INDEPENDENT TRIAL

2005
Higher School Certificate
Trial Examination

Mathematics

General Instructions

- Reading time – 5 minutes
- Working time – 3 hours
- Board approved calculators may be used
- Write using black or blue pen
- A table of standard integrals is provided at the back of the paper
- All necessary working should be shown in every question
- Write your student number and/or name at the top of every page

Total marks – 120

- Attempt Questions 1 – 10
- All questions are of equal value

This paper MUST NOT be removed from the examination room

STUDENT NUMBER/NAME: ....

STUDENT NAME / NUMBER ....

Question 1 (12 marks) **Marks**

(a) Express $\frac{1}{\sqrt{5}-2}$ with a rational denominator. **2**

(b) The thickness of a cat's whisker is $0.0000598\,\text{m}$. Write this in scientific notation correct to 2 significant figures. **2**

(c) Simplify: $\frac{3}{x-1}-\frac{2}{x+1}$ **2**

(d) Solve the pair of simultaneous equations: **2**

$$x-2y=9$$
$$2x+y=8$$

(e) Differentiate with respect to x: $\log_e 3x-\frac{3}{x}$ **2**

(f) Solve: $x^3 = 4x$ **2**

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Question 2 (12 marks) ***Start a new page*** **Marks**

(a) (i) Evaluate: $\int_0^{\frac{\pi}{3}} \cos 3x \, dx$ **2**

(ii) Find: $\int \frac{3x^2}{x^3-5}\,dx$ **2**

(b) Differentiate with respect to x.

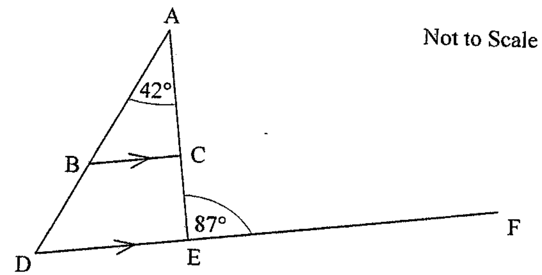
(i) $\left(e^x-3\right)^4$ **2**

(ii) $x^2 \cos x$ **2**

(c) In the diagram below, ADE is a triangle. B and C lie on AD and AE respectively such that BC is parallel to DE. Line DE is produced to F. **3**

$\angle AEF = 87°$ and $\angle DAE = 42°$

Find the size of $\angle ABC$, giving reasons

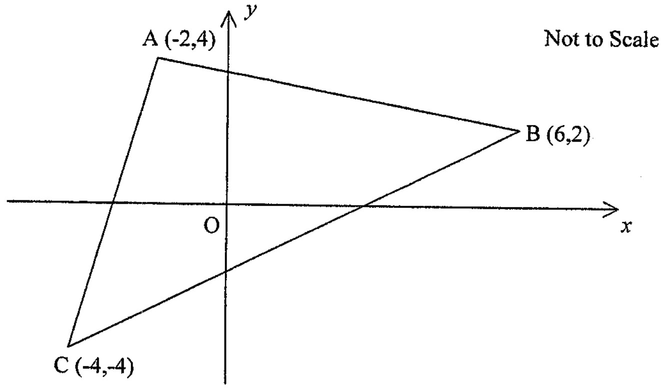


(d) Evaluate: $\sum_{r=1}^{4} 2^{1-r}$ **1**

STUDENT NAME / NUMBER ....

Question 3 (12 marks) ***Start a new page*** **Marks**

(a) The diagram below shows the points A (-2, 4), B (6, 2) and C (-4, -4). Copy or trace the diagram onto your worksheet.



(i) Calculate the length of the interval BC. **1**

(ii) Find the gradient of BC. **1**

(iii) Find the coordinates of M, the midpoint of BC. **1**

(iv) Show that the equation of l, the perpendicular bisector of BC, is $5x+3y-2=0$. **2**

(v) Show that l passes through A **1**

(vi) Hence or otherwise find the area of triangle ABC. **2**

(b) Solve: $\sqrt{3}\tan x=-1$ for $0\le x\le 2\pi$ **2**

(c) Solve: $|3-2x|\le 5$ **2**

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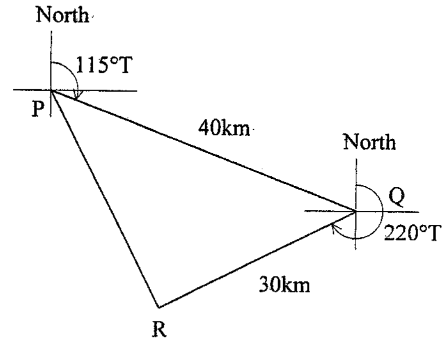
Question 4 (12 marks) ***Start a new page*** **Marks**

(a) From P the bearing of a lighthouse Q, 40 kilometres distant from P, is 115°T. From Q the bearing of a headland R, 30 kilometres from Q, is 220°T. This is illustrated in the diagram below.



(i) Find the size of $\angle PQR$. **1**

(ii) Show that the distance PR is given by: $PR^2=100(25-24\cos 75°)$ **1**

(iii) Find the bearing of R from P. **2**

(b) (i) On the same set of axes sketch the graphs $y=4-x^2$ and $y-3=0$ **2**

(ii) The graph $y-3=0$ cuts the parabola at A and B. Find the coordinates of A and B. **2**

(iii) Calculate the area enclosed by the graphs $y=4-x^2$ and $y-3=0$ **2**

(c) Find the value of k if the sum of the roots of $x^2-(k-1)x+2k=0$ is equal to the product of the roots. **2**

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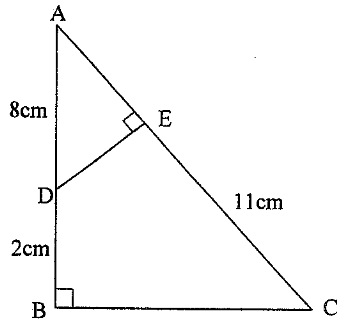
Question 5 (12 marks) ***Start a new page*** **Marks**

(a) ABC is a right-angled triangle in which $\angle ABC = 90°$. Points D and E lie on AB and AC respectively such that AC is perpendicular to DE.
AD = 8cm, EC = 11cm and DB = 2cm.



Not to Scale

(i) Prove that ΔABC is similar to ΔAED. **3**

(ii) Find the length of AE. **1**

(b) Tom is an enthusiastic gardener. He planted a silky oak tree three years ago when it was 80 centimetres tall. At the end of the first year after planting, it was 130 centimetres tall, that is it grew 50 centimetres. Each years growth was then 90% of the previous years.

(i) What was the growth of the silky oak in the second year? **1**

(ii) How tall was the silky oak after three years? **1**

(iii) Assuming that it maintains the present growth pattern, explain why it will never reach a height of 10 metres. **2**

(iv) In which year will the silky oak reach a height of 5 metres? **2**

(c) For what values of k does $x^2 - (2+k)x + 4 = 0$ have real roots? **2**

STUDENT NAME / NUMBER ....

Question 6 (12 marks) ***Start a new page*** **Marks**

(a) For the function; $f(x) = 8x^3 - 8x^2$

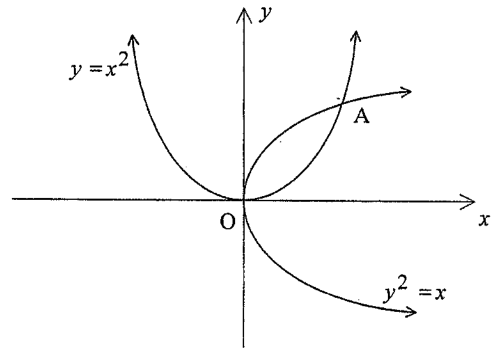
(i) Find the stationary points and determine their nature. **3**

(ii) Sketch the graph of the function $y = f(x)$ showing these stationary points. **2**

(iii) Show the point(s) at which $y = f(x)$ cuts the x-axis. **1**

(iv) Determine the values of x for which $f(x)$ is positive. **1**

(b) The curves $y = x^2$ and $y^2 = x$ are shown in the diagram below. They intersect at the origin and at the point A.



Not to Scale

(i) Show that A has coordinates (1,1) **1**

(ii) The area bounded by the two curves is rotated about the y-axis. Determine the volume of the solid generated. **2**

(c) Solve $x^4 - 4x^2 - 5 = 0$ **2**

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Question 7 (12 marks) ***Start a new page*** **Marks**

(a) Plants infected with a fungus are sprayed with a newly developed fungicide. In field tests with this fungicide, it was found that the probability that the fungus was eliminated was 4 in 5.
Two plants are selected at random from a group of sprayed plants. What is the probability that:

(i) the fungus has been eliminated from both plants? **1**

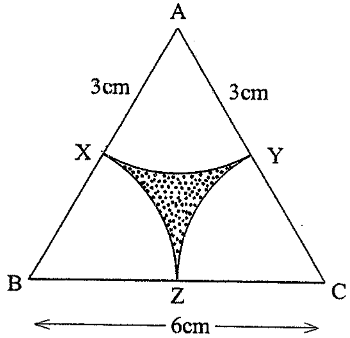
(ii) exactly one plant is still infected with the fungus? **2**

(iii) both plants are still infected with the fungus? **1**

(iv) at least one plant is still infected with fungus? **2**

(b) Find the equation of the tangent to the curve $y = e^{2x}$ at the point on the curve where $x = 1$ (leaving your answer in terms of e). **2**

(c) ABC is an equilateral triangle with sides of length 6 cm. An arc, centre A, and radius 3cm cuts AB and AC at X and Y respectively. This is repeated at B and C, as shown in the diagram.



Not to Scale

(i) Explain why $\angle ABC = \frac{\pi}{3}$ radians **1**

(ii) Find the shaded area enclosed by arcs XY, YZ and ZX **3**

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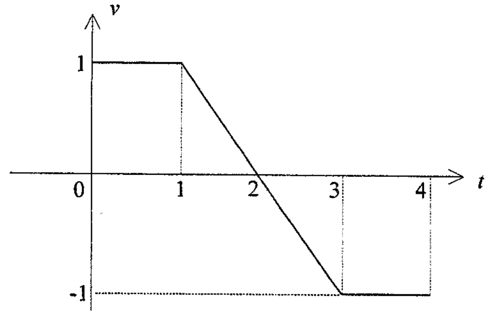
Question 8 (12 marks) ***Start a new page*** **Marks**

(a) (i) Show that $\sin\theta\tan\theta = \sec\theta - \cos\theta$ **2**

(ii) Hence, or otherwise, solve: $\sin\theta\tan\theta = 0$ for $0 \le \theta \le 2\pi$ **2**

(b) The graph below shows the velocity, v, of a particle as a function of time t for $0 \le t \le 4$ as it moves along the x-axis. When $t = 0$ the particle is at the origin ($x = 0$).



Not to Scale

(i) At what time is the particle furthest from the origin? **1**

(ii) Briefly describe the motion of the particle for $0 \le t \le 4$. **2**

(c) Using Simpson's Rule, with three function values, find an approximate value for: **3**

$$\int_0^1 \frac{4dx}{x^2+1}$$

(d) Given $y = e^{kx}$ and $\frac{dy}{dx} = 3y$, find the value of k. **2**

STUDENT NAME / NUMBER ....

Question 9 (12 marks) ***Start a new page*** **Marks**

(a) Ella borrowed $180 000 to finance an extension on her home. She agreed to pay off the loan in equal monthly instalments of $P, paid at the end of each month, at an interest rate of 6% per annum, compounded monthly.

(i) Show that after the first instalment is paid, the amount owing on the loan is: **1**

$$\$[180\,000(1.005) - P]$$

(ii) Show that after three months she owes: **2**

$$\$\left[180\,000(1.005)^3 - P\left((1.005)^2 + (1.005) + 1\right)\right]$$

(iii) If the loan is repaid after 8 years, find the value of P, the monthly instalment. **3**

(b) A particle moves in a straight line so that its distance x in metres from a fixed point O is given by:

$$x = 2t + e^{-2t} \text{ where } t \text{ is measured in seconds}$$

(i) What is the velocity of the particle when $t = \frac{1}{2}$ sec ? **2**

(ii) Show that initially the particle is at rest. **1**

(iii) As t increases, find the limiting velocity of the particle. **1**

(iv) Draw a neat sketch of the graph of the velocity as a function of time. **1**

(v) Using v as the velocity and a as the acceleration, show that $a = 4 - 2v$ **1**

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Question 10 (12 marks) ***Start a new page*** **Marks**

(a) A flat circular metal plate is being heated so that the rate of increase of the area, A metres^2, after t hours, is given by:

$$\frac{dA}{dt} = \frac{1}{50}\pi t$$

Initially the plate has a radius of 5 metres.

Leaving all answers in exact form;

(i) Find the initial area. **1**

(ii) Find an expression for the area after t hours **2**

(iii) Calculate the radius after 5 hours. **1**

(iv) How long does it take for the area to increase by 20%? **1**

(b) At the beginning of a drought, the number of sheep on a property was 285 000. Six months after the drought commenced this number had reduced to 202 000. Sheep numbers have continued to decrease so that at any time t, the number of sheep, S, is given by the formula:

$$S = Ae^{-kt}$$

where A and k are constants and t is the number of months since the drought commenced.

(i) Find the values of A and k. **2**

(ii) Show that $\frac{dS}{dt} = -kS$. **1**

(iii) How many sheep will there be one year after the drought started? **1**

(iv) When will the flock reach one-third of its original size? **2**

(v) Find the rate of decrease in the number of sheep at this time. **1**

End of Paper

NSW INDEPENDENT TRIAL EXAMS - 2005

Mathematics (2 Unit) 2005

Suggested Solutions

Marks

Question 1.

(a) $\frac{\sqrt{5}+2}{5-4} = \sqrt{5}+2$ — 2

(b) 6.0×10^{-5} — 2

(c) $\frac{3x+3-2x+2}{x^2-1}$

$= \frac{x+5}{x^2-1}$ — 2

(d) $x = 5,\ y = -2$ — 2

(e) $\frac{3}{x} + 3x^{-2}$

$= \frac{3}{x} + \frac{3}{x^2}$ — 2

(f) $x(x^2-4) = 0$

$x = 0, \pm 2$ — 2

Question 2.

(a)(i) $\left[\frac{1}{3}\sin 3x\right]_0^{\pi/3}$

$= \frac{1}{3}(\sin\pi - \sin 0)$

$= 0$ — 2

(ii) $\ln(x^3-5) + C$ — 2

(b)(i) $4e^x(e^x-3)^3$ — 2

(ii) $2x\cos x - x^2\sin x$ — 2

(c) $\angle BCE = 87°$ (Alternate $\angle$'s $BC \parallel OE$)

$\angle ABC + 42° = 87°$ (Exterior $\angle$'s Thm).

$\therefore \angle ABC = 45°$ — 3

Marks

(d) $2^0 + 2^{-1} + 2^{-2} + 2^{-3}$

$= 1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8}$

$= \frac{15}{8}$ — 1

Question 3.

(a)(i) $BC = \sqrt{(6+4)^2 + (2+4)^2}$

$= \sqrt{136}$

$= 2\sqrt{34}$ — 1

(ii) $m = \frac{2+4}{6+4} = \frac{3}{5}$ — 1

(iii) $M = \left(\frac{6-4}{2}, \frac{2-4}{2}\right)$

$= (1, -1)$ — 1

(iv) $y + 1 = -\frac{5}{3}(x-1)$

$3y + 3 = -5x + 5$

$5x + 3y - 2 = 0$ — 2

(v) $5(-2) + 3(4) - 2$

$= -10 + 12 - 2$

$= 0$ — 1

(vi) $AM = \sqrt{(-2-1)^2 + (4+1)^2}$

$= \sqrt{34}$

$\therefore$ Area $= \frac{1}{2} \cdot 2\sqrt{34} \cdot \sqrt{34}$

$= 34$ units^2 — 2

(b) $\tan x = -\frac{1}{\sqrt{3}}$

$\therefore x = \frac{5\pi}{6}, \frac{11\pi}{6}$ — 2

The Trial HSC examination, marking guidelines /suggested answers and 'mapping grid' have been produced to help prepare students for the HSC to the best of our ability.

Individual teachers/schools may alter parts of this product to suit their own requirements.

Marks

(c) $-5 \le 3 - 2x \le 5$

$-8 \le -2x \le 2$

$4 \ge x \ge -1$

$-1 \le x \le 4$ — 2

Question 4.

(a)(i) $P\hat{Q}R = 25° + 50° = 75°$ — 1

(ii) $PR^2 = 40^2 + 30^2 - 2.40.30\cos 75°$

$= 2500 - 2400\cos 75°$

$= 100(25 - 24\cos 75°)$ — 1

(iii) $\frac{43.35}{\sin P\hat{Q}R} = \frac{30}{\sin R\hat{P}Q}$

$\sin \angle RPQ = \frac{30\sin 75°}{43.35}$

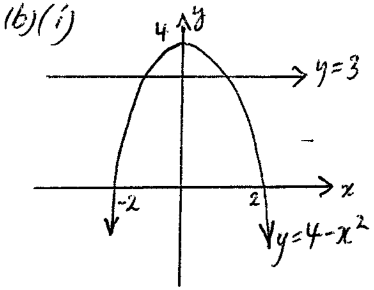
$\angle RPQ = 41°57'$

$\doteqdot 42°$

$\therefore$ Bearing is $115 + 42$

$= 157°T$ — 2

(b)(i)



— 2

(ii) $4 - x^2 = 3$

$x^2 = 1$

$x = \pm 1$

$A(1,3),\ B(-1,3)$ — 2

Marks

(iii) $A = 2\int_0^1 (4-x^2) - 3\ dx$

$= 2\left[x - \frac{x^3}{3}\right]_0^1$

$= 2\left(1 - \frac{1}{3}\right)$

$= 4/3$ units^2 — 2

(c) $(k-1) = 2k$

$k = -1$ — 2

Question 5.

(a)(i) $\angle A$ is common.

$\angle AED = \angle ABC = 90°$ ($DE \perp AC$, $\angle ABC = 90°$)

$\therefore \angle ADE = \angle ACB$ (Angle sum Δ is $180°$)

$\therefore \Delta ABC \parallel\parallel \Delta AED$ (ie. equiangular) — 3

(ii) $\frac{AD}{AC} = \frac{AE}{AB}$

Let $x = AE$

$\frac{8}{11+x} = \frac{x}{10}$

$80 = x^2 + 11x$

$x^2 + 11x - 80 = 0$

$(x+16)(x-5) = 0$

$x = 5$ as $x \ne -16$

ie. $AE = 5$ unit — 1

(b)(i) 90% of $50 = 45$ cm — 1

(ii) Growth 3 $= 90\%$ of 45

$= 40.5$ cm

$\therefore HT = 80 + 50 + 45 + 40.5$

$= 215.5$ cm — 1

Marks Marks

(iii) G.P. with $a = 50$, $r = 0.9$

$S_\infty = \frac{50}{1-0.9}$

$= 500$ cm

$= 5$m

$\therefore$ Maximum HT = 5.8m

$\therefore$ Never gets close to 10m. 2

(iv) $500 = 80 + (50 + 45 + \ldots + T_n)$

$= 80 + \frac{50(1-(0.9)^n)}{1-0.9}$

$0.9^n = \frac{80}{500} = 0.16$

$n = \frac{\ln 0.16}{\ln 0.9}$

$= 17.39$

ie. in the 18th year. 2

(c) $\Delta \geq 0$

$(2+k)^2 - 4.1.4 \geq 0$

$k^2 + 4k - 12 \geq 0$

$(k+6)(k-2) \geq 0$

$\therefore k \leq -6$ or $k \geq 2$ 2

Question 6.

(a)(i) $f'(x) = 24x^2 - 16x = 0$

$8x(3x-2) = 0$

$x = 0, \ 2/3$

$f''(x) = 48x - 16$

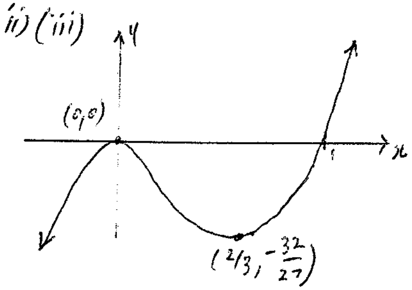
when $x = 0$, $y = 0$, $f''(x) = -16 < 0$

$\therefore (0,0)$ max T.P.

when $x = 2/3$, $y = -\frac{32}{27}$, $f''(x) = 16 > 0$

$\therefore (2/3, -\frac{32}{27})$ Min T.P. 3

(ii)(iii)



2

1

(iv) $f(x) > 0$ when $x > 0$ 1

(b) (i) $y = x^2$, $y^2 = x$

$\therefore x^4 = x$

$x^3(x-1) = 0$

$x = 0, 1$

At A, $x \neq 0$ $\therefore A(1,1)$ 1

(ii) $V = \pi \int_0^1 (x - x^4)\, dx$

$= \pi \left[\frac{x^2}{2} - \frac{x^5}{5} \right]_0^1$

$= \frac{3\pi}{10}$ units^3 2

(c) Let $X = x^2$

$X^2 - 4X - 5 = 0$

$(X-5)(X+1) = 0$

$X = 5$ as $X \neq -1$

$\therefore x = \pm\sqrt{5}$ 2

Question 7.

(a)(i) $P(NI, NI) = \left(\frac{4}{5}\right)^2 = \frac{16}{25}$ 1

(ii) $P(1I) = \frac{4}{5} \cdot \frac{1}{5} + \frac{1}{5} \cdot \frac{4}{5}$

$= 8/25$ 2

(iii) $P(I, I) = \left(\frac{1}{5}\right)^2 = \frac{1}{25}$ 1

(iv) $P(\text{at least } 1I) = 1 - P(NI, NI)$

$= 1 - 16/25$

$= 9/25$ 2

Marks Marks

(b) $x = 1$, $y = e^2$

$y' = 2e^{2x}$

$m = 2e^2$

$\therefore y - e^2 = e^2(x-1)$

$y = e^2 x$ 2

(c)(i) All angles equal

$\therefore \angle ABC = \frac{1}{3}\pi$

$= \pi/3$ 1

(ii) Sector $AXY = \frac{1}{2} \cdot 3^2 \cdot \frac{\pi}{3}$

$= \frac{3\pi}{2}$

There are 3 sectors $= 9\pi/2$

Δ Area $= \frac{1}{2} \cdot 6 \cdot 6 \cdot \sin \pi/3$

$= 18 \sin \pi/3$

$= \frac{18\sqrt{3}}{2}$

$= 9\sqrt{3}$

$\therefore$ Shaded area $= 9\sqrt{3} - \frac{9\pi}{2}$ 3

Question 8.

(a)(i) LHS $= \sin\theta \cdot \frac{\sin\theta}{\cos\theta}$

$= \frac{1 - \cos^2\theta}{\cos\theta}$

$= \frac{1}{\cos\theta} - \cos\theta$

$= \sec\theta - \cos\theta$

$=$ RHS 2

(ii) $\sec\theta - \cos\theta = 0$

$\frac{1}{\cos\theta} = \cos\theta$

$\cos^2\theta = 1$

$\cos\theta = \pm 1$ 2

$\therefore \theta = 0, \pi, 2\pi$

(b)(i) Velocity is +ve until $t = 0$ $\therefore$ Moving away. 1

(ii) First second moving away from 0 at a constant speed of 1. 2nd second slows down and stops at 2 seconds. Then moves in reverse direction evenly accelerating to 3 seconds. Then moves back towards 0 at a constant speed of 1. 2

(c) $\int_0^1 \frac{4\,dx}{x^2+1} \doteqdot \frac{1-0}{6}\left(4 + 4 \cdot \frac{16}{5} + 2\right)$

$= \frac{1}{6} \times 18.8$

$= 3.1\dot{3}$ 3

(d) $y' = ke^{kx}$

$\therefore 3e^{kx} = ke^{kx}$

$\therefore k = 3$ as $e^{kx} \neq 0$ 2

Question 9.

(a)(i) $A_1 = 180000 + 0.5\%$ of \$180000 $- P$

$= (1.005)(180000) - P$ 1

(ii) $A_2 = 1.005(180000(1.005) - P) - P$

$= 180000(1.005)^2 - P(1.005 + 1)$

So, $A_3 = ((1.005)A_2 - P)$

$= 180000(1.005)^3 - P(1.005^2 + 1.005 + 1)$ 2

	Marks		Marks

(iii) 8 yrs = 96 months and $A = 0$.

$$0 = 180000(1.005)^{96} - P\left\{\frac{1(1.005)^{96} - 1}{1.005 - 1}\right\}$$

$$P = \frac{180000(1.005)^{96} \times 0.005}{(1.005)^{96} - 1}$$

$= \$2365.46$ or $\$2365$ — **3**

(b) $x = 2t + e^{-2t}$

$v = 2 - 2e^{-2t}$

$a = 4e^{-2t}$

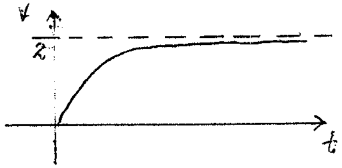
(i) $t = \frac{1}{2}$, $v = 2 - 2e^{-1}$

$= 2 - 2/e$ m/s — **2**

(ii) $t = 0$, $v = 2 - 2e^{0}$

$= 0$ — **1**

(iii) As $t \to \infty$, $v \to 2$ — **1**

(iv) 

— **1**

(v) $a = 4e^{-2t}$

$v = 2 - 2e^{-2t}$

$\therefore 2e^{-2t} = 2 - v$

$\therefore 4e^{-2t} = 4 - 2v$

$a = 4 - 2v$ — **1**

Question 10.

(a)(i) $A = \pi . 5^2 = 25\pi$ — **1**

(ii) $A = \frac{1}{50}\pi \frac{t^2}{2} + c$

when $t = 0$, $A = 25\pi$ $\therefore c = 25\pi$

$\therefore A = \frac{1}{100}\pi t^2 + 25\pi$ — **2**

(iii) $t = 5$

$A = \frac{1}{100} . \pi . 25 + 25\pi$

$= \pi/4 + 25\pi$

$\pi r^2 = \frac{101\pi}{4}$

$r = \frac{\sqrt{101}}{2}$ m — **1**

(iv) $A = 25\pi + 20\%$ of 25π

$= 30\pi$ m²

$30\pi = \frac{1}{100}\pi t^2 + 25\pi$

$5 = t^2/100$

$t = \sqrt{500}$

$= 10\sqrt{5}$ hours — **1**

(b)(i) $A = 285000$

$202000 = 285000\, e^{-k6}$

$\frac{202}{285} = e^{-6k}$

$k = \ln \frac{202}{285} / -6$

$= 0.0574$ — **2**

(ii) $\frac{dS}{dt} = -k(Ae^{-kt})$

$= -kS$ — **1**

(iii) $t = 12$, $S = 285000\, e^{-.0574 \times 12}$

$= 143120.8217$

$= 143120$ sheep — **1**

(iv) $\frac{1}{3} \Rightarrow 95000$

$t = \frac{\ln \frac{1}{3}}{-0.0574} = 19.13\ldots$

$= 19$ months. — **2**

(v) $\frac{dS}{dt} = -0.0574 \times 95000$ — **1**

Decreasing at 5453 sheep/month