Question 1 **Marks**

(a) Evaluate $\sqrt{\frac{40}{3}-\sqrt{12}}$, correct to three significant figures. 1

(b) Find the exact value of $\sin\frac{4\pi}{3}$. 1

(c) Differentiate $\frac{1}{e^x}+\sqrt{x}$ with respect to x. 2

(d) Solve for x, $5=\frac{6x}{x+1}$ 2

(e) Find the primitive of $3\sin x$ 2

(f) Solve the inequality $|x-1|>3$. 2

(g) Given $\log_a 3=1.6$ and $\log_a 7=2.4$, find $\log_a(21a)$ 2

Question 2

(a) Find the equation of the normal on the curve $y=\ln(x+2)$ at the point $(0,\ln 2)$ 3

(b) Differentiate the following:

(i) $x^2\tan 5x$ 2

(ii) $\frac{x}{1-3x}$ 2

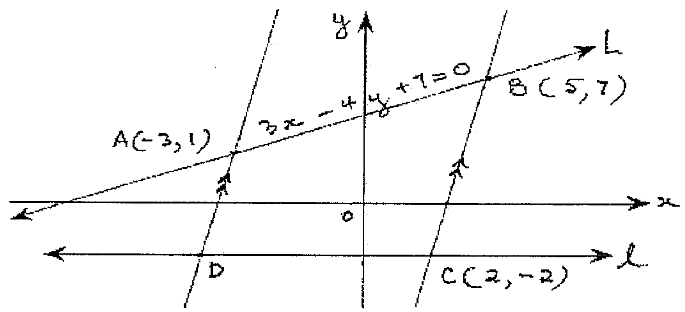
(iii) $\sin^3 x$ 1

(c) The angle subtended at the centre, O, of a sector is 42° and whose radius is 10 cm. find the arc length to the nearest centimetre. 2

(d) State the domain and range of the function $f(x)=2\sqrt{x-1}+3$ 2

Question 3 **Marks**

(a)



The points $A(-3,1)$ and $B(5,7)$ lie on the line L with the equation $3x-4y+7=0$.
The line l is parallel to the x-axis.
The points $C(2,-2)$ and D are two points on l such that $DA \parallel CB$

(i) Find the distance AB. 1

(ii) Find the perpendicular distance of C to the line L. 2

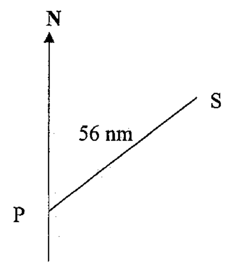
(iii) Find the angle of inclination that line L makes with the x-axis (to nearest degree). 2

(iv) Show that the equation of the line passing through A and D is $y=3x+10$. 2

(v) Find the coordinates of point D. 1

(vi) Find the area of the quadrilateral $ABCD$ by joining AC. 2

(b)



A ship S sails from port P on a bearing of N60°E for 56 nautical miles, as shown in the diagram, while a boat B leaves port P on a bearing of 110°T for 48 nautical miles. Calculate the distance from S to B (correct to one decimal place) 2

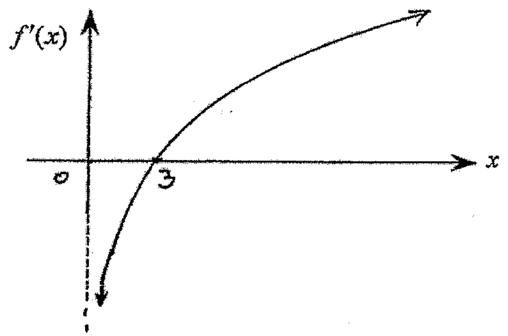
Question 4 — **Marks**

(a) (i) Find $\int \frac{3x^3 - 1}{x}\,dx$. — 2

(ii) Evaluate $\int_0^{\frac{1}{2}} \cos(\pi x)\,dx$. — 2

(b) Solve $\cos 2x = \frac{1}{\sqrt{2}}$ for $0 \le x \le \pi$. — 2

(c) The sketch of the curve $y = f'(x)$ is given below. — 3



Sketch the curve $y = f(x)$, given $f(3) = 0$

(d) The rate of water flowing, R litres per hour, into a pond is given by

$$R = 65 + 4t^{\frac{1}{3}}$$

(i) Calculate the initial flow rate — 1

(ii) Find the volume of water in the pond when 8 hours have elapsed, if initially there was 15 litres in the pond. — 2

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Question 5 — **Marks**

(a) The roots of the equation $x + \frac{1}{x} = 5$ are α and β.

Find the value of

(i) $\alpha + \frac{1}{\alpha}$ — 1

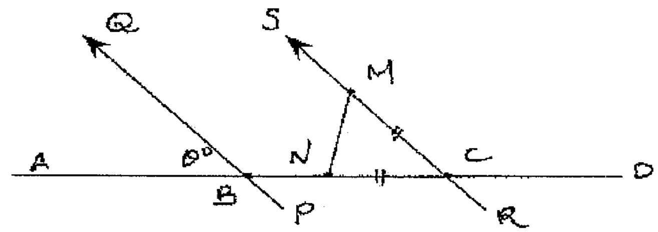
(ii) $\alpha + \beta$ — 2

(iii) $\alpha^2 + \beta^2$ — 2

(b) (i) Find the discriminant of $3x^2 + 2x + k$ — 1

(ii) For what values of k does the equation $3x^2 + 2x + k = 0$, have real roots? — 2

(c) — 3



Given $PQ \parallel RS$, $CN = CM$ and $\angle ABQ = \theta°$.

Find angle NMS in terms of $\theta°$, giving reasons.

(d) Given the equation of a parabola is $(x-3)^2 = 4y + 8$,

(i) Find the coordinates of the vertex. — 1

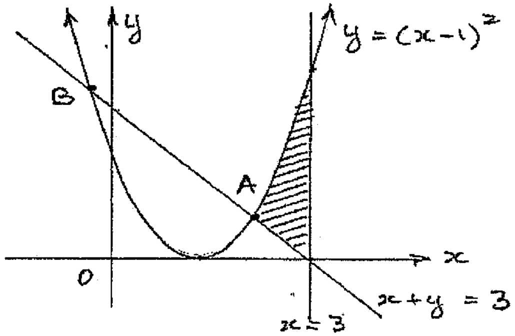
(ii) Find the coordinates of its directrix — 1

Question 6 **Marks**

(a) (i) Solve the equation $x^2-3x-18=0$ 2

(ii) Hence, or otherwise find all real solutions to $(x^2+1)^2-3(x^2+1)-18=0$ 2

(b) Given the curves $y=(x-1)^2$ and $x+y=3$ intersect at A and B.



(i) Verify that coordinates of $A=(2,1)$ 1

(ii) Hence find the area enclosed by the curve $y=(x-1)^2$, and the lines $x+y=3$ and $x=3$ 2

(c) Given $\frac{dy}{dx}=e^{1-x}$ and when $x=1$, $y=3$, find y as a function of x 2

(d) A metal ball is fired into a tank filled with a thick viscous fluid.
The rate of decrease of velocity is proportional to its velocity v cm s^{-1}

Thus $\frac{dv}{dt}=-kv$, where k=0.07 and t is time in seconds.

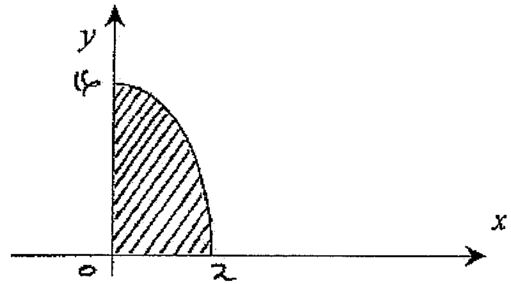
The initial velocity of the ball when it enters the liquid id 85 cm s^{-1}

(i) Show that $v=85e^{-0.07t}$ satisfies the equation $\frac{dv}{dt}=-kv$ 1

(ii) Calculate the rate when t=5 2

Question 7 **Marks**

(a) Consider the shaded area of that part of the sketch of the curve $y=16-x^4$, for $0\le x\le 2$, as shown. 3



This area is rotated about the y-axis.
Calculate the exact volume of the solid of revolution.

(b) In a game of chess between two players X and Y, both of approximately equal ability, the player with the white pieces, having the first move, has a probability of 0.5 of winning, and the probability that the player with the black pieces, for that game, winning is 0.3

(i) What is the probability that the game ends in a draw? 1

(ii) The two players X and Y play each other in a chess competition, each player having the white pieces once.
In the competition the player who wins a game scores 3 points, a player who loses a game scores 1 point and in draw each player receives 2 points.
By drawing a probability tree diagram or otherwise, find the probability that as a result of these two games

(α) X scores 6 points 1

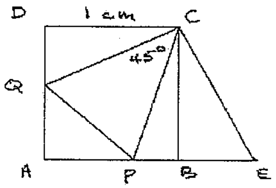
(β) X scores less than 4 points 2

(c) (i) State a formula for the interior angle sim of an n-sided convex polygon. 1

(ii) The interior angles of a convex polygon are in arithmetic sequence. The smallest angle is 120° and the common difference is 5°. Find the number of sides of the polygon. 4

Question 8 **Marks**

(a) In the diagram, $ABCD$ is a square of side length 1 cm.



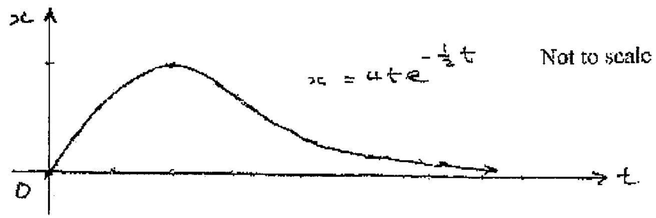
Not to scale

Points P and Q lie on AB and AD respectively, and $\angle PCQ = 45°$.
AB is produced to E such that $BE = DQ$ as shown.

(i) State which test confirms $\Delta CBE \equiv \Delta CDQ$ 1
(ii) Prove that PC bisects $\angle QCE$, giving reasons 2
(iii) Deduce that $PC \perp QE$ (justify) 2

(b) A particle is moving in straight-line motion. The particle starts from the origin and after a time of t seconds it has a displacement of x metres from O given by

$x = 4te^{-\frac{1}{2}t}$ as shown in the diagram.



Its velocity, v m/s, is given by $v = 2(2-t)e^{-\frac{1}{2}t}$

(i) What is the initial velocity? 1
(ii) When and where will the particle be at rest? 2
(iii) At what time will the particle be travelling at constant velocity? Give reasons. 3
(iv) When will the particle be accelerating? 1

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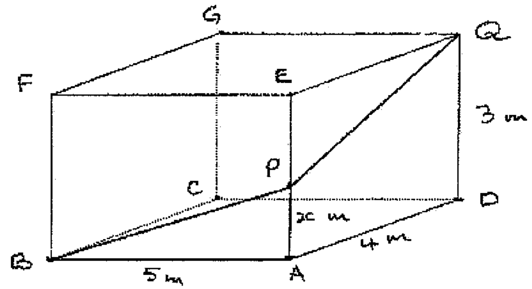
Question 9 **Marks**

(a) Show that $\frac{d}{d\theta}\left[\frac{1}{\cos\theta}\right] = \sec\theta\tan\theta$. 2

(b) Fibre cabling is to be laid in a rectangular room along BP and PQ from the corner B of the floor $ABCD$ as shown in the diagram.



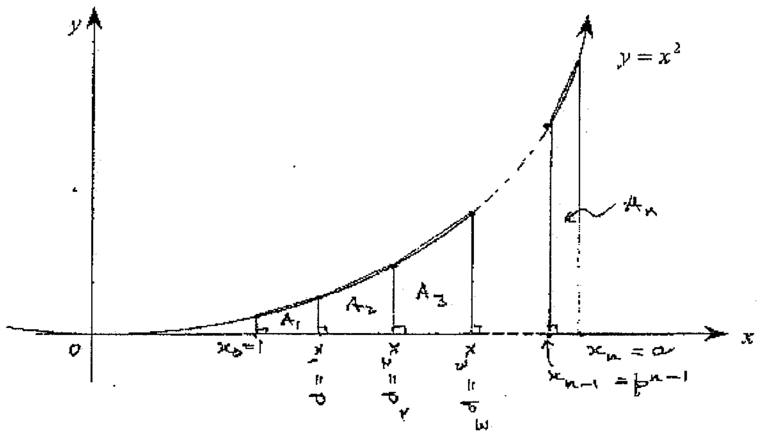
Given the dimensions of the room are $AB = 5$ m, $AD = 4$ m and the height of the room $AE = 3$ m.

Suppose $AP = x$ m,

(i) State the length of BP in terms of x. 1
(ii) Show that the length of PQ is $\sqrt{25-6x+x^2}$ m. 1
(iii) Hence state the total length, L m, of the cabling (in terms of x) 1
(iv) Find the value of AP when the total length L is to be minimum 7

Question 10 **Marks**

Consider the curve $y = x^2$ for $x \geq 0$, and let $I = \int_1^a x^2 dx$, where $a > 1$.



Divide the interval $1 \leq x \leq a$ into n parts where the divisions are not of equal length, so that $x_0 = 1$, $x_1 = p$, $x_2 = p^2$, ..., $x_k = p^k$ and $x_n = a$, where $p^n = a$ and where p > 1.

Let A_n be the area of the n^{th} trapezium, as shown in the diagram.
Let S_n be the sum of the areas of the first n trapezia.

(a) Using the trapezoidal rule, find S_1, the area of the first trapezium (in terms of p). 2

(b) Given $A_1 = S_1$, show that

(i) $S_2 = S_1 + \frac{1}{2}p^3(p-1)(1+p^2)$ and *hence* 2

(ii) $S_3 = \frac{1}{2}(p-1)(1+p^2)(1+p^3+p^6)$ 2

(c) Find an expression for S_n and *hence* show that 3

$S_n = \frac{1}{2}(1+p^2)\left(\frac{p^{3n}-1}{p^2+p+1}\right)$, when simplified.

(d) Show that $p \to 1$ as $n \to \infty$. 1

Hence, evaluate I, using $I = \lim_{p \to 1} S_n$ 2

Question 1

(a) 3.14

(b) $-\frac{\sqrt{3}}{2}$

(c) $-e^{-x} + \frac{1}{2\sqrt{x}}$

(d) $x = 5$

(e) $-3\cos x + C$

(f) $x < -2$ or $x > 4$

(g) $\log_a 21a = \log_a 3 + \log_a 7 + \log_a a$

$\log_a 21a = 1.6 + 2.4 + 1$

$= 5$

Question 2

(a) $y = \ln(x+2)$

$\frac{dy}{dx} = \frac{1}{x+2}$

when $x = 0$, $\frac{dy}{dx} = \frac{1}{2}$

$\therefore m_{normal} = -2$

let the equation of the normal be $y - y_1 = m(x - x_1)$

where $x_1 = 0$, $y_1 = \ln 2$, $m = -2$

$\therefore 2x + y - \ln 2 = 0$

(b) (i) $5x^2 \sec^2 5x + 2x \tan 5x$

(ii) $\frac{1}{(1-3x)^2}$

(iii) $3\sin^2 x \cos x$

(c) $l = r\theta$

$= 10(\frac{42\pi}{180})$

$= 7.3cm$

(d) $\{x : x \geq 1\}$

$\{y : y \geq 3\}$

Question 3

(a) (i) $AB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$
$= \sqrt{6^2 + 8^2}$
$= 10 units$

(ii) $d = \left| \dfrac{ax_1 + by_1 + c}{\sqrt{a^2 + b^2}} \right|$
$= \dfrac{6+8+7}{5}$
$= \dfrac{21}{5} units$

(iii) $m_2 = -\dfrac{a}{b} = \dfrac{3}{4}$
$m_{x\text{-}axis} = 0$
$\tan\theta = \left| \dfrac{m_1 - m_2}{1 + m_1 m_2} \right|$
$= \dfrac{3}{4}$
$\therefore \theta \approx 37^0$

(iv) $m_{BC} = \dfrac{7 - -2}{5-2} = 3$
$m_{AD} = m_{BC}$
$\therefore m_{AD} = 3$
let the equation of AD be $y - y_1 = m(x - x_1)$
where $x_1 = -3$, $y_1 = 1$ and $m = 3$
$\therefore y - 1 = 3(x+3)$
$\therefore y = 3x + 10$

(v) now D lies on $y = 3x + 10$ and $y = -2$
$\therefore D(-4, -2)$

(vi)

(b) $SB^2 = PS^2 + PB^2 - 2(PS)(PB)\cos\angle SPB$
$SB^2 = 56^2 + 48^2 - 2(56)(48)\cos 50°$
$\therefore SB = 44.54$ nautical miles

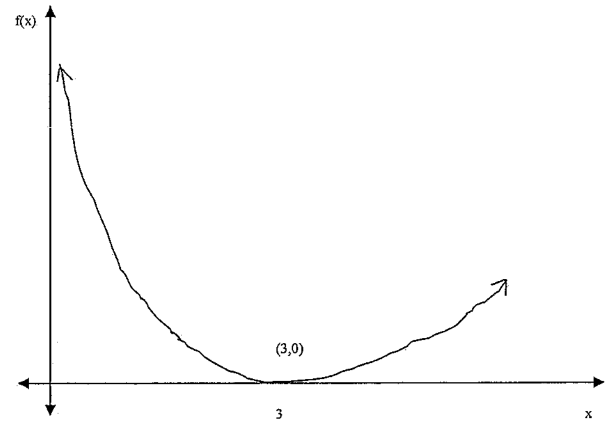
Question 4

(a) (i) $\int \dfrac{3x^3 - 1}{x} dx = \int (3x^2 - \dfrac{1}{x}) dx$
$= x^3 - \ln x + C$

(ii) $\int_0^{\frac{1}{2}} \cos(\pi x) dx = \left[\dfrac{1}{\pi} \sin(\pi x) \right]_0^{\frac{1}{2}}$
$= \dfrac{1}{\pi}$

(b) $\cos 2x = \dfrac{1}{\sqrt{2}}$
$\therefore 2x = \dfrac{\pi}{4} or \dfrac{7\pi}{4}$
$\therefore x = \dfrac{\pi}{8} or \dfrac{7\pi}{8}$

(c)



(d) (i) $R = 65 + 4t^{\frac{1}{3}}$
when $t = 0$, $R = 65 + 4(0)^{\frac{1}{3}} = 65$

(ii) now $R = \dfrac{dv}{dt} = 65 + 4t^{\frac{1}{3}}$
$\therefore V = 65t + 3t^{\frac{4}{3}} + C$
when $t = 0$, V = 15, $\therefore C = 15$
$\therefore V = 65t + 3t^{\frac{4}{3}} + 15$
when $t = 0$, V = 583 litres

Question 5

(a) (i) $\alpha + \frac{1}{\alpha} = 5$

(ii) $\alpha + \beta = -\frac{b}{a} = 5$

(iii) $\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta$
$= (5)^2 - 2(1)$
$= 23$

(b) (i) $\Delta = b^2 - 4ac = 4 - 4(3)(k)$
$= 4 - 12k$

(ii) for real roots $\Delta \geq 0$
$\therefore 4 - 12k \geq 0$
$\therefore k \leq \frac{1}{3}$

(c) $\angle ABQ = \angle ACS = \theta°$ (corresponding angles on $PQ \| RS$ are equal)
now $\angle CNM = \angle NMC$ (equal angles opposite equal sides in isosceles triangle PRM)
$\therefore \theta° + \angle CNM + \angle NMC = 180°$ (angle sum triangle CNM is 180°)
$\therefore 2 \times \angle NMC = 180° - \theta°$ ($\angle NMC = \angle CNM$)
$\therefore \angle NMC = \frac{180° - \theta°}{2}$
$\angle NMS + \angle NMC = 180°$ (adjacent angles on a straight line are supplementary)
$\therefore \angle NMS = 180° - \frac{180° - \theta°}{2}$
$\therefore \angle NMS = \frac{180° + \theta°}{2}$

(d) (i) vertex: (h,k)
(3,-2)

(ii) directrix: $y = -a + k$
$\therefore y = -3$

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Question 6

(a) (i) $x^2 - 3x - 18 = 0$
$(x-6)(x+3) = 0$
$\therefore x = -3$ or $x = 6$

(ii) $(x^2+1)^2 - 3(x^2+1) - 18 = 0$
let $U = x^2 + 1$
$\therefore U^2 - 3U - 18 = 0$
$(U-6)(U+3) = 0$
$\therefore U = -3$ or $U = 6$

$\therefore x^2 + 1 = -3$
$x^2 = -4$
no real solution

$\therefore x^2 + 1 = 6$
$x^2 = 5$
$x = \pm\sqrt{5}$

$\therefore x = \pm\sqrt{5}$

(b) (i) $y = (x-1)^2$ _(1)
$x + y = 3$___(2)
$\therefore x = -1 or 2$
$\therefore y = 1 or 4$
the curves intersect at (2,1) and (-1,4)

(ii) Area $= \int_2^3 [(x-1)^2 - (3-x)]dx$
$= \int_2^3 (x^2 - x - 2)dx$
$= \frac{11}{6}$ units^2

(c) $\frac{dy}{dx} = e^{1-x}$
$y = \int e^{1-x} dx$
$\therefore y = -e^{1-x} + C$
when x=1, y=3
$\therefore 3 = -1 + C$
$\therefore C = 4$
$\therefore y = -e^{1-x} + 4$

(d) (i) $V = 85e^{-0.07t}$
$\frac{dV}{dt} = 85 \times -0.07 \times e^{-0.07t}$
$= -0.07 \times 85e^{-0.07t}$
$= -kV$

(ii) when t = 5, $\frac{dV}{dt} = -0.07 \times 85e^{-0.07 \times 5}$
$= -4.19$ cm/s^2

http://danielslake.virtualave.net/jrexams.html

Question 7

(a)
$$V = \pi\int_a^b x^2\,dy$$

$$V = \pi\int_0^{16} (16-y)^{\frac{1}{2}}\,dy$$

$$V = -\pi\int_0^{16} -(16-y)^{\frac{1}{2}}\,dy$$

$$V = -\pi\left[\frac{2(16-y)^{\frac{3}{2}}}{3}\right]_0^{16}$$

$$V = \frac{128\pi}{3}\text{ units}^3$$

(b) -

(c) (i) $(n-2)\times 180°$

(ii) $S_n = \frac{n}{2}(2a+(n-1)d)$

$= \frac{n}{2}(240° + 5°(n-1))$

$= \frac{n}{2}(235°) + \frac{5°n^2}{2} = (n-2)\times 180°$

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$\therefore n = 6 or 19$

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Question 8

(a) (i) SAS

(ii) $\angle DCQ + \angle QCP + \angle PCB = 90°$ (interior angle of a square is a right angle)

$\therefore \angle DCQ + \angle PCB = 45°$

now $\angle DCQ = \angle BCE$ (corresponding angles in $\Delta CBE \equiv \Delta CDQ$)

$\therefore \angle BCE + \angle PCB = 45°$

$\therefore \angle QCP = \angle PCE = 45°$

$\therefore$ PC bisects $\angle QCE$

(iii)-

(b) (i) when $t = 0$, $v = 2(2-0)e^0$

$= 4m/s$

(ii) particle is at rest when v = 0

$\therefore 2(2-t)e^{-\frac{t}{2}} = 0$

$\therefore t = 0$

when t = 2, $x = 4(2)e^{-1}$

$= \frac{8}{e}m$

the particle will be at rest when t = 2, and at $x = \frac{8}{e}m$

(iii)–

(iv)particle accelerates when $\frac{d^2x}{dt^2} > 0$

ie when t > 4

Question 9

(a) $\frac{d}{d\theta}(\frac{1}{\cos\theta}) = \frac{(\cos\theta)(0)-(1)(-\sin\theta)}{\cos^2\theta}$

$= \sec\theta\tan\theta$

(b) (i) $BP^2 = AB^2 + AP^2$ (by Pythagoras)

$BP^2 = 5^2 + x^2$

$\therefore BP = \sqrt{25+x^2}$ (BP > 0)

(ii) $AE = AP + PE$

$PE = AE - AP$

$PE = 3 - x$

now $PQ^2 = PE^2 + EQ^2$ (by Pythagoras)

$= (3-x)^2 + 4^2$

$= 25 - 6x + x^2$

$\therefore PQ = \sqrt{25-6x+x^2}$ (PQ > 0)

(iii)total cabling = BP + PQ

$L = (\sqrt{25+x^2} + \sqrt{25-6x+x^2})$

(iv) $\frac{dL}{dx} = \frac{1}{2}(25+x^2)^{-\frac{1}{2}} \times (2x) + \frac{1}{2}(25-6x+x^2)^{-\frac{1}{2}} \times (2x-6)$

$= \frac{x}{\sqrt{25+x^2}} + \frac{x-3}{\sqrt{25-6x+x^2}} = 0$ (for stationary points)

$\therefore x = \frac{5}{3} or 15$

now $0 \le x \le 3$

$\therefore x = \frac{5}{3}$

Test

x	1	$\frac{5}{3}$	2
$\frac{dL}{dx}$	-0.25	0	0.129
	\	*MIN*	/

Since the function is continuous in the domain $0 \le x \le 3$, $x = \frac{5}{3}$ is a local minimum and there is only one turning point in the domain, $x = \frac{5}{3}$ is also the absolute minimum

$\therefore AP = \frac{5}{3}$ metres

Question 10

(a) $\int_1^p x^2 dx \approx \frac{p-1}{2}(1+p^2)$

$= \frac{p-1}{2} + \frac{p^2(p-1)}{2}$

$= \frac{p-1}{2}(p^2+1)$

(b)

(i) $S_2 = S_1 + A_2$

$= S_1 + \frac{p^2-p}{2}(p^2+p^4)$

$= S_1 + \frac{p^4+p^6-p^3-p^5}{2}$

$= S_1 + \frac{p^3}{2}(p^3-p^2+p-1)$

$= S_1 + \frac{1}{2}p^3(p-1)(1+p^2)$

(ii) $S_3 = S_2 + A_3$

$= \frac{(p^2+1)(p-1)}{2} + \frac{p^3(p-1)(1+p^2)}{2} + \frac{p^3-p^2}{2}(p^4+p^6)$

$= \frac{(p^2+1)(p-1)}{2}[1+p^3+p^6]$

(c) $S_n = \frac{1}{2}(p-1)(1+p^2)[1+p^3+p^6+...+p^{3(n-1)}]$

$= \frac{1}{2}(p-1)(1+p^2) \times \frac{[1\times(p^3)^n - 1]}{p^3-1}$

$= \frac{1}{2}(1+p^2)\left[\frac{p^{3n}-1}{p^2+p+1}\right]$

(d) –

(e) -