2}}}{\cancel{2t}} = t$

$t = t$

LHS = RHS

1, Sub of t-result formulae
2, Correct simplification to LHS = RHS

d) 8 units

No. of ways $= 7! \times 2$

$= 10080$

1, Correct use of 7!

2, Correctly taking into account 2

4d)