Year 11 Ext 1 Mathematics 2012	Calculus and Trigonometry
Time Allowed: 1 period	Marks: 34
Show all working to gain maximum marks	Marks will be deducted for poor or illegible work
Name: ____________	Teacher: HRK RDS JJA RABS

PART A (8 marks) START A NEW BOOKLET HRK

1. Find from first Principles the derivative of $y = 3x^2 - 2x + 4$ 3

2. Find $\frac{d}{dx}\left(\frac{1}{x^3\sqrt{x}}\right)$ 2

+ Soln's

3. Evaluate $f'(8)$ if $f(x) = \frac{\sqrt{x}+\sqrt{2}}{\sqrt{x}-\sqrt{2}}$ 3

PART B (8 marks) START A NEW BOOKLET RABS

1. The curve $y = ax^2 + bx + 5$ passes through the point (1, 7). The tangent at that point is parallel to the x-axis. Find the values of a and b. 4

2. The curves $y = 2x^3$ and $y = 2x^2$ met at the origin and at point P.

   a. Find the co-ordinates of point P. 1

   b. Find the acute angle between the two curves at point P. 3

PART C (9 marks) START A NEW BOOKLET RDS

1. Find the exact value of $\tan 15^\circ$ 3
(leave your answer with a rational denominator)

2.
   a. Using the compound angle formula for cosine, show that $\cos 2\theta = 2\cos^2\theta - 1$ 1

   b. Hence or otherwise solve the following equation $7\cos x + \cos 2x - 3 = 0$ in the domain $-180^\circ \le x \le 180^\circ$. 3

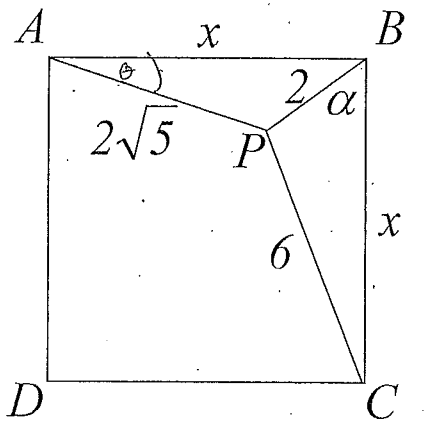
3. Show that $\frac{\tan\theta - \cos\theta\sin\theta}{\cos\theta\sin\theta} = \tan^2\theta$ 2

PART D (9 marks)	START A NEW BOOKLET	IIA

1.

a. Find the exact value of $\tan 315° + \cos^2 30°$, leaving answer with a common rational denominator. **2**

b. Solve $4\cos^2 x = 3$ for $0 \le x \le 360$. **2**



NOT TO SCALE

2. The diagram shows a square $ABCD$ of x cm, with a point P within the square, such that $PC = 6$ cm, $PB = 2$ cm and $AP = 2\sqrt{5}$ cm. Let $\angle PBC = \alpha$.

a. Using the cosine rule in triangle PBC, show that: **1**

$$\cos\alpha = \frac{x^2 - 32}{4x}$$

b. By considering triangle PBA, show that: **1**

$$\sin\alpha = \frac{x^2 - 16}{4x}$$

c. Using the identity, $\sin^2\alpha + \cos^2\alpha = 1$ or otherwise, show that the value of x is a solution of $x^4 - 56x^2 + 640 = 0$. **1**

d. Find x. Give reasons for your answer. **2**

* FROM 1ST PRINCIPLES MEANS FROM 1ST PRINCIPLES !!
No marks awarded for using the quick rule!

** SETTING OUT OF THIS NEEDS ATTENTION

b) INDICES NEEDS PRACTICE ☹ YR 8/9 WORK

$$\frac{1}{x^3 \times \sqrt{x}} = \frac{1}{x^3 \times x^{\frac{1}{2}}}$$

$3 + \frac{1}{2} = 3\frac{1}{2}$ YR 3 WORK!

$$= \frac{1}{x^{7/2}}$$

$$= x^{-\frac{7}{2}}$$ & see Soln

Now differentiate

c) The differentiation of a constant IS ZERO!

eg $\frac{d}{dx}(6) = 0$ $\sqrt{2}$ IS A CONSTANT!

SO $\frac{d}{dx}\sqrt{2} = 0$

Yes this is fiddly! BUT NOT difficult if set out clearly and methodically (see solution), also if $f'(8)$ is the question we can substitute in the 8 when the derivative is only partially simplified

PART A Q1 EXT 1 TEST MAY 201

(3) $y = 3x^2 - 2x - 4$

$$\frac{dy}{dx} = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \to 0} \frac{3(x+h)^2 - 2(x+h) - 4 - (3x^2 - 2x - 4)}{h}$$

$$= \lim_{h \to 0} \frac{\cancel{3x^2} + 6xh + 3h^2 - \cancel{2x} - 2h - \cancel{4} - \cancel{3x^2} + \cancel{2x} + \cancel{4}}{h}$$

$$= \lim_{h \to 0} \frac{\cancel{h}(6x + 3h - 2)}{\cancel{h}}$$

$$= 6x - 2$$

Q2

(2) $$\frac{d}{dx} x^{-\frac{7}{2}} = -\frac{7}{2}x^{-\frac{9}{2}}$$

$$= -\frac{7}{2\sqrt{x^9}}$$

Q3

$$f(x) = \frac{\sqrt{x} + \sqrt{2}}{\sqrt{x} - \sqrt{2}} \quad \begin{matrix} u \\ v \end{matrix}$$ (1)

OR $\frac{(\sqrt{x} + \sqrt{2})^2}{x - 2}$

(3) $$f'(x) = \frac{vu' - uv'}{v^2}$$

$$= \frac{(\sqrt{x} - \sqrt{2})\frac{1}{2}x^{-\frac{1}{2}} - (\sqrt{x} + \sqrt{2})\frac{1}{2}x^{-\frac{1}{2}}}{(\sqrt{x} - \sqrt{2})^2}$$

$$= \frac{\cancel{\sqrt{x}} - \sqrt{2} - \cancel{\sqrt{x}} - \sqrt{2}}{2\sqrt{x}(\sqrt{x} - \sqrt{2})^2}$$ (1)

$$f'(8) = \frac{-\cancel{2}\sqrt{2}}{\cancel{2}\sqrt{8}(\sqrt{8} - \sqrt{2})^2}$$ (1)

$$\frac{-\cancel{2}\sqrt{2}}{4\cancel{\sqrt{2}}(2)}$$

$$= -\frac{1}{4}$$

PTO FOR COMMENTS

EXT1 CALC + TRIG TEST SOLNS

PART B

1. WHEN $x=1$, $y=7$

$$7 = a.1^2 + b.1 + 5$$
$$7 = a + b + 5$$
$$2 = a + b \quad \text{——— } (1) \ \checkmark$$

IF $y = ax^2 + bx + 5$

$$\frac{dy}{dx} = 2ax + b$$ ...MARK FOR WORKING IF YOU GOT THIS FAR

PARALLEL TO x-AXIS, $m = 0$ & $x = 1$

$$0 = 2a + b \quad \text{——— } (2) \ \checkmark$$
$$2 = a + b \quad \text{——— } (1)$$
$$(2)-(1): \ -2 = a \ \checkmark$$

SUB INTO (1): $b = 4$ ✓

- MANY STUDENTS HAD DIFFICULTY WITH THIS.
- MANY DID NOT UTILISE THE PROPERTY THAT m OF X-AXIS = 0.
- SOME INCORRECTLY USED GRAD. FORMULA:

$$0 = \frac{y-7}{x-1}$$ ...THIS WORKING IS NOT SOUND! THINK: $\frac{0}{0}$ ∴

THEN USED $x=1$ & $y=7$ . . .

2.

a. $y = 2x^3$ $\qquad$ $y = 2x^2$

$$2x^3 = 2x^2 \qquad \text{(EQUATE THE TWO)}$$
$$2x^3 - 2x^2 = 0$$
$$2x^2(x-1) = 0$$
$$\therefore x = \cancel{0}, 1$$

ALREADY KNOW IT PASSES THRU $(0,0)$

SUB $x = 1$ INTO EITHER:

POINT $P = (1, 2)$ ✓

- SOME STUDENTS SIMPLY GAVE THE ANSWER, NO WORKING. THIS IS NOT AN ADEQUATE DISPLAY OF UNDERSTANDING.

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b. FIND THE 2 GRADIENTS:

$y = 2x^3$ $\qquad$ $y = 2x^2$

$\frac{dy}{dx} = 6x^2$ $\qquad$ $\frac{dy}{dx} = 4x$

AT $x = 1$: $m_1 = 6$ & $m_2 = 4$. ✓

$$\tan\theta = \left|\frac{m_1 - m_2}{1 + m_1 m_2}\right|$$
$$= \left|\frac{6-4}{1+24}\right|$$

$\therefore \tan\theta = \frac{2}{25}$ ✓

$\therefore \theta = 4.573^\circ \ldots$

OR

$= 4^\circ 34' 26''$ ✓

- ANSWERED QUITE WELL. ☺

- SOME PROBS WERE REMEMBERING THE $\tan\theta$ FORMULA CORRECTLY!

PART C

Ext 1 Solutions Term 2, Test 1 2012

1) $\tan(15^\circ) = \tan(45 - 30^\circ)$

$= \dfrac{\tan 45 - \tan 30^\circ}{1 + \tan 45 \tan 30^\circ}$ (1)

$= \dfrac{1 - \frac{1}{\sqrt{3}}}{1 + \frac{1}{\sqrt{3}}}$ (1)

$= \dfrac{\sqrt{3} - 1}{\sqrt{3} + 1} \times \dfrac{\sqrt{3} - 1}{\sqrt{3} - 1}$

$= \dfrac{(\sqrt{3} - 1)^2}{2}$ (1)

- This is the most common way to test compound angles

Ext students should know this.

Many students forgot to rationalise.

2a) $\cos(2\theta) = \cos(\theta + \theta)$

$= \cos\theta\cos\theta - \sin\theta\sin\theta$

$= \cos^2\theta - \sin^2\theta$

As $\sin^2\theta = 1 - \cos^2\theta$

then $\cos 2\theta = \cos^2 - (1 - \cos^2\theta)$

$= 2\cos^2\theta - 1$ (1)

This was well done.

Needed do show the $1 - \cos^2\theta = \sin^2\theta$ substitution

b) $7\cos x + \cos 2x - 3 = 0$ (1)

$7\cos x + (2\cos^2 x - 1) - 3 = 0$

$7\cos x + 2\cos^2 x - 4 = 0$

let $u = \cos x$

$2u^2 + 7u - 4 = 0$

MANY students ignored a) ...

$(2u-1)(u+4) = 0$

$\therefore \cos x = \frac{1}{2}$ & $\cos x = -4$

① $(\cos x \neq -4)$

$\therefore x = \cos^{-1}\left(\frac{1}{2}\right)$

✓ ✓

$x = 60°, 300°$ → Not in domain

$\therefore x = 60°, -60°$ ①

Remember, trig equations need one trig function only. Always look to remove so that only sin, cos or tan remain.

3) $\dfrac{\tan\theta - \cos\theta \sin\theta}{\cos\theta \sin\theta} = \tan^2\theta$

$$LHS = \frac{\sin\theta}{\cos\theta} - \frac{\cos\theta\sin\theta}{1} \div (\cos\theta\sin\theta)$$

$$= \frac{\sin\theta - \cos^2\theta\sin\theta}{\cos\theta} \times \frac{1}{\cos\theta\sin\theta}$$

$$= \frac{\cancel{\sin\theta}(1-\cos^2\theta)}{\cos^2\theta\cancel{\sin\theta}}$$

$$= \frac{\sin^2\theta}{\cos^2\theta}$$

$$= \tan^2\theta = RHS.$$

Important to split the fraction or do the top ÷ bottom.

Learn to handle fractions within fractions!

PART D

1. a) $\tan 315° + \cos^2 30°$

$= -\tan 45° + \left(\frac{\sqrt{3}}{2}\right)^2$

$= -1 + \frac{3}{4}$

$= -\frac{1}{4}$

b) $\cos^2 x = \frac{3}{4}$

$\cos x = \frac{\pm\sqrt{3}}{2}$

$= 30°, 150°, 210°, 330°$

2. a) $\cos\alpha = \dfrac{a^2+b^2-c^2}{2bc}$

$= \dfrac{x^2+z^2-6^2}{2\times2\times x}$

$= \dfrac{x^2-32}{4x}$

b) $\cos(90-\alpha) = \dfrac{x^2+2^2-(2\sqrt{5})^2}{2\times2\times x}$

$= \dfrac{x^2-16}{4x}$

c.) $\left(\dfrac{x^2-16}{4x}\right)^2 + \left(\dfrac{x^2-32}{4x}\right)^2 = 1$

$x^4 - 32x^2 + 256 + x^4 - 64x^2 + 1024 = 16x^2$

$2x^4 - 112x^2 + 1280 = 0$

$x^4 - 56x^2 + 640 = 0$

$(x^2-40)(x^2-16) = 0$

$x = \sqrt{40}, 4$

$x = 4$ not possible as $\cos\alpha = \dfrac{x^2-16}{4x}$

$\cos\alpha = 0$

$\alpha = 0$

$\therefore x = \sqrt{40}$

the only