ST ANDREW'S CATHEDRAL SCHOOL
Founded 1885

HSC Mathematics Extension 1
Assessment Task 3.
2007

Time Allowed: 55 minutes

Weighting: 20%

Instructions

- Write your student number on each page.
- Take a new sheet of paper for each question
- Write in blue or black pen (pencil may be used for diagrams)
- Write on one side of the sheet of paper only
- Be sure to make any sketches a reasonable size – approximately a fifth to a quarter of a page
- You may use scientific calculators

Question 1. 12 Marks. Start a new page.

(a) Find the exact values of:

(i) $\tan^{-1}\left(\dfrac{1}{\sqrt{3}}\right)$ **1**

(ii) $\cos\left[\sin^{-1}\left(\dfrac{40}{41}\right)\right]$ **2**

(b) Find the general solution to $\sqrt{3}\tan x - 1 = 0$.
Express your answer in terms of π. **2**

(c) If $f(x) = \tan^{-1}(\log_e x)$ evaluate $f'(e)$ **2**

(d) Evaluate $\displaystyle\int_0^{\frac{\pi}{4}} \sin^2 2x \; dx$ **3**

(e) Fully simplify $-\log_{1/x} x - \log_{1/x} x^2 - \log_{1/x} \sqrt{x}$ **2**

Question 2. 11 Marks. Start a new page.

(a) Evaluate:

(i) $\int \frac{1}{\sqrt{9-x^2}}\,dx$ **1**

(ii) $\int_0^{\frac{3}{4}} \frac{1}{9+16x^2}\,dx$ **3**

(b) Using the substitution $u=1+2\sin x$, or otherwise, evaluate $\int \frac{\cos x}{1+2\sin x}\,dx$ **3**

(c) (i) Show that $\frac{d}{dx}\left(x\cos^{-1}x-\sqrt{1-x^2}\right)=\cos^{-1}x$ **2**

(ii) Hence, find the area bounded by $y=\cos^{-1}x$, $x=-1$, $x=1$ and the $x-$axis. **2**

Question 3. 13 Marks. Start a new page.

(a) Consider the function $f(x)=\tan^{-1}x+\tan^{-1}\left(\frac{1}{x}\right)$

(i) State any values of x for which $f(x)$ is undefined. **1**

(ii) Show that $f(1)=\frac{\pi}{2}$ **1**

(iii) Show that $f'(x)=0$ **2**

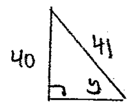
(b) Evaluate $\int_0^2 \frac{8x}{\sqrt{1+2x^2}}\,dx$, using the substitution $u=1+2x^2$. **3**

(c) Sketch the graph of $y=4\cos^{-1}(2x-1)$, and write down its domain and range. **3**

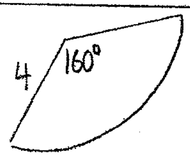
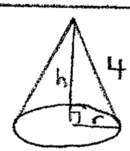
(d) The net of the curved surface of a cone is the sector of a circle.
A sector of angle $160°$ is cut out of a circular filter paper of diameter 8cm. This sector is folded to make a cone. Find:

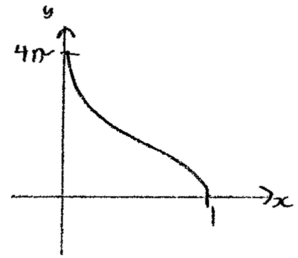
(i) The radius of the base of the cone **2**

(ii) The height of the cone **1**

Q	Solution Ext1 Task 3 2007.	Marking Scheme
1	(a) (i) $\tan^{-1}\left(\frac{1}{\sqrt{3}}\right) = \pi/6$ (ii) $\cos\left(\sin^{-1}\left(\frac{40}{41}\right)\right)$ let $y = \sin^{-1}\frac{40}{41}$ $\sin y = \frac{40}{41}$  9 by pythag $\therefore \cos y = 9/41$	1 correct answer 1 correct method with minor error 2 correct solution
	(b) $\tan x = 1/\sqrt{3}$ $x = n\pi + \tan^{-1}\frac{1}{\sqrt{3}}$ $= n\pi + \pi/6$	1 correct general express with $\tan^{-1}$ 2 correct solution
	(c) $f(x) = \tan^{-1}(\log_e x)$ $f'(x) = \frac{1}{1+(\ln x)^2}\cdot\frac{1}{x}$ $f'(e) = \frac{1}{1+(\ln e)^2}\cdot\frac{1}{e}$ $= \frac{1}{2}\cdot\frac{1}{e}$ $= \frac{1}{2e}$	1 correct integration 2 correct solution
	(d) $\int_0^{\pi/4} \sin^2 2x\,dx$ $\cos 2x = 1-2\sin^2 x$ $\sin^2 x = 1/2(1-\cos 2x)$ $\sin^2 2x = 1/2(1-\cos 4x)$ $=\frac{1}{2}\int_0^{\pi/4}(1-\cos 4x)\,dx$ $=\frac{1}{2}\left[x - 1/4\sin 4x\right]_0^{\pi/4}$ $=\frac{1}{2}\left[\pi/4 - 1/4\sin\pi - (0 - 1/4\sin 0)\right]$ $=\frac{1}{2}\left[\pi/4\right]$ $= \pi/8$	1 correct substitution 2 correct integration 3 correct solution

Q	Solution	Marking Scheme
1	(e) $-\log_{1/x} x - \log_{1/x} x^2 - \log_{1/x}\sqrt{x}$ $= -\log_{1/x} x - 2\log_{1/x} x - 1/2\log_{1/x} x$ $= -7/2\log_{1/x} x$ $= -7/2(-1)$ $= 7/2$ $y = \log_{1/x} x$ $(1/x)^y = x$ $(1/x)^y = (1/x)^{-1}$ $y = -1.$	1 correct expression in $\log_{1/x}$ simplified 2 correct solution
2	(a) (i) $\sin^{-1}\frac{x}{3} + C$ (ii) $\int_0^{3/4}\frac{1}{9+16x^2}\,dx$ $=\int_0^{3/4}\frac{1}{16}\cdot\frac{1}{9/16+x^2}\,dx$ $=\frac{1}{16}\left[4/3\tan^{-1}\frac{4x}{3}\right]_0^{3/4}$ $=\frac{1}{16}\left[4/3\tan^{-1}1 - 4/3\tan^{-1}0\right]$ $=\frac{1}{16}\cdot\frac{4\pi}{12}$ $=\frac{\pi}{48}$	1 correct answer. 1 correct change of integrand 2 correct integral 3 correct solution
	(b) $\int\frac{\cos x}{1+2\sin x}\,dx$ $u = 1+2\sin x$ $du = 2\cos x\,dx$ $=\frac{1}{2}\int\frac{2\cos x\,dx}{1+2\sin x}$ $=\frac{1}{2}\int\frac{du}{u}$ $=\frac{1}{2}\ln u + C$ $=\frac{1}{2}\ln(1+2\sin x)+C$	1 correct substitution 2 correct simplification and integration 3 correct solution

Q	Solution	Marking Scheme
2	(c) (i) $\frac{d}{dx}\left(x\cos^{-1}x - (1-x^2)^{\frac{1}{2}}\right)$ $= \cos^{-1}x + \frac{-x}{\sqrt{1-x^2}} - \frac{1}{2}(1-x^2)^{-\frac{1}{2}}.-2x$ $= \cos^{-1}x + \frac{-x}{\sqrt{1-x^2}} + \frac{x}{\sqrt{1-x^2}}$ $= \cos^{-1}x.$	1 correct differentiation 2 correct solution
	(ii) $A = \int_{-1}^{1} \cos^{-1}x \, dx$ $= \left[x\cos^{-1}x - \sqrt{1-x^2}\right]_{-1}^{1}$ $= \left(1\cos^{-1}1 - \sqrt{0}\right) - \left(-1\cos^{-1}(-1) - 0\right)$ $= 0 - 0 + \pi + 0$ $= \pi$ units^2	1 correct sub. 2 correct solution
3	(a)   $160^\circ = \frac{8\pi}{9}$ i) $C_{base} = l$ $2\pi r = 4.\frac{8\pi}{9}$ $2r = \frac{32}{9}$ $r = \frac{16}{9}$ ii) $4^2 = h^2 + r^2$ $h = \sqrt{16 - (\frac{16}{9})^2}$ $= \sqrt{\frac{1040}{81}}$ $= \frac{4\sqrt{65}}{9}$	1 correct link of radius and arc length 2 correct solution 1 correct answer

Q	Solution	Marking Scheme
3	(b) $\int_0^2 \frac{8x}{\sqrt{1+2x^2}}$ $u = 1+2x^2$, $du = 4x\,dx$; $x=0$, $u=1$; $x=2$, $u=9$ $= 2\int_1^9 \frac{du}{\sqrt{u}}$ $= 2\left[2\sqrt{u}\right]_1^9$ $= 4(\sqrt{9} - \sqrt{1})$ $= 4(2)$ $= 8$	1 mark correct limits 2 marks correct limits and integration 3 correct solution
	(c) $y = 4\cos^{-1}(2x-1)$  $D: -1 \le 2x-1 \le 1$ $0 \le 2x \le 2$ $0 \le x \le 1$ $R: 0 \le \frac{y}{4} \le \pi$ $0 \le y \le 4\pi$	1 correct D&R 2 correct shape 3 correct solution
	(d) $f(x) = \tan^{-1}x + \tan^{-1}\left(\frac{1}{x}\right)$ (i) $x \ne 0$ (ii) $f(1) = \tan^{-1}1 + \tan^{-1}(1)$ $= \pi/4 + \pi/4$ $= \pi/2$ (iii) $f'(x) = \frac{1}{1+x^2} + \frac{1}{1+\frac{1}{x^2}} \times -x^{-2}$ $= \frac{1}{1+x^2} - \frac{1}{1+\frac{1}{x^2}} \times \frac{1}{x^2}$ $= \frac{1}{1+x^2} - \frac{1}{x^2+1}$ $= 0$	1 correct answer 1 correct solution 1 correct diff 2 correct solution