YEAR 11 — INDEX LAWS - WORKSHEET — 2 UNIT

(1) Write each of the following with positive indices.

(i) gh^{-2}

(ii) $\dfrac{a^{-1}}{b^{-2}}$

(iii) $\dfrac{2^{-3}t^{5}}{3^{-1}v^{-2}}$

(iv) $(5y^{-3}z)^{-1}$

(2) Express in surd form

(i) $a^{\frac{1}{2}}$ (ii) $5^{-1}x^{-4/3}$

(iii) $p^{\frac{1}{5}}q^{-\frac{3}{5}}$ (iv) $a^{\frac{1}{4}}b^{-\frac{2}{3}}$

(3) Express the following without root signs and with positive indices.

(i) $\sqrt[5]{c^{2}}$

(ii) $\sqrt[3]{8h^{-1}k^{2}}$

(i) $\dfrac{\sqrt[4]{g^{-1}h^{3}}}{j^{-5}}$

(iv) $\dfrac{\sqrt{(x-3y)^{-2}}}{x^{-3/2}y^{3}}$

(4) Find the numerical values of:

(i) (a) 9^{-2} (b) 7^{0}

(c) $\dfrac{1}{4^{-3}}$ (d) $\dfrac{5^{-1}}{3^{-2}}$

(e) $27^{\frac{2}{3}}$ (f) $16^{1.25}$

(4) (ii) (a) $(11\frac{1}{9})^{\frac{1}{2}}$

(b) $(6\frac{1}{4})^{-1.5}$

(c) $(1\frac{9}{16})^{-\frac{3}{2}}$

(d) $(0.001)^{-\frac{2}{3}}$

(5) Simplify the following

(i) $y^{\frac{3}{5}}\cdot y^{-\frac{2}{5}}\cdot y^{\frac{4}{5}}$

(ii) $\dfrac{9b^{-\frac{3}{4}}\times 2b^{\frac{1}{2}}}{6b^{-\frac{1}{4}}}$

(iii) $\left[6x^{-\frac{1}{2}}\div 2x^{\frac{2}{3}}\right]\times 3x^{\frac{5}{6}}$

(6) Simplify

(i) $\dfrac{2^{x}\cdot 4^{x+1}}{8^{x-2}}$

(ii) $\dfrac{3^{-n}\cdot 9^{2}}{3^{3n-2}\cdot 27}$

(iii) $\dfrac{25^{2n-1}\times 5^{1-n}}{25^{1-n}\times (5^{n})^{3}}$

(iv) $\dfrac{12^{3x}\cdot 8^{-1}}{9^{-x}\cdot 4^{-2x}\cdot 6^{x}}$

(7) Solve these equations

(i) $9^{x}\cdot 27^{x-2}=3^{-x}$

(ii) $9^{x}=\sqrt{3}$

(iii) $(0.01)^{x}=100\,(10^{-x})$

(iv) $(\frac{1}{9})^{2x-1}=3\,(27^{-x})$

(8) Solve for x, by first forming a quadratic equation.

(i) $2^{2x}-3.2^{x}+2=0$

(ii) $49^{x}-6.7^{x}=7$

(9) Simplify, by expanding each of the following:

(i) $x^{\frac{1}{3}}(x^{\frac{2}{3}}+5x^{-\frac{1}{3}})$

(ii) $(x^{-1}+y^{-1})^{2}$

(iii) $(x^{\frac{1}{3}}-1)(x^{\frac{2}{3}}+x^{\frac{1}{3}}+1)$

(10) Simplify

(i) $x^{-1}-y^{-1}$

(ii) $\dfrac{1}{x^{-1}-y^{-1}}$

(iii) $\dfrac{x^{-1}-y^{-1}}{x^{-2}-y^{-2}}$

CHALLENGE QUESTIONS

[1] Show that

$$(xy^{-1}-yx^{-1})\div(x^{2}y^{-2}-y^{2}x^{-2})=\frac{xy}{x^{2}+y^{2}}$$

[2] If $z=2+\sqrt{3}$, find the values of $z+z^{-1}$, $z^{2}+z^{-2}$ in simplest surd form

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(1)

(i) $\frac{g}{h^2}$ (ii) $\frac{b^2}{a}$ (iii) $\frac{3t^5v^2}{8}$

(iv) $\frac{y^3}{5z}$

(2) (i) $\sqrt{a}$ (ii) $\frac{1}{\sqrt[5]{l^4}}$

(iii) $\sqrt[5]{\frac{p}{q^3}}$ (iv) $\sqrt[4]{a} \cdot \sqrt[3]{\frac{1}{b^2}}$

(3) (i) $c^{\frac{2}{5}}$ (ii) $\frac{2k^{\frac{2}{3}}}{h^{1/3}}$ (iii) $\frac{j^5 h^{\frac{3}{4}}}{g^{\frac{1}{4}}}$

(iv) $\frac{x^{\frac{3}{2}}}{(x-3y)y^3}$

(4) (i) (a) $\frac{1}{81}$ (b) 1

(c) 64 (d) $\frac{9}{5} = 1\frac{4}{5}$

(e) 9 (f) 32

(ii) (a) $3\frac{1}{3}$ (b) $\frac{8}{125}$

(c) $\frac{64}{125}$ (d) 100

(5) (i) y (ii) 3 (iii) $\frac{9}{x^{1/3}}$

(6) (i) 256 (ii) 3^{3-4n} (iii) 5^{2n-3}

(iv) $2^{9x-3}\ 3^{4x}$

(7) (i) $x = 1$ (ii) $x = \frac{1}{4}$

(iii) $x = -2$ (iv) $x = 1$

(8) (i) $x = 0$ or 1

(ii) $x = 1$

(9) (i) $x + 5$ (ii) $x^{-2} + y^{-2} - 2x^{-1}y^{-1}$

$= \frac{1}{x^2} + \frac{1}{y^2} - \frac{2}{xy}$

(iii) $x - 1$

(10) (i) $\frac{y-x}{xy}$ (ii) $\frac{xy}{y-x}$

(iii) $\frac{xy}{x+y}$

CHALLENGE QUESTIONS

[1] L.H.S. $= \frac{\left(\frac{x}{y} - \frac{y}{x}\right)}{\left(\frac{x^2}{y^2} - \frac{y^2}{x^2}\right)}$

$= \left(\frac{x^2-y^2}{yx}\right) \div \left(\frac{x^4-y^4}{y^2x^2}\right)$

$= \frac{x^2-y^2}{xy} \times \frac{(xy)^2}{(x^2+y^2)(x^2-y^2)}$

Note: Difference of two squares.

$= \frac{xy}{x^2+y^2}$ = R.H.S. Q.E.D.

[2] $z + z^{-1} = z + \frac{1}{z}$

$= (2+\sqrt{3}) + \frac{1}{2+\sqrt{3}} \times \frac{2-\sqrt{3}}{2-\sqrt{3}}$

$= (2+\sqrt{3}) + \frac{2-\sqrt{3}}{4-3} = (2+\sqrt{3}) + (2-\sqrt{3})$

$= \underline{4}$

$z^2 + z^{-2} = z^2 + \frac{1}{z^2} = \underbrace{\left(z+\frac{1}{z}\right)^2 - 2}$

Check this step!

$= (4)^2 - 2$ (from above)

$= 16 - 2$

$= \underline{14}$