Exercise 2E Exam Practice

1 Given that

$$a = 3^{5x} \text{ and } b = 3^{x+2},$$

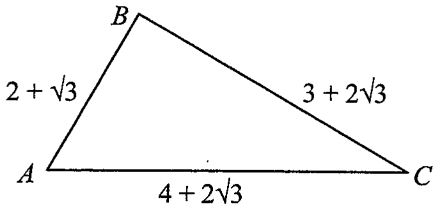
a express each of the following in the form 3^y, where y is a function of x:

i $3a$

ii b^3 **(2 marks)**

b find the value of x for which $3a = b^3$. **(3 marks)**

2



In triangle ABC, $AB = 2 + \sqrt{3}$, $AC = 4 + 2\sqrt{3}$ and $BC = 3 + 2\sqrt{3}$.

a Show that $\angle ABC = 90°$. **(4 marks)**

b Find in its simplest form the value of $\tan(\angle BAC)$. **(4 marks)**

3 Given that

$$25^{3x+1} = 5^{y+4}$$

a find an expression for y in terms of x. **(3 marks)**

Given also that

$$4^{3x-1} = 8^z$$

b show that $y = 3z$. **(3 marks)**

4 Given that

$$\frac{x}{\sqrt{2}-1} + 7 = 4x$$

find x in the form $a + b\sqrt{2}$ where a and b are integers. **(5 marks)**

5 A solid right-circular cylinder has base radius of $\dfrac{1}{\sqrt{5}-2}$ cm and height of $(3\sqrt{5} + 1)$ cm.

Show that the surface area of the cylinder can be expressed in the form $\pi(p + q\sqrt{5})$ cm^2 where p and q are integers to be found. **(6 marks)**

Exercise 2E Exam Practice

1 **a i** 3^{5x+1} **ii** 3^{3x+6} **b** $^5/_2$

2 **b** $\sqrt{3}$

3 **a** $y = 6x - 2$

4 $3 + \sqrt{2}$

5 $\pi(52 + 22\sqrt{5})$

Ex. 2E (Exam Practice)

2. a) $AB^2 + BC^2 = (2+\sqrt{3})^2 + (3+2\sqrt{3})^2$

$= 4 + 4\sqrt{3} + 3 + 9 + 12\sqrt{3} + 12$

$= 28 + 16\sqrt{13}$ ✓

$AC^2 = (4+2\sqrt{3})^2$

$= 16 + 16\sqrt{3} + 12$

$= 28 + 16\sqrt{3}$ ✓

$AB^2 + BC^2 = AC^2$ ✓

$\therefore \angle ABC = 90$ ✓

b) $\tan(\angle BAC) = \dfrac{3+2\sqrt{3}}{2+\sqrt{3}} \times \dfrac{2-\sqrt{3}}{2-\sqrt{3}}$ ✓

$= \dfrac{6 - 3\sqrt{3} + 4\sqrt{3} - 6}{1}$

$= \sqrt{3}$ ✓

4. $\dfrac{x}{\sqrt{2}-1} + 7 = 4x$

$\dfrac{x}{\sqrt{2}-1} \times \dfrac{\sqrt{2}+1}{\sqrt{2}+1} + 7 = 4x$

$x\sqrt{2} + x + 7 = 4x$

$x\sqrt{2} - 3x = -7$ ✓

$x(\sqrt{2}-3) = -7$

$x = \dfrac{-7}{\sqrt{2}-3} \times \dfrac{\sqrt{2}+3}{\sqrt{2}+3}$ ✓

$= \dfrac{-7(\sqrt{2}+3)}{-7}$

$= \sqrt{2}+3$

5. $S.A = 2\pi r^2 + 2\pi r \times \text{height}$ ✓

$= \left(\dfrac{4}{4\sqrt{5}-8}\right)^2 \pi + \dfrac{2}{2\sqrt{5}-4}\pi \times (3\sqrt{5}+1)$

problem area, try again

$= \dfrac{4}{20-16\sqrt{5}+16}\pi + \dfrac{6\sqrt{5}+2}{2\sqrt{5}-4}\pi$

$= \pi\left(\dfrac{4}{36-16\sqrt{5}} + \dfrac{6\sqrt{5}+2}{2\sqrt{5}-4}\right)$

$= \pi\left(\dfrac{4}{36-16\sqrt{5}} \times \dfrac{36+16\sqrt{5}}{36+16\sqrt{5}} + \dfrac{6\sqrt{5}+2}{2\sqrt{5}-4} \times \dfrac{2\sqrt{5}+4}{2\sqrt{5}+4}\right)$

$= \pi\left(\dfrac{144+64\sqrt{5}}{16} + \dfrac{60+24\sqrt{5}+4\sqrt{5}+8}{4}\right)$

$= \pi\left[\dfrac{16(9+4\sqrt{5})}{16} + \dfrac{68+28\sqrt{5}}{4}\right]$

$= \pi\left(9+4\sqrt{5} + \dfrac{4(17+7\sqrt{5})}{4}\right)$

$= \pi(9+4\sqrt{5}+17+7\sqrt{5})$

$= \pi(26+11\sqrt{5})\ cm^2$

3. a) $25^{3x+1} = 5^{y+4}$

$(5^2)^{3x+1} = 5^{y+4}$ ✓

$5^{6x+2} = 5^{y+4}$

$6x+2 = y+4$

$y = 6x-2$ ✓

b) $4^{3x-1} = 8^z$

$(2^2)^{3x-1} = (2^3)^z$ ✓

$2^{6x-2} = 2^{3z}$ ✓

$3z = 6x-2$

$\therefore y = 3z$ ✓