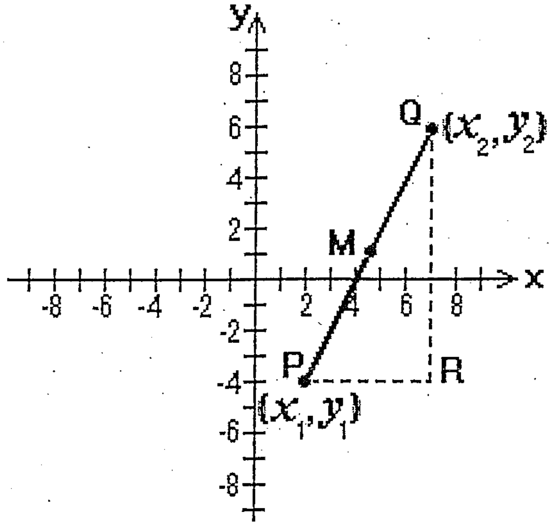
# INTERVALS

Any two points on the number plane define an *Interval*.




The *Length* of this interval PQ can be found using pythagoras' rule:-

that is: $PQ^2 = PR^2 + RQ^2$ So: $PQ = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

The *Mid-point* , M, of the interval PQ is simply the point on the line PQ whose co-ordinates are the middle (or average) of the x and y values of the points P & Q.

So . . . .

$$M = (\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2})$$

The *Gradient* (slope or incline) of the interval PQ is the ratio of its height difference (*rise*) divided by its horizontal difference (*run*), as we proceed from point P to point Q. We use the letter "$m$" to denote gradient.

So . . . .

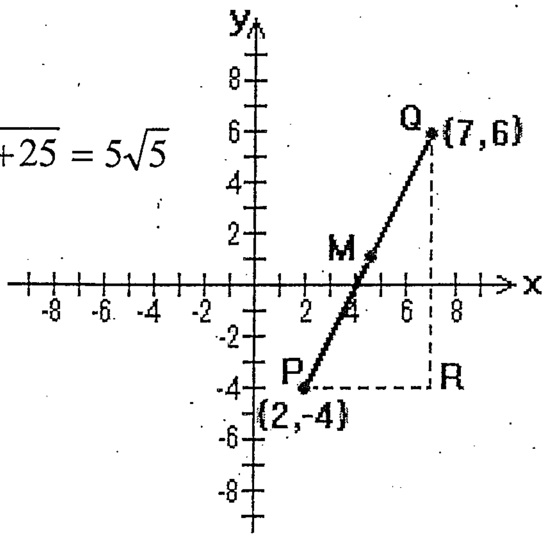
$$m = \frac{(y_2) - (y_1)}{(x_2) - (x_1)}$$

In the example opposite, P=(2,-4) & Q=(7,6)

Length is: $PQ = \sqrt{(6 - -4)^2 + (7-2)^2} = \sqrt{100+25} = 5\sqrt{5}$

Midpoint is: $M = \left(\frac{2+7}{2}, \frac{-4+6}{2}\right) = (4 \cdot 5, 1)$

Gradient is: $m = \frac{(6)-(-4)}{(7)-(2)} = \frac{10}{5} = \frac{2}{1} = 2$



# EXERCISE 31 – Length, Mid-point & Gradient

1. Given P = (2,5) & Q = (6,2) find the length, mid-point and gradient of PQ.

| LENGTH | MID-POINT | GRADIENT |
|---|---|---|
| | | |

2. Given P = (5,4) & Q = (1,-4) find the length, mid-point and gradient of PQ.

| LENGTH | MID-POINT | GRADIENT |
|---|---|---|
| | | |

3. Given P = (-4,0) & Q = (8,5) find the length, mid-point and gradient of PQ.

| LENGTH | MID-POINT | GRADIENT |
|---|---|---|
| | | |

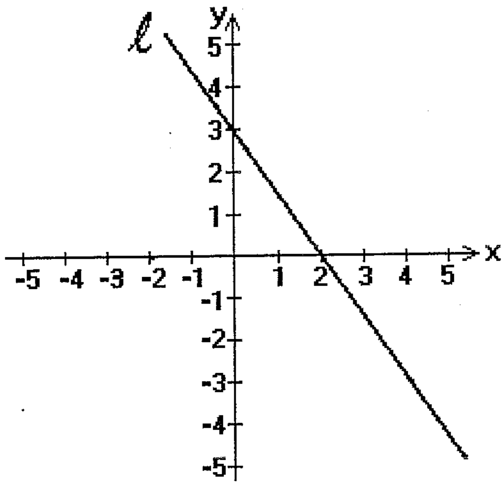
4. Given P = (-2,1) & Q = (4,-7) find the length, mid-point and gradient of PQ.

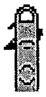
| LENGTH | MID-POINT | GRADIENT |
|---|---|---|
| | | |

# EXERCISE 30 – The Intercept Method
(for graphing lines)

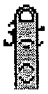
The quickest and easiest way to graph the solution of a linear equation (a straight line), is to plot its intercepts and then draw the line passing through these points.

For the linear equation: $3x + 2y = 6$




1st place your pen or finger over the term containing $y$ (set $y = 0$) $3x +$  $= 6$

- this gives you the $x$-intercept $x = 2$

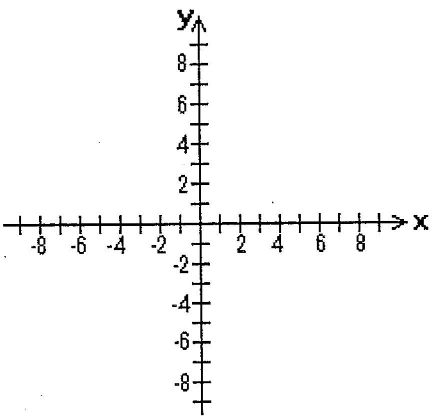
2nd, place your pen or finger over the term containing $x$ (set $x = 0$)  $+ 2y = 6$

- this gives you the $y$-intercept $y = 3$

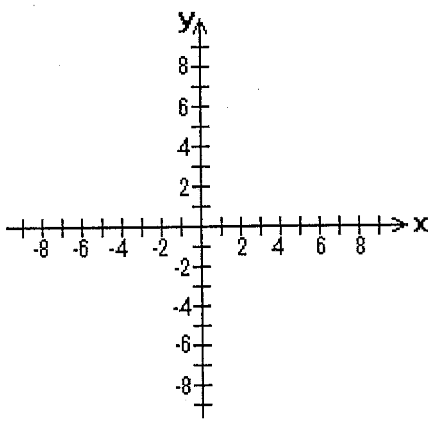
3rd Then simply get your ruler and draw a line through these two points.

Draw the graphs of the solutions to the following linear equations:

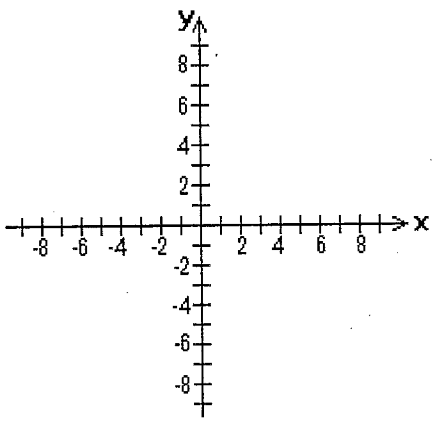
(1) $2x + y = 8$



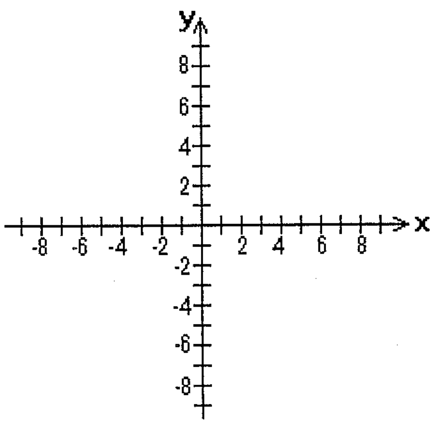
(2) $x + 3y = 6$



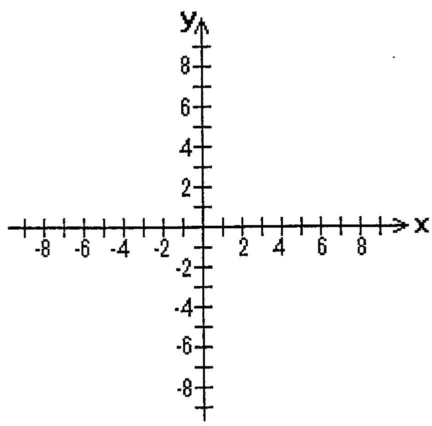
(3) $2x - y = 8$



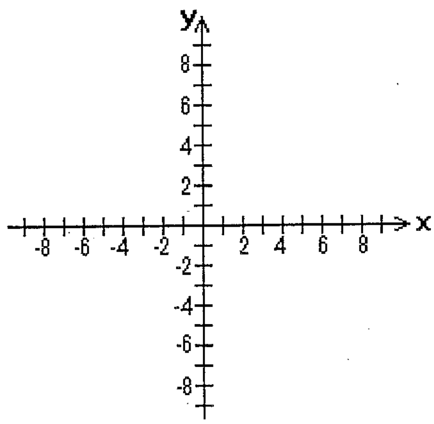
(4) $3x - 4y = 12$



(5) $y = 2x - 5$



(6) $\frac{x}{3} - y = 2$



# EXERCISE 32 – Lines And Their Equations

## Gradient of a Line:

All straight lines have a linear equation of the form: $y = mx + c$
- where "$m$" is the gradient of the line, and "$c$" is the $y$-intercept (set $x$=$0$).

For example: $y = 5x - 3$ has gradient "$5$" and $y$-intercept "$-3$".
$y = 4 - x$ has gradient "$-1$" and $y$-intercept "$4$".

Ex.1 Find the gradient and $y$-intercept of the lines whose linear equations are:

(a) $y = 5 + x$ $m=$ $c =$

(b) $y = 2 - 5x$ $m=$ $c =$

(c) $2y = x - 3$ $m=$ $c =$

(d) $4x + y = -3$ $m=$ $c =$

(e) $y = \dfrac{x+4}{2}$ $m=$ $c =$

(f) $5x - 2y + 4 = 0$ $m=$ $c =$

(g) $3x + 2y - 5 = 0$ $m=$ $c =$

(h) $\dfrac{x}{2} - \dfrac{y}{3} = 5$ $m=$ $c =$

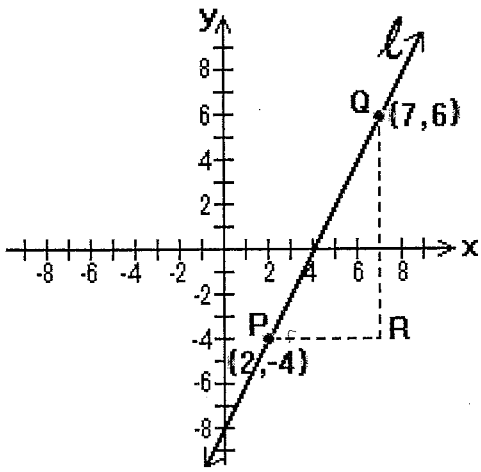
## Finding the equation of a straight line:

So to find the equation of a given line, l, we need first to find its gradient. We do this by simply selecting any two "friendly" points on its graph and use the method from the previous section on intervals – this will give us the value of $m$ in its equation. If the value of the $y$-intercept is not known, then we substitute any point on the line, l, into the equation $y = mx + c$, with our new value of $m$.

For the example shown here we have already found the gradient to be $m = 2$
Hence the equation is $y = 2x + c$

Substituting $x$ = 7 and $y$ = 6 we get:
$6 = 2\times 7 + c$ hence $c$ = -8

So the final equation is l: $y = 2x - 8$

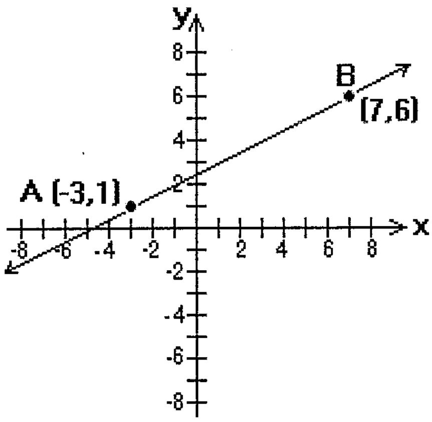


The equations of lines can be given in two different forms:

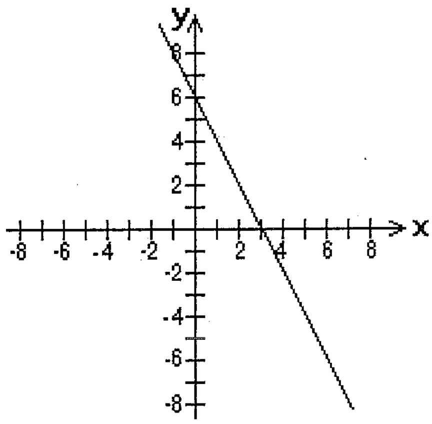
1. The *gradient intercept form*, sometimes called the "explicit" form:
   $y = mx + c$
2. The *general form* - with all terms on the left side of the equation equal to 0.
   $ax + by + c = 0$

Ex.2 Find the equations in *general form* of the graphs below:-

(a)




(b)




(c) The line passing through (2,-5) with gradient $m$ = -4

(d) The line through the points A=(-4,6) and B=(6,2)

# EXERCISE 33 – Parallel and Perpendicular Lines

Parallel lines – two lines with the *same gradients* $m_1$ and $m_2$ will be parallel.
– ie. $\boxed{m_1 = m_2}$

Perpendicular lines – two lines will be perpendicular if their gradients $m_1$ and $m_2$ multiply to give "-1",
– ie. $\boxed{m_1 \times m_2 = -1}$

1. Which of the following lines are (i) parallel to or (ii) perpendicular to the line $y = 7 - 2x$ ?

   (A) $2x - y = 1$　　　　(B) $x + 2y = 4$

   (C) $2x + y - 7 = 0$　　　　(D) $3x - 6y + 5 = 0$

2. Find the equation of the line through (-2,5) and parallel to the line $6x+2y = 7$

3. Find the equation of the line through (-2,5), perpendicular to $6x+2y = 7$

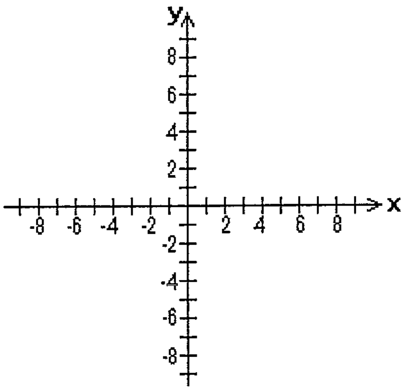
4. Show that the points A=(-1,6) , B=(4,2) and C=(9,-2) are collinear.

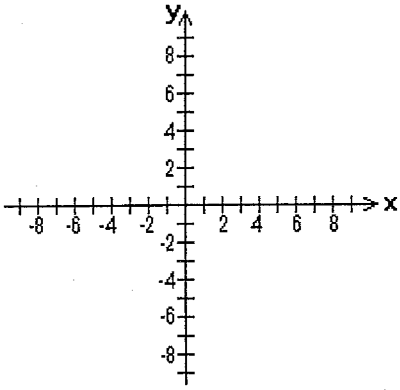
5. If $2x + y - 5 = 0$ cuts the $x$ and $y$ axes at the points A and B respectively, find the length of the interval AB.

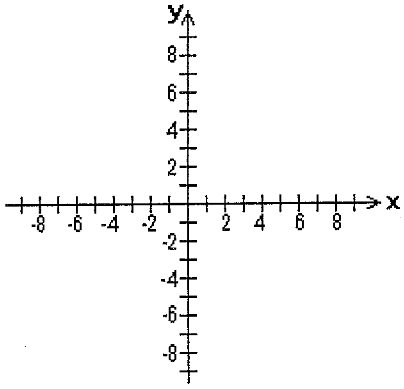
# HOMEWORK SHEET (15)

1. Sketch the graphs of the linear equations below:

   (a) $y = 5 - 2x$ (b) $x + 5y = 10$ (c) $\frac{x}{4} - \frac{y}{3} = 2$




2. What is the $y$-intercept of each of the graphs in question 1 above?

   (a) (b) (c)

3. Find the equations in general form of the . . . .

   (a) line through the points A=(1,8) and B=(-1,2)

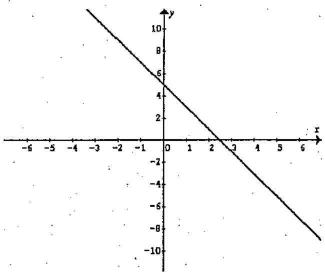
   (b) line through (-4,3) perpendicular to $x - 2y = 7$

4. Find the point of intersection of the 2 lines mentioned in question 3 above.

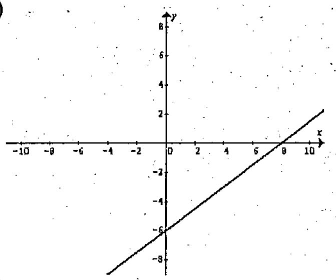
# ANSWERS – Line Graphs

## Homework 15

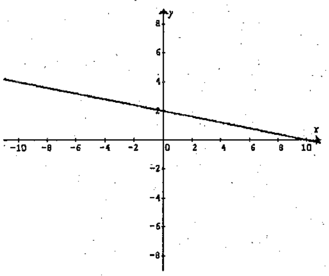
1. a)



b)



c)



2. a)

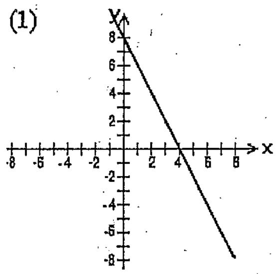
3. a) $3x - y + 5 = 0$

   b) $2x + y + 5 = 0$

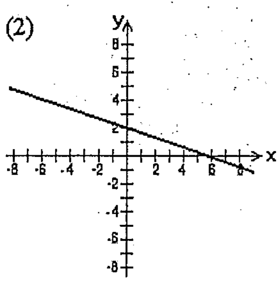
4. (-2 , -1)

## Exercise 30

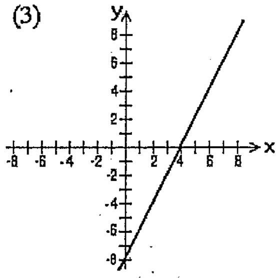
(1)



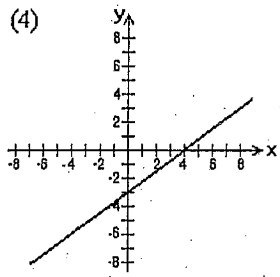
(2)



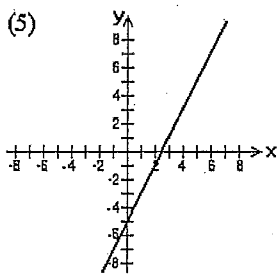
(3)



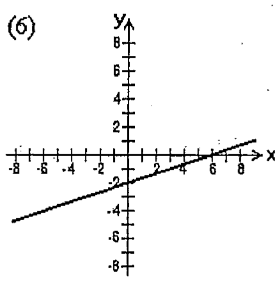
(4)



(5)



(6)



## Exercise 31

| | | |
|---|---|---|
| 1.(a) 5 | (b) (4,3·5) | (c) $-\frac{3}{4}$ |
| 2.(a) $4\sqrt{5}$ | (b) (3 , 0) | (c) 2 |
| 3.(a) 13 | (b) (2,2·5) | (c) $\frac{5}{12}$ |
| 4.(a) 10 | (b) (1 ,-3) | (c) $-\frac{4}{3}$ |

## Exercise 32

1.(a) m = 1 , c = 5 (b) m = -5 , c = 2

(c) m = ½, c = -1·5 (d) m = -4 , c = -3

(e) m = ½ , c = 2 (f) m = 2·5 , c = 2

(g) m = -1·5 , c = 2·5 (h) m = 1·5 , c = -15

2.(a) $x$-2$y$+5 = 0 (b) 2$x$+$y$-6 = 0

(c) 4$x$+$y$-3 = 0 (d) 2$x$+5$y$-22 = 0

## Exercise 33

1. (C) is parallel ; (D) is perpendicular
2. 3$x$+$y$+1 = 0 3. $x$-3$y$+17 = 0
4. gradient AB = gradient BC = $-\frac{4}{5}$
5. A=(2·5,0) B=(0,5) $\therefore$ AB = $\frac{5\sqrt{5}}{2}$