Inequalities and regions

A statement involving $>, \geq, <$ or $\leq$ is called an **inequality**.

Equations remain true if 'you do the same to both sides'.

The rule is not so simple for inequalities:

- the inequality remains true if we add the same number to both sides or subtract the same number;
- it also remains true if we multiply or divide by a positive number;
- however, if we **multiply or divide by a negative number, the direction of the inequality reverses.**

Check that you can follow the solutions of these inequalities.

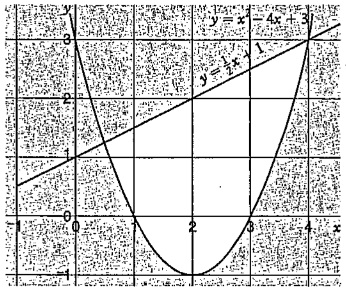
Solving linear equations ► page 22

	$x > 8 + 2x$		$2x^2 - 18 \leq 0$
Subtract $2x$ from both sides.	$^-x > 8$	Add 18 to both sides.	$2x^2 \leq 18$
Multiply both sides by $^-1$, remembering to reverse the sign.	$x < {^-8}$	Divide both sides by 2.	$x^2 \leq 9$
		There are positive and negative solutions.	$^-3 \leq x \leq 3$

Check by substituting, say, $x = {^-9}$ into both sides of the original inequality.

Check by substituting, say, $x = {^-3}$ and $x = 1$ into the original inequality.

Inequalities can be shown on a graph.
Check that the coordinates of all points above the line $y - \frac{1}{2}x = 1$ satisfy the inequality $y - \frac{1}{2}x > 1$ and points below the line satisfy $y - \frac{1}{2}x < 1$.
The line $y - \frac{1}{2}x = 1$ is the boundary between the two **regions** $y - \frac{1}{2}x > 1$ and $y - \frac{1}{2}x < 1$.
From the graph it is possible to see that points in the untinted region satisfy both the inequalities $y - \frac{1}{2}x < 1$ and $y > x^2 - 4x + 3$.
It is usually best to shade out the regions you do *not* want.



1 Solve these inequalities.

(a) $5x + 17 > 7$ (b) $x + 7 < 3x + 2$ (c) $3 - 2y < 11$ (d) $\frac{1}{2}(2 - 5n) \leq 11$

(e) $5 + 3x > \frac{x}{2}$ (f) $10 < 2 - 4x$ (g) $5(a - 2) - 8a \geq 0$ (h) $3x^2 - 12 > 0$

2 On graph paper, draw x- and y-axes, and mark each axis from $^-3$ to 5 using a scale of 2 centimetres to represent 1 unit.
On your diagram draw and label clearly the region which satisfies all of these inequalities.

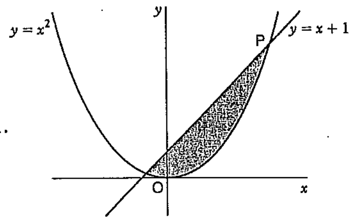
$y \geq {^-2}$
$y \leq 3x + 1$
$2x + y \leq 5$

MEG

3 The values of x satisfy the inequality $3x + 1 \leq 27 \leq 5x - 6$.
(a) (i) Find the largest possible value of x.
(ii) Find the smallest possible value of x.
(b) Write down all the possible integer values of x. MEG

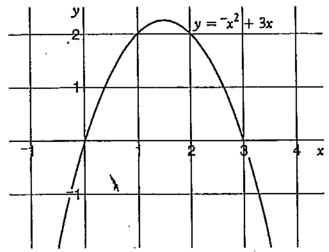
4 (a) What can you say about x if $13 - 6x$ is less than 25?
(b) If $y - z \leq 12$ and $z \leq 7$, what is the greatest value of y?

5 The diagram shows the graphs of $y = x + 1$ and $y = x^2$.
(a) Write down two inequalities that describe the shaded area.
(b) At the point P, explain why $x^2 = x + 1$.



MEG (SMP)

6 Use this graph of $y = {^-x^2} + 3x$ to help you decide which of these statements are correct.
(a) $^-x^2 + 3x \leq 0$ for $0 < x < 3$
(b) $^-x^2 + 3x \geq 0$ for $0 \leq x \leq 3$
(c) $^-x^2 + 3x > 0$ for $0 \leq x \leq 3$
(d) $^-x^2 + 3x > 2$ for $1 \leq x \leq 2$
(e) $^-x^2 + 3x \geq 2$ for $1 \leq x \leq 2$



7 Nita has two types of fish in a tank: loach and guppy.
There are x loach and y guppy.
(a) Nita has at least 10 guppy.
Write this information as an inequality.
(b) The tank will accommodate up to 30 fish.
Write this information as an inequality.

Draw a grid with values of x and y from 0 to 30 using a scale of 2 cm to 5 units.
(c) On the grid indicate the region which satisfies both these inequalities.

Nita has three times as many guppy as loach.
(d) Draw a line on the grid to show this information.
(e) How many of each sort of fish could Nita have?
Circle the points on the grid which show all the possible answers. MEG/ULEAC (SMP)

Answers and hints ► page 109

Inequalities and regions (page 26)

1 (a) $x > {}^-2$

(b) *You need to collect terms in x on one side of the equation. Here are two alternative methods.*

$x + 7 < 3x + 2$ or $3x + 2 > x + 7$

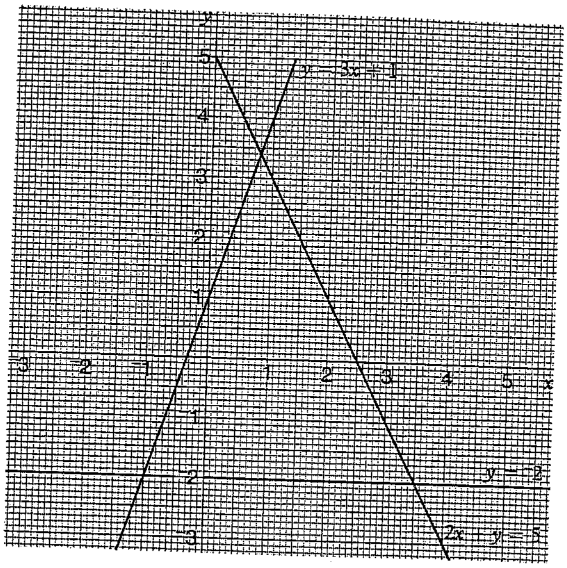
$^-2x < {}^-5$ $2x > 5$

$x > \frac{5}{2}$ $x > \frac{5}{2}$

(c) $y > {}^-4$ (d) $n \geq {}^-4$ (e) $x > {}^-2$

(f) $x < {}^-2$ (g) $a \leq 3\frac{1}{3}$ (h) $x < {}^-2$ or $x > 2$

2 The unshaded region satisfies all the inequalities, including the lines forming the sides of the triangle.



3 (a) Start by solving the two inequalities $3x + 1 \leq 27$ and $5x - 6 \geq 27$.

So $x \leq 8\frac{2}{3}$ and $x \geq 6\frac{3}{5}$.

(i) The largest value of x is $8\frac{2}{3}$.

(ii) The smallest value of x is $6\frac{3}{5}$.

(b) The integer values of x are 7 and 8.

4 (a) Solving the inequality $13 - 6x < 25$ gives $x > {}^-2$.

(b) $y = 19$

y has its maximum value when z is a maximum (that is 7) and $y - z = 12$.

5 (a) $y < x + 1$ or $y \leq x + 1$ and $y > x^2$ or $y \geq x^2$

(b) At P both $y = x + 1$ and $y = x^2$, so $x^2 = x + 1$.

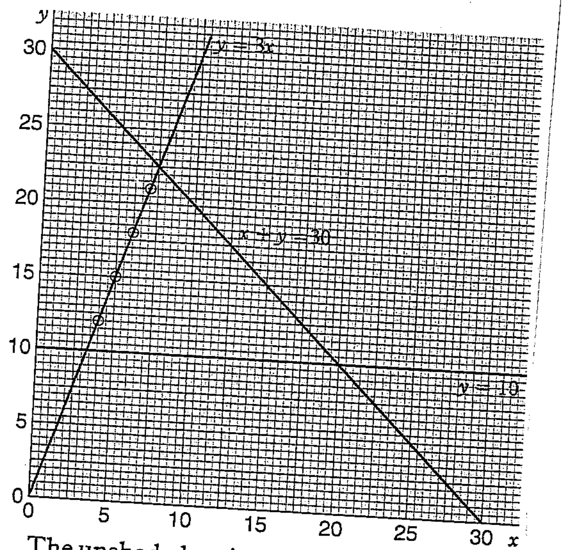
6 (a) False (b) True (c) False

(d) False (e) True

7 (a) $y \geq 10$ or $y > 9$

(b) $x + y \leq 30$ or $x + y < 31$

(c)



The unshaded region satisfies both inequalities.

(d) The line with equation $y = 3x$; see the graph.

(e) There are four possible answers:

(4, 12), (5, 15), (6, 18) and (7, 21).

More help or practice

Solving inequalities ► Book Y5 pages 92 to 97

Inequalities and regions ► Book Y5 pages 121 to 124

Regions with two boundaries ► Book Y5 pages 125 to 126